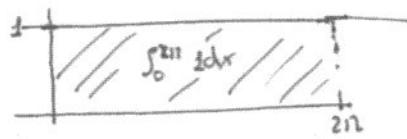


MMI PRÁCTICA-12

Nombre y apellidos.....

1.- Calcula las siguientes integrales:

$$1_1 - \int_0^{2\pi} 1 dx = 2\pi \quad \text{y A que}$$



$$1_2 - \int_0^{2\pi} \cos^2 nx dx = \frac{\cos^2 a}{2} = \frac{1 + \cos 2nx}{2} dx = \frac{1}{2} x + \frac{\sin 2nx}{4n} \Big|_0^{2\pi} = \pi$$

$$1_3 - \int_0^{2\pi} \sin^2 nx dx = \int_0^{2\pi} 1 - \cos^2 nx dx = \int_0^{2\pi} 1 dx - \int_0^{2\pi} \cos^2 nx dx = 2\pi - \pi = \pi$$

$$1_4 - \int_0^{2\pi} \cos nx \cos mx dx, \text{ para } n \neq m.$$

(Indicación: Usa que $\cos A \cos B = 1/2(\cos(A+B) + \cos(A-B))$).

$$\int_0^{2\pi} \cos nx \cos mx dx = \int_0^{2\pi} \frac{1}{2} (\cos((n+m)x) + \cos((n-m)x)) dx =$$

$$= \left[\frac{1}{2} \frac{\sin(n+m)x}{n+m} + \frac{1}{2} \frac{\sin(n-m)x}{n-m} \right]_0^{2\pi} = 0$$

$$1_5 - \int_0^{2\pi} \cos nx \sin mx dx.$$

(Indicación: Usa que $\cos A \sin B = 1/2(\sin(A+B) - \sin(A-B))$).

$$\int_0^{2\pi} \cos nx \sin mx dx = \int_0^{2\pi} \frac{1}{2} (\sin((n+m)x) - \sin((n-m)x)) dx =$$

$$= \left[-\frac{1}{2} \frac{\cos(n+m)x}{n+m} + \frac{1}{2} \frac{\cos(n-m)x}{n-m} \right]_0^{2\pi} = 0$$

$$1_6 - \int_0^{2\pi} \sin nx \sin mx dx, \text{ para } n \neq m.$$

(Indicación: Usa que $\sin A \sin B = 1/2(\cos(A-B) - \cos(A+B))$).

$$\int_0^{2\pi} \sin nx \sin mx dx = \int_0^{2\pi} \frac{1}{2} (-\cos((n-m)x) - \cos((n+m)x)) dx =$$

$$= \left[\frac{1}{2} \frac{\sin(n-m)x}{n-m} - \frac{1}{2} \frac{\sin(n+m)x}{n+m} \right]_0^{2\pi} = 0$$

2.- Dada una función f continua en $[0, 2\pi]$, se definen sus coeficientes de Fourier por:

$$\frac{a_0}{2} = \frac{1}{2\pi} \int_0^{2\pi} f(x) dx, \quad a_n = \frac{1}{\pi} \int_0^{2\pi} f(x) \cos(nx) dx, \quad y \quad b_n = \frac{1}{\pi} \int_0^{2\pi} f(x) \sin(nx) dx, \quad n \in \mathbb{N}.$$

Y se llama serie de Fourier de f a la expresión: $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(nx) + b_n \sin(nx)$.

2.1.- Calcula la serie de Fourier de la función $f(x) = x^2$

(Indicación: Las integrales se calculan por partes).

$$-\frac{a_0}{2} = \frac{1}{2\pi} \int_0^{2\pi} x^2 dx = \frac{1}{2\pi} \cdot \frac{x^3}{3} \Big|_0^{2\pi} = \frac{8\pi^3}{21\pi \cdot 3} = \frac{4}{3}\pi^2$$

$$\begin{aligned} -a_n &= \frac{1}{\pi} \int_0^{2\pi} x^2 \cos nx dx = \frac{1}{\pi} \left[\frac{x^2 \sin nx}{n} \Big|_0^{2\pi} - \frac{1}{n} \int_0^{2\pi} 2x \sin nx dx \right] \\ &= \frac{2}{\pi n} \int_0^{2\pi} x(-\sin nx) dx = \frac{2}{\pi n} \left[\frac{x \sin nx}{n} \Big|_0^{2\pi} - \frac{1}{n} \int_0^{2\pi} \sin nx dx \right] \end{aligned}$$

$$= \frac{2}{\pi n} \frac{2\pi (-\sin 2\pi)}{n} = \frac{4}{n^2}$$

$$\begin{aligned} -b_n &= \frac{1}{\pi} \int_0^{2\pi} x^2 \sin nx dx = \frac{1}{\pi} \left[-\frac{x^2 \cos nx}{n} \Big|_0^{2\pi} + \frac{2}{n} \int_0^{2\pi} x \cos nx dx \right] \\ &= \frac{1}{\pi} \frac{(-4\pi^2)}{n} + \frac{2}{\pi n} \int_0^{2\pi} x \cos nx dx = -\frac{4\pi}{n} + \frac{2}{\pi n} \left[\frac{x \sin nx}{n} \Big|_0^{2\pi} - \frac{1}{n} \int_0^{2\pi} \sin nx dx \right] \\ &= -\frac{4\pi}{n} \end{aligned}$$

Luego la serie Fourier de x^2 en $[0, 2\pi]$

$$Es \quad \frac{4}{3}\pi^2 + \sum_{n=1}^{\infty} \frac{4}{n^2} \cos nx - \frac{4\pi}{n} \sin nx.$$

2.2.- Supuesto que $x^2 = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(nx) + b_n \sin(nx)$, para todo $x \in (0, 2\pi)$ ¿cuánto suma la

serie $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}$? Si la igualdad se da $\forall x \in (0, 2\pi)$ en

parte visualizada $x = \pi$ se tiene que

$$\pi^2 = \frac{4}{3}\pi^2 + \sum_{n=1}^{\infty} \frac{4}{n^2} \cos n\pi - \frac{4\pi}{n} \sin n\pi = \frac{4}{3}\pi^2 + \sum_{n=1}^{\infty} \frac{4(-1)^n}{n^2}$$

$$\text{entonces } \left(\pi^2 - \frac{4}{3}\pi^2 \right) \frac{1}{4} = -\frac{1}{12}\pi^2 = \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$$

$$\text{por tanto } \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} = \frac{\pi^2}{12}.$$