Translating Theoretical Mathematics into Biomedical Solutions

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Abstract: Understanding how the cells forming tissues interact at different levels is essential to fight diseases derived from alterations in tissue activity. This is a challenging goal to which the development of adequate mathematical models and methods can contribute. At the cellular scale, cell metabolism and interactions can be represented by models based on agents, such as cellular automata, active vertex-Voronoi, immerse boundary or cellular Potts models [1, 2, 3]. These descriptions are often hybrid, that is, they are coupled to continuous equations for the dynamics of concentrations, signals, flows and stresses. The high computational cost of solving numerically the resulting systems confers particular relevance to the analysis of reduced models and continuum limits, whose conclusions provide qualitative insight on the observed behaviors, helping us to guide experiments and simulations, and, eventually, propose pharmacological therapies. Popular approximations lead to kinetic equations or multiphasic conservation laws for densities of different cells and biological agents, coupled to reaction-diffusion systems and equations for flows and stresses. These systems are often set in moving domains that vary with time, which raises a number of issues regarding well posedness and proper discretization [4, 5, 6, 7]. We will focus on recent results regarding epithelial monolayers, bone tissue and bacterial biofilms. After discussing agent based approaches, we will consider continuum limits admitting travelling wave and selfsimilar solutions of particular relevance. We will illustrate the methods analyzing the invasion of healthy tissue by cancerous cells [3], osteoporosis progression in bone tissue and the eradication of persistent biofilms by antibiotic cocktails [8]. Under adequate hypotheses, we formulate reduced models for the effect of drugs on these tissues, that are well posed, and for which we introduce control strategies to obtain a desired tissue response to medicine [8].

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