Symplectic Dynamics
at the Institute for Advanced Study

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Board of Trustees of the Institute for Advanced Study Meeting
October 2011

Origin of Word Symplectic

- Introduced by former **IAS Professor Hermann Weyl**.

- It derives from Greek word meaning **complex**.

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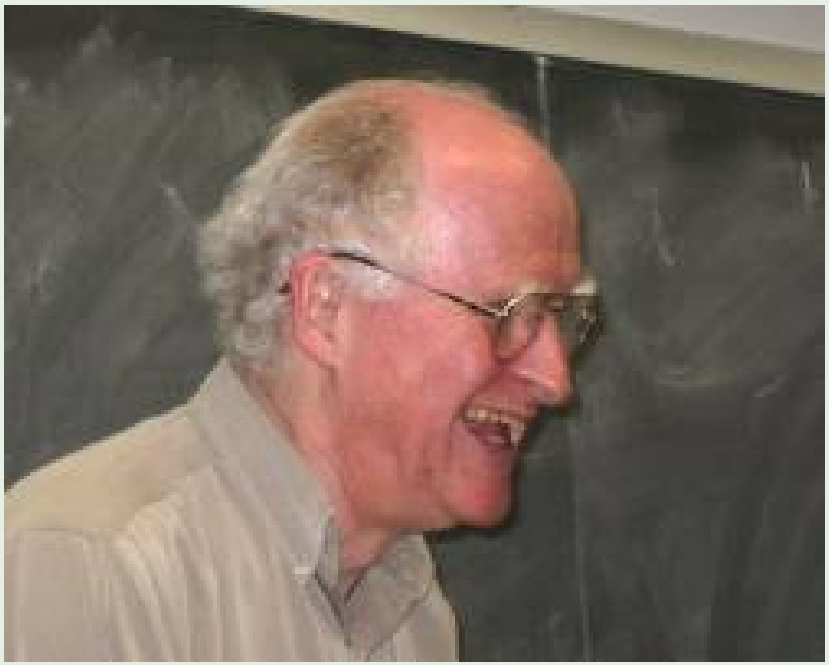
1. **Dynamical Systems**: studies motions of **planets, space shuttles, molecules, cars, water waves** ...

2. **Symplectic Geometry**: studies a sophisticated notion of area in 4 dimensions, which can be even negative!

3. Symplectic Dynamics Program Organizers (2011-2012)

Helmut Hofer (IAS) and John Mather (Princeton)



4. 100 Years after Poincaré: Rationale Behind Special Year

The modern fields of

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evolved from Poincaré's (1854-1912) work in celestial mechanics as

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Developments After Poincaré

- Fields developed independently.
- **Rationale behind Special Year at IAS:**
  recover common core!

5. Symplectic Geometry: Bigger Picture

Where else is Symplectic Geometry?

- **Applied**: physics, chemistry, engineering eg:
  - Molecular spectroscopy
  - Fluids
  - Plasma physics
  - Elasticity theory
  - Mirror symmetry
  - Robotics ...
- **Pure**: connected at a **core** level with major subjects:
  - Representation theory
  - Complex algebraic geometry
  - Fourier theory
  - Microlocal analysis ...

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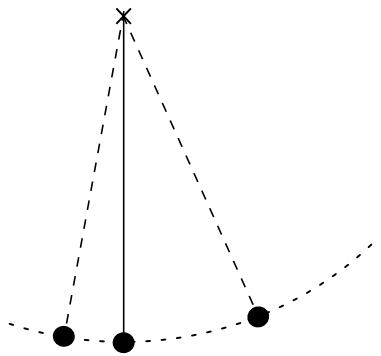

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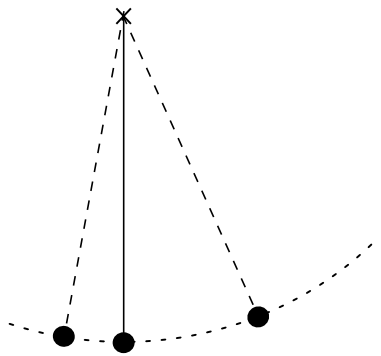

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Goal of Dynamical Systems Theory

To understand **qualitative properties** of orbits:

- **shape**: are the orbits circles, straight lines, parabolas ...?
- **long term behavior**: do the orbits escape?

7. An Application of Symplectic Dynamics

Symplectic Dynamics can be used to **find optimal orbits** which minimize fuel consumption for space missions

This represents a huge improvement of the current methods, which find orbits by trial and error!

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7. Belbruno's idea: ignite **MUSES-A** at the right time and place to send it to Lagrange point (gravitational fields of earth and sun cancel out). There was enough fuel for this.
8. Belbruno's methods can be greatly improved using Symplectic Dynamics developed by Hofer and collaborators (SFT).

8. A Famous Type of Dynamical Systems: Integrable

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An important example is the **Shallow Water Wave**
It took 100 years to know it is integrable!

9. My work at IAS since January 2011

Solution to Isospectral Problem for Toric Systems

Paper: Isospectrality for quantum toric integrable systems
authors: Charles, Pelayo, and Vũ Ngọc

Goes back to work in the 1970s by:

- Colin de Verdiere (Institut Fourier)
- Guillemin (MIT) and Sternberg (Harvard)

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10. Explanation of the Isospectral Conjecture

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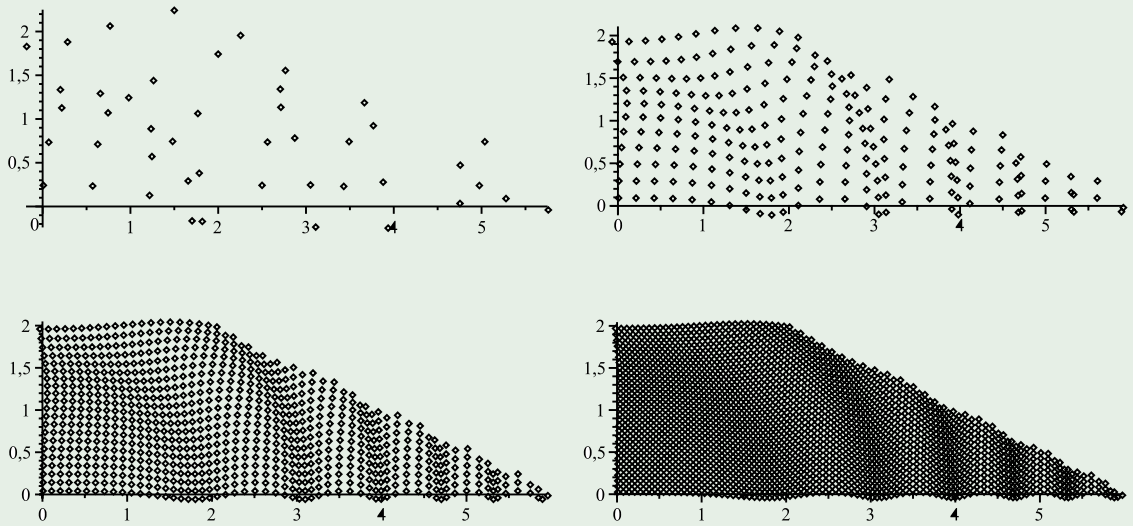

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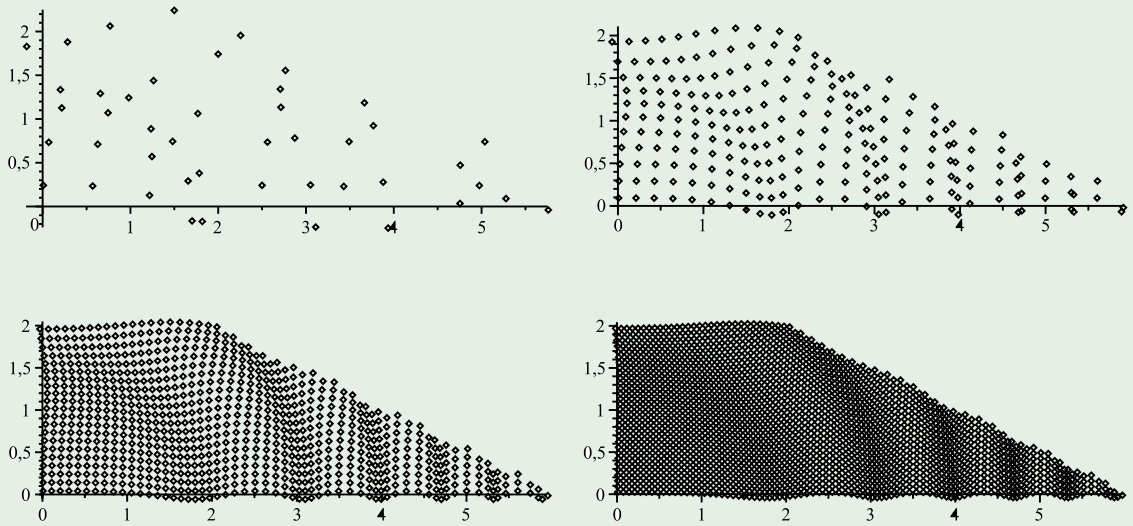

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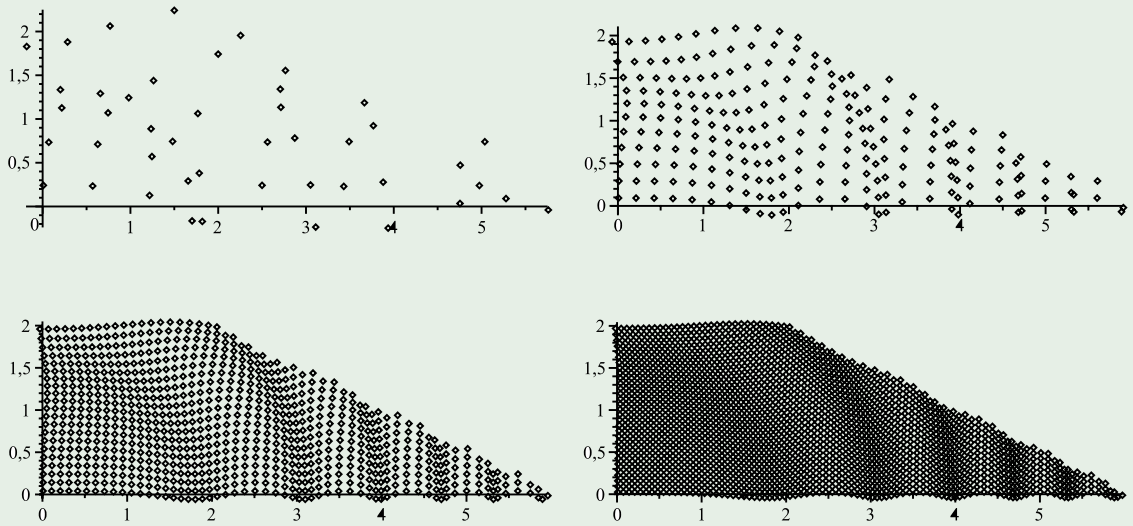

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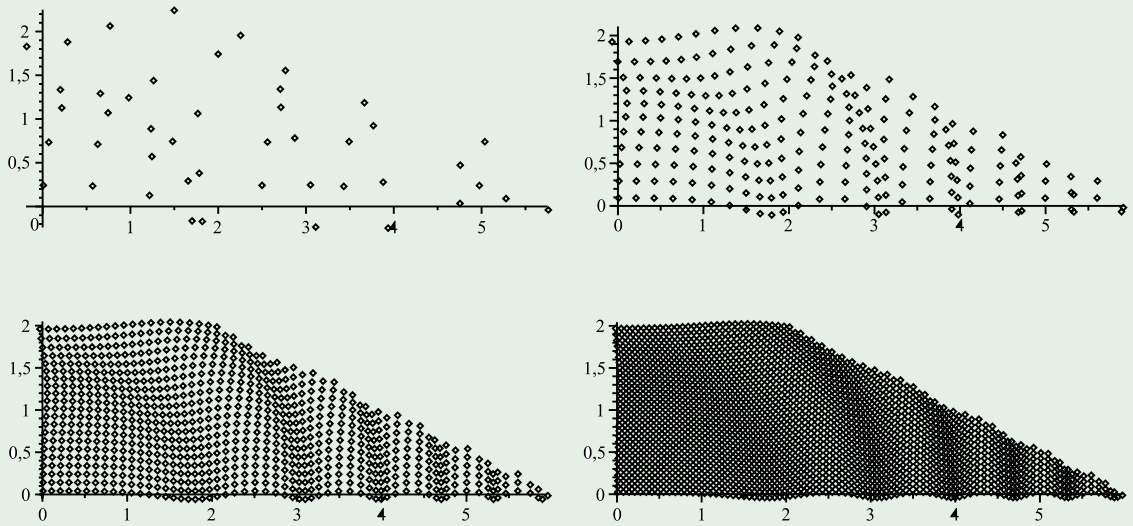

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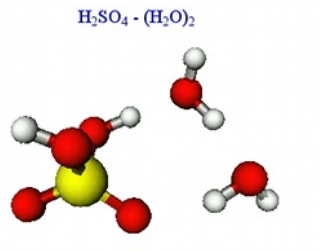
ISOSPECTRAL THEOREM (Charles, Pelayo, Vũ Ngọc)

If the system is integrable and periodic, then you can.

12. ANOTHER Application of Isospectral Theorem



Question in Quantum Molecular Spectroscopy

Can you hear:

1. **the shape of molecules?**
2. **the orbits of molecules in 3D space?**

Question arose from works by chemists:

- *Mark Child's group in Oxford (UK)*
- *Jonathan Tennyson's at University College London (UK)*
- *Frank De Lucia's at Ohio State University (USA)*
- *Boris Zhilinskii's in Dunkerque (France)*
- *Marc Joyeux's in Grenoble (France)*
- ....