

Analysis of fluid-solid mixture models of biofilm spread

A. Carpio (UCM), G. Duro (UAM)

March 26, 2024

We investigate multiphase systems of conservation laws describing bacterial biofilm spread on surfaces [1, 5, 6]. These systems are set in moving domains, which brings in numerous challenges. A quasistationary approximation allows us to remove time dependence from the transport and concentration submodels [3, 4]. We stabilize the quasistationary displacement-pressure submodel by deriving an additional equation for the velocity. This closure involves an additional parabolic problem set in a moving domain. We establish an existence, uniqueness and stability theory for parabolic problems with moving domains whose boundaries move according to specific vector fields [3, 2]. Fixed point arguments allow us to construct a solution to the full system of coupled submodels. These results assume that the motion of the boundary of the biofilm is prescribed. Addressing the free boundary problem, which couples this system to an additional integrodifferential equation for the motion of the biofilm boundary, remains a challenge.

References

- [1] B. Birnir, A. Carpio, G. Duro, Driving biofilms to finite time extinction by antibiotic cocktails, *Communications in Nonlinear Science and Numerical Simulation*, 152, 109362, 2026
- [2] A. Carpio, G. Duro, On the solution of boundary value problems set in domains with moving boundaries, *Mathematical Methods in Applied Sciences*, 48(10), 10427-10441, 2025
- [3] A. Carpio, G. Duro, Well posedness of fluid-solid mixture models for biofilm spread, *Applied Mathematical Modelling* 124, 61-85, 2023
- [4] A. Carpio, G. Duro, Analysis of a two phase flow model of biofilm spread, *Nonlinear Analysis* 224, 113538, 2024

- [5] A. Carpio, E. Cebrián, Incorporating cellular stochasticity in solid-fluid mixture biofilm models, *Entropy* 22(2), 188, 2020
- [6] A. Carpio, E. Cebrián, P. Vidal, Biofilms as poroelastic materials, *International Journal of Non-linear Mechanics* 109, 1-8, 2019