ABOUT HARTSHORNE CONJECTURE ON COMPLETE INTERSECTIONS

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In the 70's, Hartshorne stated his famous conjecture that a smooth n-dimensional projective variety of $\mathbb{P}^N$ must be a complete intersection if $n > 2/3N$.

In this talk, we will show how a conjecture of this type could be solved if we were able to understand how to extend vector bundles from a subvariety to an ambient space. In fact, the conjecture can be divided into two different natural problems:

1) When is it possible to extend the normal bundle of a subvariety X of an ambient variety Y? In other words, when is it possible to express X as the zero locus of a section of a vector bundle over Y whose rank is the codimension of X in Y? When the codimension is one, the answer is always positive, while in codimension two the so-called Hartshorne-Serre correspondence essentially asserts that the answer is positive if and only if the determinant of the normal bundle extends to Y. We will discuss the kind of results one could expect for this problem.

2) When does a vector bundle of rank r over $\mathbb{P}^N$ splits as a direct sum of line bundles? We will show that on could expect this to be true when $N > 2r$, using that, in this range, one could expect the vector bundle to extend to $\mathbb{P}^{N+1}$.

Putting together both speculations, we will conclude that one could expect Hartshorne's conjecture to be true when $n > 3/4(N-1)$.