

## Hyperbolic problems set in moving domains

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In contrast with the huge amount of results available for the theoretical and numerical study of initial value problems for partial differential equations set in spatial regions whose boundaries do not change with time [5], little is known when boundaries move. The few available results are typically constrained to parabolic problems [2]. We consider here hyperbolic problems, such as wave and transport equations. First, we will show how to construct solutions of initial value problems for linear wave equations in domains whose boundaries deform with time following a smooth vector field [2, 4]. Next, we will address more complex nonlinear transport problems arising a cellular studies which can be solved numerically by physically inspired changes of variables to produce travelling wave solutions [1, 3, 6, 7]. Finally, we will show how to handle different biologically inspired boundary dynamics by means of characteristic curves and discuss the limitations of the presented approaches.

### References

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