



Capi Corrales Rodríguez

is a member of the Mathematics
Department at Universidad
Complutense de Madrid.

A Walk For Mathematicians Through a Very Special Triangle: Prado, Reina Sofía, Thyssen-Bornemisza

Space and time. Terms used in philosophy to describe the structure of nature. They are sometimes described as containers in which all natural events and processes occur, and sometimes, as relations which connect such events.

(Collier's Encyclopedia.)

The second sentence in this definition tells us that Space and Time are at times thought of as *containers*, and at times thought of as *relations*. Curiously, these two words, *containers* and *relations* describe, respectively, the idea of space held during the seventeenth century, when the notion of space was explicitly mentioned for the first time in mathematics (Newton in [Ne], 1687), and the idea held in contemporary mathematics (Hausdorff in [Ha], 1914).

The first mathematical notion of space – the space as a cubic container of the 17th century – reflected how Physical Space, the space where physical events take place, was thought of then. In those times, mathematicians associated Space and the Physical Universe, and they considered that the model which describes Space as an immense and homogeneous shoebox is an exact reproduction of the universe around us. Nowadays, mathematicians describe a space (notice that it is no more “Space” but “a space”) as an abstract entity consisting of two things: any set of objects, and a list of relations among these objects. The evolution that takes us from space thought of as a container, to space defined as a web of relations – a net of relations –, can be traced along the centuries in the paintings that were made while the mathematics were developed. In these pages we invite you to jump from the box into the net as you walk with us amongst three of the painting museums of Madrid: *Prado*, *Reina Sofía* and *Thyssen-Bornemisza*. We will start by identifying the position of our

museums – which from now on we will respectively call, as Spaniards do, *el Prado*, *el Reina* and *el Thyssen* – in a map of the city. It is not difficult to find one's way on a map of Madrid, since it has a vertical axis that, under the successive names of Paseo de la Castellana, Paseo de Recoletos and Paseo del Prado, runs down the city from North to South, and a horizontal one which, starting at Parque del Oeste and ending at Plaza de las Ventas, the bullfighting arena, takes us from West to East. Paseo de la Castellana starts at the Northern entrance to the city, marked by the square known as Plaza de Castilla, easily identified by the two black *Puerta de Europa* towers by the architects Philip Johnson and John Burgee. Paseo del Prado ends at Plaza del Emperador Carlos V. This spacious and open square, with the Atocha railway station in its South-East corner, stands in the middle of the Scientific Axis conceived in the 17th century by king Carlos III and the architects Sabatini and Villanueva. The Scientific Axis developed around four projects: an Astronomical Observatory – built in the South-West end of Retiro Park –, the Botanical Gardens, a Science Museum – now *el Prado* –, and a Medical Center, whose main Hospital Building eventually became *el Reina*. The third of our museums, *el Thyssen*, stands practically across the street from *el Prado*, a little further up North.

We start our stroll in *el Thyssen*, the only one of our three museums that holds a collection covering, time wise, the evolution of western painting from the 14th century up to the end of the 20th. (*el Prado* goes up to the middle of the 19th century, and *el Reina* from then on). The building itself, one of the oldest in the city, reflects the passage of time. The original palace was built in the 16th century, and one hundred years later its Italian owner Pico de la Mirándola modified it in the Italian style of the period, covering the facade with

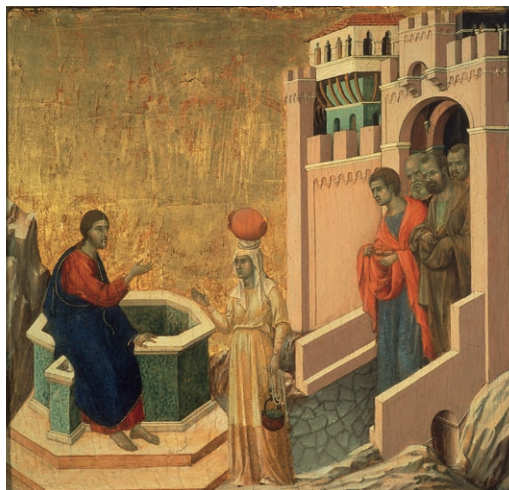
all kind of decorations. In those days Spaniards had more sober taste than Italians – we just have to observe some of the buildings constructed around that period by Spanish architects, such as *la Puerta de Alcalá*, the Astronomical Observatory, the Exhibitions Pavilion in the Botanical Gardens, *el Prado* or *el Reina*. As soon as a Spanish family bought the palace in the 18th century, the decorations were removed from the facade, leaving it as we see it now. In 1990, once it was decided to turn it into a painting museum for the Thyssen Collection, the architect Rafael Moneo not only restored the palace, but he also turned it into an extraordinary building from a museological point of view. This museum is now required reference among architects when it comes to implementing technology to control the environment and conditions of paintings.

After visiting the permanent collection, we focus on a painting from the first room of the museum, and another one from the newest wing of the building, opened in 1994: *Christ and the woman of Samaria*, executed by Duccio di Buoninsegna in 1311, and *New York with moon*, painted by Georgia O'Keeffe in 1925.

In both canvases, the buildings play a central role; in both canvases, the buildings seem to be painted cardboard cutouts; in both canvases, what is far away appears smaller, and what is close by, bigger. Despite these similarities, we notice how differently the buildings are represented!

Duccio di Buoninsegna (1255-13-19) is an external and unrelated observer who describes the city from a distance. Georgia O'Keeffe (1887 – 1986), on the other hand, not only places herself on the streets, but also compares her size with that of the buildings around her, thus establishing a relation between herself – and consequently us, when we look at the painting –, and them. Quite a different point of view! Six hundred years separate the external observer of the XIV century and the involved or implicated observer of the XX. Halfway between them stands Velázquez, and exactly halfway between Velázquez and O'Keeffe, we find Goya. In search of their paintings, we leave *el Thyssen*, walk around the fountain of Neptune and cross the street to approach *el Prado's* North entrance.

El Prado is a neoclassical construction, built in the 18th century imitating the style of classical Greece and Rome. In the original plans of the Scientific Axis, the building appears as a Natural Sciences Museum, which explains why the rooms in the lower ground are so dark and have so few windows: they were conceived as laboratories. The War of Independence against Napoleon delayed construction of the Museum. The works resumed after the war, and Carlos III's daughter in law



Duccio di Buoninsegna
Christ and the Samaritan Woman (1310–1311)
With kind permission of Museo Thyssen-Bornemisza, Madrid.



O'Keeffe, Georgia
New York with Moon (1925)
With kind permission of Colección Carmen Thyssen-Bornemisza.

decided to use the building as a painting museum, donating part of the royal family's collection for the project. *El Prado* opened as such in 1819. We promenade leisurely along its rooms until we reach the area dedicated to Velázquez.

Velázquez (1599 – 1660) worked in the XVII century, a time in which painters aimed to represent precisely what the eye saw. Canvases were constructed in such a way that, when looking at them, one could see exactly what the painter was seeing when painting it. Keeping this idea in mind, we look at *Las meninas*. The mastery of the standard rules and tricks with which to construct Euclidean Space is complete. It is impossible to stand in front of this painting without asking oneself whose eyes are describing the scene. Answer: the eyes of the king and queen reflected in the mirror at the back, a mirror that, masterfully, also brings into the painting the space not in the painting, the space where we, the audience, are. But



Velázquez, Diego **Las meninas (1656)**
With kind permission of Museo Nacional del Prado.



Goya, Francisco **The Shootings of May 3rd, 1808 (1814)**
With kind permission of the Museo Nacional del Prado.

then, how could Velázquez be placed in the middle of the scene, if he was painting exactly what he was seeing?

Answer: he placed a huge mirror in the back, behind the king and queen, and he painted what he saw reflected in it. Was Velázquez left-handed then, since he holds a brush in his right hand? No, he was not. How did he do it, then? Here is a little puzzle for the viewers to solve.

Velázquez remains as an external observer, although with his mirrors and great mastery it may seem otherwise at first. His description follows the point of view of the king and the queen and they, as the audience, remain outside the room. The relation that Velázquez establishes between himself and the painting by placing himself in the middle of the scene is just an astute trick. It does not change the fact that the observer whose point of view is described remains outside the room. Who performed the jump? Who placed himself directly on the bodies, opening the way for O'Keefe and her contemporaries? Answer: Francisco de Goya (1746 – 1828), contemporary to Carl Friedrich Gauss (1777 – 1855), the mathematician who first considered surfaces as bodies by themselves, and not just as boundaries of bodies.

Between 1812 and 1816, Gauss, then director of the Astronomical Observatory of Göttingen, carried out a topographical study of the land covered by the kingdom of Hannover. Until then, the shape of the Earth had been deduced by studying the movement of the celestial bodies. Reflecting on the data he was obtaining, Gauss realized and proved that, in fact, it is sufficient to take geodesic measures on the surface of the Earth in order to deduce its exact shape. Such measurements suffice to compute the curvature of the Earth at a specific place, and, in fact, such a strategy can be followed on any surface. In 1827 Gauss published his ideas and results on this subject in *Disquisitiones generales circa superficies curvas*, a text that stands as the origin of modern differential geometry. In it, Gauss initiated the study of surfaces as bodies in themselves, that is, from an intrinsic point of view,



Goya, Francisco **La Romería de San Isidro (1820 – 1823) (fragment)**
With kind permission of the Museo Nacional del Prado.

proving that in order to describe a surface with precision it is not necessary to immerse it in any external space or environment.

Reflecting on these issues, we reach *The Shootings of May 3, 1808* and *The Black Paintings*, all of them canvases in which Goya describes the war taking place around him.

The scene in *The shootings* is startling. The soldiers, with their backs to the audience, stand as dark flat surfaces on one side, while the victims, led by the man dressed in white, play the main role.

"The subject is as current nowadays as it was in 1814 when Goya painted *The shootings of May 3rd, 1808*. Many of the 20th century painters turned their gaze back to Goya when they depicted the horror of war all over the world. One of them was Picasso, who used the same subject to describe the Spanish Civil War. Even the disturbances in Northern Ireland, the students uprising in Beijing in 1989 and the war in the former Yugoslavia have been described keeping the image of Goya's painting in mind." (Translated by Swedish mathematician Göran Bergqvist.)

Linköping TV-guide, Sweden, May 3, 2005.

In his black paintings Goya goes a step further and, besides focusing on the victims, places himself directly on their faces and bodies, the only reference in the powerful description he gives of the brutal consequences of war. Unlike in *The shootings*, in these paintings we see no weapons or soldiers, no bloody corpses or special effects. Like his contemporary Gauss, Goya does not need to refer to the surrounding space in order to describe, clearly and precisely, the tragedy and desperation of the war victims.

Goya and Gauss taught us to place ourselves directly on the bodies we want to describe. Once we have them in front of us, once we get on the different bodies, it is only natural that we start establishing relations. First, we establish relations only among these bodies, later, also among them and ourselves, as O'Keefe does in her painting. Establishing relations between the different shapes

and bodies is precisely the type of work that we will find in the paintings of Juan Gris and Pablo Picasso, and looking for their work we leave *el Prado* and approach *el Reina*, on the southwest corner of Plaza del Emperador Carlos V.

We all like to improve the neighborhood in which we live, and that is precisely what king Carlos III tried to do with his Scientific Axis. He was then living in the park *El Retiro*, and as part of his project to improve his neighborhood he thought of building the largest medical center of his kingdom in it. The architect Sabatini designed the complex and the main hospital, which was to be immense and have seven patios. Only one of them had been built when the king died and construction was stopped. It has remained thus, functioning as a hospital for two hundred years until it closed when the large medical complex *Ciudad Sanitaria Provincial* opened in the city. In 1988, after a subtle renovation, the building reopened as Centro de Arte Reina Sofía. On entering the museum we look for the room dedicated to the work of Juan Gris (1887 – 1927), a painter born in Madrid.

"Juan Gris called this way of painting 'poetic' and in his exhibits Gris would stand by the paintings and would decorate them with 'rhymes and metaphors', as he called them, pointing out to the astonished observer similarities that the observer had never noticed before. 'Don't you think the mouth of this jar looks like a pear, the one lying by it; the pentagram like guitar strings; the cup like that Ace of Hearts? Things are linked by relations'."

D. H. Kahnweiler, "Der Weg zum Kubismus"
Editions Gallimard, 1963.

We place our selves in front of his three paintings *The open window*, *Guitar in front of the sea* and *Guitar with inlaid work*.

The first two ones are very similar: in each of them we see a table with a few things on it and, behind, a window open onto the sea with a chain



Gris, Juan **The open window** (1921)
With kind permission of Archivo Fotográfico Museo Nacional Centro de Arte Reina Sofía, Madrid.

of mountains in the distance. One of them is made out of ovals; the other of triangles. How many things can we draw out of ovals? How many out of triangles? Which mountains look more like mountains, those made out of ovals or those made out of triangles? Which fruit-bowl looks more like a fruit-bowl? Which guitar is more of a guitar? Speaking of guitars, ... The same instrument stands behind the triangles of the second painting and the egg-shaped curves of the third one, a beautiful Spanish guitar with ebony inlay in its rim. Could we think of other shapes that we could use as a basic scaffold to draw this guitar with? What should we include in our drawing so that anyone can recognize a guitar? What should we include in our drawing so that anyone can recognize this guitar?

Ovals, triangles, egg-shaped closed curves, ... Gris' work illustrates very well the period of intense analysis in the most abstract essence of shapes that took place both in painting and mathematics – but not only in painting and mathematics – during the first half of the 20th century.

Juan Gris established relations among different bodies and figures, and the same type of work can be recognized in many of Pablo Picasso's work, for example in his six versions of *Las meninas* (which can be seen at Picasso Museum in Barcelona). But



Gris, Juan **Guitar in front of the sea** (1925)
With kind permission of Archivo Fotográfico Museo Nacional Centro de Arte Reina Sofía, Madrid.



Gris, Juan **Guitar with inlaid work** (1925)
With kind permission of Archivo Fotográfico Museo Nacional Centro de Arte Reina Sofía, Madrid.

Mathematics and Society

A Walk For Mathematicians Through
A Very Special Triangle: Prado, Reina
Sofía, Thyssen-Bornemisza



Picasso, Pablo *Woman sitting* (1939)

With kind permission of Archivo Fotográfico Museo Nacional Centro de Arte Reina Sofía, Madrid.

Picasso (1881–1973) pushed his analysis a little further. He also studied how to establish relations between the different parts of a body. He did so in order to carry on the difficult task of combining in a coherent way on the flat surface of a canvas, volumes and different points of view that an observer moving around a body can take. As we get closer to the bodies we want to describe, we work more and more locally. When we look at the left eye of a face, the right one will appear out of focus. In order to see the second eye we must move,

and then we will lose clear sight of the left eye, the nose profile or the mouth. Once we are close to it, in order to get a complete view of a body we must move around it and on doing so we get several partial views, which we call local data. The manner in which we combine and relate the different pieces of local information in order to give a coherent global description (the search of a Hasse Principle, one could say) is the kind of work that many of Picasso's paintings illustrate. We finish our little walk in front of his painting *Woman sitting*.

One of the woman's eyes and her right hand are described standing in front of her; the other eye and a nostril, are depicted from her left side; the second nostril and the mouth, are painted as seen standing at her right; as for the hat, the blue is seen from her back, and the red part from above her head. Front, back, left, right and above. Cool!

"Digital systems will kill the point of view. Shooting in video is so cheap compared to the price of film, that nowadays many directors place cameras all over in order to get a lot of material to choose from in post-production later on. Since post-production often depends more on the producer and less on the director, the final result is no point of view and an infinity of perspectives." José Luis Alcaine, photography director in *Bad Education* (Pedro Almodóvar, 2004) El País, March 28, 2004.

Bibliography:

- [Ga] Gauss, Carl F.: Disquisitiones generales circa superficies curvas, Göttingen, 1827.
- [Ha] Hausdorff, Felix: Grundzüge der Mengenlehre, Leipzig, 1914.
- [Ne] Newton, Sir Isaac: Philosophiae Naturalis Principia Mathematica, London, 1687.

Revista Matemática Complutense

Editor-in-Chief:
Fernando Cobos

Revista Matemática Complutense is an international journal which publishes carefully selected (original or survey) papers in all areas of Mathematics. It is remarkable the figure of the *Santaló Lecturer* of the Faculty of Mathematics, who each year is asked to submit in this journal a survey on his/her invited lecture. The Editorial Staff includes 33 associate editors from all over the world. See the web page indicated below for details; information on the editorial process can be also found there. A volume is published per year, divided into two issues.



ISSN: 1139-1138

Facultad de Ciencias Matemáticas
Universidad Complutense de Madrid
28040 Madrid — Spain
Revista_Matematica@mat.ucm.es



Facultad de Ciencias Matemáticas UCM

www.mat.ucm.es/serv/revista