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Modeling edema dynamics in soft tissue using mixture theory

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Tissues are formed by a mixture of cells, liquid in which chemical substances are dissolved, and an extracellular matrix made of polymer threads that keep the cells bound together. When tissues are damaged due to mechanical or chemical causes, they react by sending a fluid flow to the part involved. The uncontrolled buildup of fluid in the involved tissue leads to edema, which, if it is not drained, affects tissue function and consequently human health. Tissues can be represented as a mixture [3, 4] with three fluid components [2] consisting of a biomass phase (cells and polymeric matrix), which is split into two components - alive and dead cells, with volumetric fractions $\phi_a(x,t)$ and $\phi_d(x,t)$ respectively, and a fluid component, $\phi_f(x,t)$, that carries nutrients, waste, and toxicants. The mixture [1] is governed by the mass and linear momentum balance laws coupled with advection-reaction-diffusion equations for the concentrations of nutrients $c_n(x,t)$ and toxicants $c_p(x,t)$. In our paper we develop and simulate a model, by using a the open acces freeFEM platform [5] that solves systems of partial differential equations through finite element methods. In particular, the effects of toxicants on tissue, the resulting fluid infiltration, and its drainage by medicine treatment are investigated.

References

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