

## Object-Based Full Waveform Inversion with Quantified Uncertainty in Stratified Media

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We consider the problem of determining inclusions in layered media from noisy measurements of wave fields on the surface. For inclusions governed by a small number of parameters, we define the posterior probability of the parameter set by means of the finite dimensional Bayes's formula [1, 2]. We determine the maximum a posteriori approximation (MAP) of the inclusions by optimizing constrained functionals involving the measurements, the available a priori information and parameter dependent solutions of wave equation constraints with changing discontinuities which we approximate with adaptive finite element meshes [1, 3, 4]. A Laplace approximation of the posterior probability about the MAP point provides basic uncertainty estimates. Markov Chain Monte Carlo (MCMC) studies allow us to quantify uncertainty in the distribution of inclusion parameters in more detail, including multimodality and asymmetry effects. However, the higher computational cost of this approach forces the use of coarse solvers defined in prefixed meshes. While we find reasonable agreement between the MAP estimates obtained in both ways, MCMC sampling unveils multimodal distributions affected by the prior knowledge and the noise to signal ratio. Educated prior information obtained by topological methods may improve the results [5].

### References

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