

HuJa VII

PROBLEM 10: a) $\begin{cases} y''' = xe^x \\ y(0) = y'(0) = y''(0) = 0 \end{cases}$ solve

que integrant terms we get:

$$\int_0^x y''' = y''(x) = \int_0^x se^s dx = se^s \Big|_0^x - \int_0^x e^s dx$$

$y''(0) = 0$ \downarrow
part II

$$= xe^x - (e^x - 1)$$

then $y''(x) = xe^x - e^x + 1$ integral. etc.

$$\therefore y'' = y' \ln \frac{y'}{x} \quad (x \neq 0) \quad \Rightarrow \quad y' = \frac{y'}{x} \ln \frac{y'}{x}$$

it comes to the variable $z = \frac{y'}{x}$ then

$$\left[z' = \frac{y''x - y'}{x^2} = \frac{1}{x}(y'' - z) = \frac{1}{x} \left\{ z \ln z - z \right\} \right] \text{L.V. of variable substitution}$$

ass $\int \frac{z'}{z \ln z - z} dx = \ln(\ln z - 1).$

$$\int \frac{1}{x} dx = \ln x + C = \ln x + C$$

thus $\ln z - 1 = Cx \Rightarrow \ln z = Cx + 1$

ass $\ln \frac{y'}{x} = Cx + 1 \Rightarrow y' = x e^{Cx+1}.$

integral $y(x) + C_1 = \int x e^{Cx+1} dx. \text{ -e2e}$

PROBLEM 11: Establish the conditions for the function to be unique in the implicit form, also it exists.

$$f: B_\delta(x_0, y_0) \rightarrow \mathbb{R} \quad \text{such that } f(x_i, y_i) = 0$$

thus $y' = f(x, y(x)).$ can f uniquely determine the existence of a unique solution y exists if f exists and $y(x_0) = y_0$

from the function $y'(x) = f(x, y(x))$ can $y(x_0) = y_0$

and $f(x_0, y_0) = f(x_0, y_0')$ i.e. $f(x_0, y_0) = f(x_0, y_0')$

i.e., the antiderivative of the function $f(x, y)$ exists in the interval $(x_0, x_0').$

HOJA VII

PROBLEMA 12: SOLUCIÓN SINGULAR

a) PARA EL PROBLEMA $F(x, y, y') = 0$ CON $F \in C^1$,

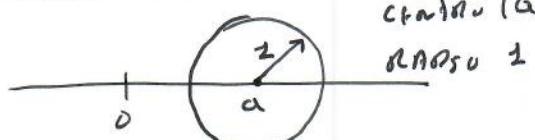
SI $\frac{\partial F}{\partial y'}(x, y, y') = 0$ EL TRONCO DE LA FUNCIÓN
IMPRESISTIBLE NOS DICE QUE EXISTE UNA $F(x, y, f(x, y)) = 0$
LUEGO SI $y' = f(x, y)$ ES SOLUCIÓN, TAMBÉN ES
UNA y_1 DE $F(x, y, y') = 0$

b) SI $\frac{\partial F}{\partial y'} = 0$, UNA SOLUCIÓN SINGULAR SE
EXISTE EN ESTE CASO COMPLEJO QUÉ:

$$\left. \begin{array}{l} F(x, y(x), y'(x)) = 0 \\ \frac{\partial F}{\partial y'}(x, y, y') = 0 \end{array} \right\} \quad \begin{array}{l} \text{ENROLLAR EN UNA} \\ \text{HORIZONTAL} \\ \text{CURVA.} \\ \text{VER GRÁFICA PREFERIDA} \end{array}$$

Ejemplo CONSIDERAMOS LA FAMILIA DE CURVAS DE

$$(x-a)^2 + y^2 = 1$$



REVISANDO EN x

$$x(x-a) + 2yy' = 0, \text{ ASÍ } a = x + yy'$$

$$\text{LUEGO } 1 = (x-a)^2 + y^2 \iff a = x + yy' \quad z = y^2y'^2 + y^2 \iff y^2(y'^2 + 1) = 1$$

$$\text{ASI } F(x, y, y') = y^2(y'^2 + 1) - 1 = 0 \quad F \in C^1(\mathbb{R}^2)$$

$$y \frac{\partial F}{\partial y'} = 2y^2y'.$$

$$\text{EN CONEXIÓN } \frac{\partial F}{\partial y'} = 0 \iff \begin{cases} y=0 \\ y'=0 \end{cases}$$

EL GRANDE CASO $y=0$ $(x-a)^2 = 1$, NOS DA UNA LINEA $y=0$
QUE ES UNA SOLUCIÓN DE (1) !

$$\text{ES EL GRAN CASO } y' = 0 \quad \left. \begin{array}{l} F(x, y, y') = 0 \\ y' = 0 \end{array} \right\} \text{ NOS DA } y = \pm 1$$

EN LOS CURVAS SINGULARES $y = 1$ $y = -1$ QUE SON
SOLUCIONES DE (1) . A CONTINUACIÓN
LOS CURVAS SINGULARES

ESTA UNA ETIQUETA DE HECHOS DE VINCERAN AL
SOLUCIONES.

HuJa VII

PROBLEMA 23 y 14)

+180 $f(y, y') = 0$
 Ejemplo $y = y^1 e^{y^1} \quad (=) \boxed{y - y^1 e^{y^1} = 0}$
 I] ss $y = f(y')$ bc cambio $y^1 = \rho$ nos da
 $\begin{cases} y^1 = \rho \\ y^1 = f'(\rho) \rho' \end{cases}$
 y nos $\rho = f'(\rho) \rho'$ E.R. u vt variacion sencilla

$\int \frac{\rho' + f'(\rho)}{\rho} = x + C$ integrando partimticas
 $\left\{ \begin{array}{l} y = f(\rho) \\ y = y^1 e^{y^1} \quad \text{ss } y^1 = \rho \end{array} \right.$

En variacion f simb $y^1 = \frac{dy}{dx} = \frac{dy}{d\rho} \cdot \frac{d\rho}{dx} = (\rho + C^\rho) \rho'$
 $y = \rho e^\rho$
 $\frac{dy}{d\rho} = \rho + \rho e^\rho$
 Lvrgu $\rho = (e^\rho + \rho e^\rho) \rho'$
 $\Leftrightarrow 1 = \left(\frac{e^\rho}{\rho} + e^\rho \right) \rho'$

en t-ganancia $\left\{ \begin{array}{l} x = t + e^\rho + \int \frac{e^\rho}{\rho} \rho' dx \\ y = \rho e^\rho \end{array} \right.$ solucion generalizada

II] ss $f(y, y') = 0$, w curva nos da los y ns y'

el cambio $\left\{ \begin{array}{l} y = \varphi(t) \\ y^1 = \psi(t) \end{array} \right.$ con $\varphi \neq \psi$ (curvas)

nsi $\varphi'(t) \frac{dt}{dx} = \psi(t) \Rightarrow \frac{\varphi'(t)}{\psi(t)} = \frac{1}{\frac{dt}{dx}}$

$\Rightarrow \int \frac{\varphi'(t)}{\psi(t)} dt = \int \frac{1}{\frac{dt}{dx}} = x + C$

$\left\{ \begin{array}{l} x = \int \frac{\psi(t)}{\varphi(t)} dt + C \\ y = \varphi(t) \end{array} \right.$ solucion generalizada

Parabel mit $y^2 = 4x$

$$\text{also } f(x, y) = 0$$

$$\text{Formel a) } ay^2 + b y^2 = x$$

Sei x st. funkt. definiert $x = f(y)$

Hinweis $y^2 \geq 0$ für y

$$x = f(p).$$

$$y \quad \boxed{y^2 = p : \frac{dy}{dx} = \frac{dy}{dp} \cdot \frac{dp}{dx} = \frac{dy}{dp} \cdot \frac{1}{\frac{dp}{dx}} = \frac{dy}{dp} \cdot \frac{1}{f'(p)}}.$$

$$\text{Ass } \left. \begin{array}{l} x = f(p) \\ y = \int p f'(p) dp + C \end{array} \right\}$$

$$\boxed{p f'(p) = \frac{dy}{dp}}$$

$$\text{Cvrt. } \int p f'(p) dp + C$$

$$\text{Formel a) } ay^2 + by^2 = x$$

$$x = ap + bp^2$$

$$\text{Vrb. } \frac{dy}{dx} = \frac{dy}{dp} \cdot \frac{dp}{dx} = \frac{dy}{dp} \cdot \frac{1}{\frac{dp}{dx}} = \frac{dy}{dp} \cdot \frac{1}{a+2bp} \quad \begin{matrix} \boxed{x = ap + bp^2} \\ \frac{dx}{dp} = a+2bp \end{matrix}$$

$$y \text{ ist } \frac{dy}{dp} = (a+2bp) p$$

$$\text{Vrb. } \int \frac{dy}{dp} dp = \frac{ap^2}{2} + \frac{2b}{3} p^3 + C$$

$$y \quad \left. \begin{array}{l} x = ap + bp^2 \\ y = \frac{ap^2}{2} + \frac{2b}{3} p^3 + C \end{array} \right\}$$

zu lvc in der
parametrische

PROBLEMA 15:

$$a) \quad y = x f(y') + g(y')$$

notarán que $y' = f(y') + x f'(y') y'' + g'(y') y''$.

$$\text{asi } (y' - f(y')) = [x f'(y') + g'(y')] y''$$

Si se multiplican las ecuaciones $\boxed{y' = y'}$ se tiene que

$$(1) \quad \frac{dy}{dx} = y', \quad \text{asi } \frac{dy}{dp} = \frac{dy}{dx} \cdot \frac{dx}{dp} = y' \frac{dx}{dp}$$

$$f(x) - f(f(x)) = [x f'(f(x)) + y'(f(x))] \frac{df(x)}{dx}$$

Si considerando la resultante de x , como $x'(p) = \frac{1}{f'(x)}$

Si se tiene

$$f(x) - f(f(x)) = [x(p) f'(p) - y'(p)] \frac{1}{x'(p)}$$

$$\text{y asi } x'(p) = \frac{f'(p)}{p - f(p)} x(p) + \frac{y'(p)}{p - f(p)}. \quad \text{Es una linea}$$

b) En el caso particular de que

$$y = x y' + g(y')$$

notarán que $y' = y' + x y'' + y'(y') y''$

$$\Leftrightarrow 0 = (x + y'(y')) y'' \quad \text{Luego}$$

o bien $y'' = 0$ (solución平凡a de recta $y = ax + b$)

o bien $x + y'(y') = 0 \Leftrightarrow y' = (y')^{-1}(-x)$.

OBSERVACION: si $f(x, y, y') = 0$ con $f(x, y, y') = x y' + g(y') - y$

$$\frac{\partial f}{\partial y'} = x + g'(y') \quad \text{es igualdad en la que}$$

$$\left. \begin{array}{l} \frac{\partial f}{\partial y'} = x + g'(y') = 0 \\ f(x, y, y') = 0 \end{array} \right\} \quad \text{SOLUCIONES GENERALES DEL ECUACION (EQUIVALENTE A LA FAMILIA DE RECTAS (VALORES CONSTANTES))}$$

$$\text{c) } y : xy' + \frac{\alpha}{2y'} \quad \text{notarán que } y' = y' + x y'' + \frac{\alpha}{2} \frac{-y''}{y'^2} \Leftrightarrow 0 = (x - \frac{\alpha}{2} \frac{1}{y'^2}) y''$$

$$- y'' = 0 \Rightarrow y = \alpha x + \beta \quad \text{rectas, pertenecientes a } y'' = 0 \text{ la recta}$$

$$\alpha x + \beta = \alpha x + \frac{\alpha}{2} \alpha \quad \text{Luego } \beta = \frac{\alpha}{2} \Rightarrow y = \alpha x + \frac{\alpha}{2} \alpha \quad \text{afijo - 1.}$$

$$- o \text{ bien } x = \frac{\alpha}{2} \frac{1}{y'^2} \Rightarrow y' = \sqrt{\frac{\alpha}{2} \frac{1}{x}} \Rightarrow y = \sqrt{\frac{\alpha}{2}} \sqrt{x} + k.$$

Integrando

HuSA VII

PROBLEMA 16: d) $y'' + y'^2 = 2e^{-y}$ E.R.O. 2: ONNA

nu agantur la
vulasarit suatu.

EL CAMBIO $y' = p$ nu nra

$$y'' = \frac{dy'}{dx} = \frac{dy}{dx} \cdot \frac{dp}{dx} = \frac{dy}{dp} \cdot p = \frac{dp}{dy} \cdot p$$

ASÉ LA E.R.O., risolto per y

$$\frac{dp}{dy} p + p^2 = 2e^{-y}$$

$$\Leftrightarrow \frac{dp}{dy} = -p + \frac{2}{p} e^{-y}$$

EL CAMBIO $z = p^2$ nu nra

$$\begin{aligned} z' &= 2p p' = 2p \left[-p + \frac{2}{p} e^{-y} \right] = -2p^2 + 4e^{-y} \\ &= -2z + 4e^{-y} \end{aligned} \quad \text{E.R.O. LINEAR}$$

f) $y'' = 2yy'$ $y(0) = y'(0) = 1$

integrammo $\int y'' dx = \int 2yy' dx$

$$\Leftrightarrow y' = y^2 + x \quad \text{E.R.O. vulasarit suatu}$$

PROBLEMA 17: si $f(t, x, x', \dots, x^n) = 0$ es θ -HOMOGÉNEA

EL CAMBIO $x = e^{\int z dt}, x' = z e^{\int z dt}, x'' = (z' + z) e^{\int z dt} \dots$
 $\dots x^n = e^{\int z dt} \phi(z, z', \dots z^{n-1})$

LUGO $f(t, x, x', \dots, x^n) = e^{\int z dt} f(t, z, z', \dots, z^{n-1}) = 0$
& θ -HOMOGÉNEA

EL ONNA NR LA E.R.O. ST RISOLVUTI CON 1

ESEMPIO: $yy'' - (y')^2 = 6x y^2$

obiettivo (>) $y > y''$ - (>) $y' \geq 0$ \Leftrightarrow

$$yy'' - (y')^2 = 6x y^2 \quad \text{LUGO H 1-HOMOGÉNEA}$$

EL CAMBIO $y = e^{\int z dt}, y' = z e^{\int z dt}, y'' = e^{\int z dt} (z' + z^2)$ nu nra

$$e^{\int z dt} (z' + z^2 - z^2) = e^{\int z dt} (6z) \Leftrightarrow z'(x) = 6x$$

ASSE $z(x) = 3x^2 + x$ y sun tanho

$$\boxed{y = e^{\int 3x^2 + x dx} + k_2 = e^{x^3 + k_1 x + k_2}} \quad \text{SOL. GENERALE}$$