CÁLCULO DE PRIMITIVAS

PROBLEMA 1:

a) $\int 3x\,dx = 3\int x\,dx = 3\frac{x^2}{2} + k$

d) $\int (3x-2)^2\,dx = \begin{cases} \frac{1}{3}\int 3(3x-2)^2\,dx = \frac{1}{3}\frac{(3x-2)^3}{3} = \frac{1}{9}(27x^3 - 6(3x)^2 + 9x4 - 8 \\ \qquad = 3x^3 - 6x^2 + 4x - 8 \\ \int 9x^2 - 12x + 4\,dx = \end{cases}$

$= 9\int x^2 - 12\int x + 4\int 1 =$

$= 9\frac{x^3}{3} - 12\frac{x^2}{2} + 4x + k$

h) $\int 2x \cos x^2\,dx = \operatorname{sen} x^2$

$(x^2)' = 2x.$

j) $\int \frac{1}{x^k}\,dx = \int x^{-k} = \frac{x^{-k+1}}{-k+1} = \frac{1}{(1-k)\,x^{k-1}}$

$k \in \mathbb{N} - \{1\}$

en particular $\int \frac{1}{x^4}\,dx = \frac{1}{-3\,x^3}$

e) $\int \frac{2x}{(x^2-3)^4}\,dx = \frac{1}{-3(x^2-3)^3}$

$(x^2)' = 2x$

ñ) $\int 3\cosh x + 2\operatorname{senh} x\,dx = 3\int \cosh x\,dx + 2\int \operatorname{senh} x\,dx = 3\operatorname{senh} x + 2\cosh x.$

o) $\int \cosh x \cosh(\operatorname{senh} x)\,dx = \operatorname{senh}(\operatorname{senh} x)$

$(\operatorname{senh})'(x) = \cosh x$

r) $\int \frac{1}{1+x^2}\,dx = \operatorname{Arctg} x$

s) $\int \frac{1}{\sqrt{1-x^2}}\,dx = \operatorname{Arc\,sen} x$

t) $\int \frac{1}{\sqrt{1+x^2}}\,dx = \operatorname{Arc\,senh} x.$

CÁLCULO DE PRIMITIVAS

PROBLEMA 2:

a) $\int \frac{1}{\sqrt{1-x^2}}\,dx = \text{Arc sen} x$

b) $\int \text{sen} x \cos^4 x\,dx = -\int -\text{sen} x \cos^4 x\,dx = -\frac{\cos^5 x}{5}$

$(\cos x)' = -\text{sen} x$

c) $\int \frac{\ln x}{x}\,dx = \frac{\ln^2 x}{2}$

$(\ln x)' = \frac{1}{x}$

d) $\int x\sqrt{1-x^2}\,dx = -\frac{1}{2}\int -2x\sqrt{1-x^2}\,dx = -\frac{1}{2}\sqrt{(1-x^2)^3}\cdot\frac{2}{3}$

$= -\frac{1}{3}(1-x^2)^{3/2}$

e) $\int \frac{dx}{\sqrt{x-1}+\sqrt{x+1}} = \int \frac{\sqrt{x-1}-\sqrt{x+1}}{(\sqrt{x-1}+\sqrt{x+1})(\sqrt{x-1}-\sqrt{x+1})}\,dx =$

$= \int -\frac{\sqrt{x-1}}{2} + \frac{\sqrt{x+1}}{2}\,dx$

f) $\int \frac{e^x+e^{2x}+e^{3x}}{e^{4x}}\,dx = \int \frac{e^x[1+e^x+(e^x)^2]}{(e^x)^4}\,dx =$ $(e^x)' = e^x$

$= \int \frac{1+u+u^2}{u^4}\,du.$

$u = e^x$

g) $\int \text{tn}^2 x\,dx = \int \text{tn}^2 x + 1 - 1\,dx = \int \text{tn}^2 x + 1\,dx - \int dx =$

$= \text{tn} x - x.$

h) $\int \frac{dx}{a^2+x^2} = \frac{1}{a^2}\int \frac{dx}{1+\left(\frac{x}{a}\right)^2}\,dx = \frac{1}{a}\left[\int \frac{1}{a}\,\frac{dx}{1+\left(\frac{x}{a}\right)^2}\right.$ $\left(\frac{x}{a}\right)' = \frac{1}{a}.$

i) $\int \frac{a^x}{b^x}\,dx = \int \left(\frac{a}{b}\right)^x dx = \int e^{x\ln\frac{a}{b}}\,dx =$

k) $\int \frac{1}{1+\text{sen} x}\,dx = \int \frac{1-\text{sen} x}{(1+\text{sen} x)(1-\text{sen} x)}\,dx = \int \frac{1-\text{sen} x}{\cos^2 x}\,dx =$

$= \int \frac{1}{\cos^2 x}\,dx + \int \frac{-\text{sen} x}{\cos^2 x}\,dx = \text{tn} x + \frac{1}{\cos x}$

l) $\int \frac{8x^2+6x+4}{x+1}\,dx = \int \frac{(x+1)(8x-2)+6}{x+1}\,dx = \int (8x-2) + \frac{6}{x+1}\,dx$

$8x^2+6x+4 \mid x+1$

$-8x^2-8x \quad 8x-2$

$-2x+4$

$2x\ \ 2$

6

CÁLCULO DE PRIMITIVAS

PROBLEMA 3: a) $\int \arctan x \, dx \underset{\text{PARTES}}{=} x \arctan x - \int \frac{x}{1+x^2} dx$

$= x \arctan x - \frac{1}{2} \int \frac{2x}{1+x^2} dx$

b) $\int \ln|x| \, dx = \begin{cases} \int \ln(-x). \\ \text{ó} \\ \int \ln x \, dx \end{cases}$

f) $\int x^3 e^{x^2} dx \underset{\text{PARTES}}{=} x^2 \frac{e^{x^2}}{2} - \int 2x e^{x^2} dx = \frac{x^2 e^{x^2}}{2} - e^{x^2}$

m) $\boxed{\int \cos(\ln x) \, dx} \underset{\text{PARTES}}{=} x \cos(\ln x) - \int x \left(-\operatorname{sen}(\ln x) \frac{1}{x}\right) =$

$= x \cos(\ln x) + \int \operatorname{sen}(\ln x) \, dx \underset{\text{PARTES}}{=}$

$= x \cos(\ln x) + x \operatorname{sen}(\ln x) - \boxed{\int \cancel{x} \cos \ln x \frac{1}{\cancel{x}}}$

DESPEJANDO $2 \int \cos(\ln x) \, dx = x \cos(\ln x) + x \operatorname{sen}(\ln x)$

ASÍ $\int \cos(\ln x) \, dx = \frac{x \cos(\ln x) + x \operatorname{sen}(\ln x)}{2}$

PROBLEMA 4: a) $\int \operatorname{arc\,sen} x \, dx = x \operatorname{arc\,sen} x - \int x \frac{1}{\sqrt{1-x^2}} dx$

$= x \operatorname{arc\,sen} x + \frac{1}{2} \int \frac{-2x}{\sqrt{1-x^2}} dx =$

$= x \operatorname{arc\,sen} x + \sqrt{1-x^2}$

b) $\int f^{-1}(x) \, dx \underset{\text{PARTES}}{=} x f^{-1}(x) - \int x (f^{-1})'(x) \, dx =$

$= x f^{-1}(x) - \int \frac{x}{f'(f^{-1}(x))} dx = x f^{-1}(x) - F(f^{-1}(x))$

$(F(f^{-1}(x)))' = F'(f^{-1}(x)) \cdot \frac{1}{f'(f^{-1}(x))} = x \frac{1}{f'(f^{-1}(x))} dx$

CÁLCULO DE PRIMITIVAS

PROBLEMA 5:

b) $$\boxed{\int \cos^n x\, dx} = \int \cos x \cos^{n-1} x\, dx = $$

PARTES

$$= \operatorname{sen} x \cos^{n-1} x - \int \operatorname{sen} x\,(n-1)\cos^{n-2} x\,(-\operatorname{sen} x)\,dx =$$

$$= \operatorname{sen} x \cos^{n-1} x + \int \operatorname{sen}^2 x\,(n-1)\cos^{n-2} x\, dx =$$

$$= \operatorname{sen} x \cos^{n-1} x + \int (1-\cos^2 x)(n-1)\cos^{n-2} x\, dx$$

$$= \operatorname{sen} x \cos^{n-1} x + (n-1)\int \cos^{n-2} x\, dx \boxed{-(n-1)\int \cos^n x\, dx}$$

ASÍ $$n\int \cos^n x\, dx = \operatorname{sen} x \cos^{n-1} x + (n-1)\int \cos^{n-2} x\, dx$$

Y $$\int \cos^n x\, dx = \frac{1}{n}\operatorname{sen} x \cos^{n-1} x + \frac{n-1}{n}\int \cos^{n-2} x\, dx$$

c) $$\int \frac{dx}{(x^2+1)^n} = -\int \frac{x^2}{(1+x^2)^n}dx + \int \frac{1}{(x^2+1)^{n-1}}dx =$$

$$\frac{1}{(x^2+1)^n} = \frac{1}{(x^2+1)^{n-1}} - \frac{x^2}{(1+x^2)^n}$$

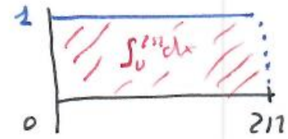
$$\underset{\text{POR PARTES}}{=} -\left[\frac{-1}{2n-2}\,\frac{x}{(1+x^2)^{n-1}} + \frac{1}{2n-2}\int \frac{1}{(1+x^2)^{n-1}}dx\right] + \int \frac{1}{(x^2+1)^{n-1}}dx =$$

$$= \frac{1}{2n-2}\,\frac{x}{(1+x^2)^{n-1}} + \left(1-\frac{1}{2n-2}\right)\int \frac{1}{(x^2+1)^{n-1}}dx =$$

$$= \frac{1}{2n-2}\,\frac{x}{(1+x^2)^{n-1}} + \frac{2n-3}{2n-2}\int \frac{1}{(x^2+1)^{n-1}}dx.$$

CÁLCULO DE PRIMITIVAS

PROBLEMA 6] a) $\int_0^{2\pi} dx = 2\pi$



$$\int_0^{2\pi} \cos^2 nx \, dx = \int_0^{2\pi} \frac{1+\cos 2nx}{2} =$$

$\cos 2x = \cos^2 x - \sin^2 x =$

$= \cos^2 x - (1-\cos^2 x)$

y despejando

$\cos^2 x = \frac{1+\cos 2x}{2}$

$$= \int_0^{2\pi} \frac{1}{2}\, dx + \int_0^{2\pi} \frac{\cos 2nx}{2} = \pi + \left(\frac{\sin 2nx}{4n} \Big|_0^{2\pi} \right) = \pi.$$

$$\int_0^{2\pi} \sin^2 nx \, dx = \int_0^{2\pi} 1-\cos^2 nx \, dx = 2\pi - \pi = \pi.$$

b) $$\int_0^{2\pi} \sin nx \cos mx \, dx = \int_0^{2\pi} \frac{1}{2}\left[\sin(n+m)x - \underbrace{\sin(m-n)x}_{=0 \text{ si } n=m}\right] dx =$$

$\cos A \sin B =$

$= \frac{1}{2}\left(\sin(A+B) - \sin(A-B)\right)$

$$= \frac{1}{2}\left[-\frac{\cos(n+m)x}{n+m}\Big|_0^{2\pi} + \cos(m-n)x \Big|_0^{2\pi} \right] = 0$$

$n \neq m$. $$\int_0^{2\pi} \sin nx \sin mx \, dx \underset{\text{PARTES}}{=} \underbrace{-\frac{\cos nx}{n} \sin mx \Big|_0^{2\pi}}_{=0} + \frac{m}{n}\int_0^{2\pi} \cos nx \cos mx \, dx \underset{\text{PARTES}}{=}$$

$$= \frac{m}{n}\left[\underbrace{\cancel{\frac{\sin nx}{n} \cos mx}}_{=0} \Big|_0^{2\pi} + \frac{m}{n}\int_0^{2\pi} \sin nx \sin mx \, dx \right] = \frac{m^2}{n^2}\int_0^{2\pi} \sin nx \sin mx \, dx$$

Así como $m \neq n \Rightarrow \frac{m^2}{n^2} \neq 1$, luego $\int_0^{2\pi} \sin nx \sin mx \, dx = 0$

Sólo puede tomar este valor.

c) $$\frac{a_0}{2} = \frac{1}{2\pi}\int_0^{2\pi} x^2 \, dx = \frac{1}{2\pi}\left(\frac{x^3}{3}\Big|_0^{2\pi} \right) = \frac{8\pi^3}{6\pi} = \frac{4\pi^2}{3}$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} x^2 \cos nx \, dx \underset{\text{PARTES}}{=} \frac{1}{\pi}\left(\underbrace{x^2 \frac{\sin nx}{n}\Big|_0^{2\pi}}_{=0} - \frac{1}{n}\int_0^{2\pi} 2x \sin nx \, dx \right) \underset{\text{PARTES}}{=}$$

CÁLCULO DE PRIMITIVAS.

PROBLEMA 6:] c) CONTINUACIÓN

$$= \frac{1}{n\pi}\left(\frac{2x\cos nx}{n}\Big|_0^{2\pi} - \frac{1}{n}\underbrace{\int_0^{2\pi} 2\cos nx\, dx}_{0} \right) =$$

$$= \frac{1}{n^2\pi} 4\pi \cos n2\pi = \frac{4}{n^2}$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} x^2 \operatorname{sen} nx\, dx \underset{\text{PARTES}}{=} \frac{1}{\pi}\left[-x^2\frac{\cos nx}{n}\Big|_0^{2\pi} + \frac{1}{n}\int_0^{2\pi} 2x\cos nx\, dx \right] =$$

$$\underset{\text{PARTES}}{=} -\frac{1}{\pi}\frac{4\pi^2}{n} + \frac{1}{n\pi}\left[\underbrace{2x\frac{\operatorname{sen} nx}{n}\Big|_0^{2\pi}}_{0} - \frac{1}{n}\underbrace{\int_0^{2\pi} 2\operatorname{sen} nx\, dx}_{0} \right] =$$

$$= -\frac{4\pi}{n}$$

LA SERIE DE FOURIER DE f ES

$$\boxed{\frac{4\pi^2}{3} + \sum_{n=1}^{\infty} \frac{4}{n^2}\cos nx - \frac{4\pi}{n}\operatorname{sen} nx.}$$

PROBLEMA 7:] $\displaystyle ¿\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx?$

SI LLAMAMOS $x = a+b-u$, COMO f ES CONTINUA,

$$\frac{dx}{du} = -1$$

$$x = a \Rightarrow u = b$$

$$x = b \Rightarrow u = a$$

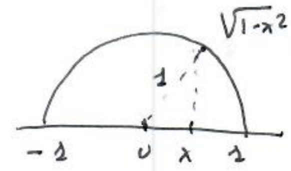
PODEMOS USAR LA FÓRMULA DE SUSTITUCIÓN Y:

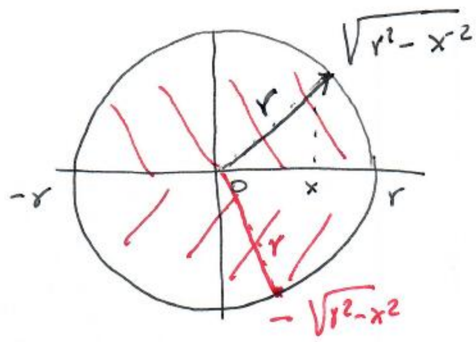
$$\int_a^b f(x)\,dx = \int_b^a f(a+b-u)\,(-1)\,du = \int_a^b f(a+b-u)\,du.$$

CÁLCULO DE PRIMITIVAS

PROBLEMA 8.] POR DEFINICIÓN

$$\pi = 2\int_{-1}^{1} \sqrt{1-x^2}\,dx$$





ÁREA DEL CÍRCULO DE RADIO r

$$\text{SERÁ} \quad \int_{-r}^{r} \sqrt{r^2-x^2}\,dx + \left(-\int_{-r}^{r} -\sqrt{r^2-x^2}\,dx\right) =$$

$$= 2\int_{-r}^{r} \sqrt{r^2-x^2}\,dx =$$

$$= 2\int_{-r}^{r} \sqrt{r^2\left(1-\left(\frac{x}{r}\right)^2\right)}\,dx = 2\int_{-r}^{r} r\sqrt{1-\left(\frac{x}{r}\right)^2}\,dx =$$

$$\underset{\downarrow}{=} \quad 2\int_{-1}^{1} r^2\sqrt{1-y^2}\,dy = r^2\left[2\int_{-1}^{1}\sqrt{1-y^2}\,dy\right] =$$

$$= r^2\pi.$$

CAMBIO DE VARIABLE

$$\left\{\begin{array}{l} \text{SI } y = \frac{x}{r} \\ \frac{dy}{dx} = \frac{1}{r} \qquad dx = r\,dy \\ \text{SI } x = r \Rightarrow y = 1 \\ \text{SI } x = -r \Rightarrow y = 1 \end{array}\right.$$