CALCULO DE PRIMITIVAS II

PROBLEMA 1:

$$\boxed{\int_2^3 \frac{\operatorname{sen} x^2}{x}\, dx} = \int_2^3 \frac{\operatorname{sen} x^2}{x} \cdot \frac{2x}{2x}\, dx$$

$\frac{\operatorname{sen} x^2}{x}$ es continua en $[2,3]$

el cambio de variable:

$y = x^2 \qquad x = 2 \Rightarrow y = 4$

$dy = 2x\, dx \qquad x = 3 \Rightarrow y = 9$

$$= \boxed{\int_4^9 \frac{\operatorname{sen} y}{2y}\, dy.}$$

PROBLEMA 2:

a) $\int e^x \operatorname{sen} e^x\, dx = -\cos e^x$

$(e^x)' = e^x$

b) $\int x e^{-x^2} dx = -\frac{1}{2} \int -2x\, e^{-x^2} dx = -\frac{1}{2} e^{-x^2}$

$(-x^2)' = -2x$

d) $\int \frac{e^x}{e^{2x} + 2e^x + 1}\, dx = \int \frac{1}{y^2 + 2y + 1}\, dy =$

$y = e^x$

$dy = e^x dx$

$$= \int \frac{1}{(y+1)^2}\, dy = -\frac{1}{y+1} = -\frac{1}{e^x + 1}$$

f) $\int \frac{x}{\sqrt{1-x^4}}\, dx = \frac{1}{2} \int \frac{2x}{\sqrt{1-x^4}}\, dx = \frac{1}{2} \int \frac{1}{\sqrt{1-y^2}}\, dy =$

$y = x^2$

$dy = 2x\, dx$

$$= 1/2 \operatorname{Arc\,sen} y = 1/2 \operatorname{Arc\,sen} x^2.$$

PROBLEMA 3: $\left(\ln\left|\operatorname{tn}\frac{x}{2}\right|\right)' = \frac{1}{\left|\operatorname{tn}\frac{x}{2}\right|} \left(\left|\operatorname{tn}\frac{x}{2}\right|\right)' =$

$$= \frac{\left(\operatorname{tn}\frac{x}{2}\right)'}{\operatorname{tn} x/2} = \frac{\left(1 + \operatorname{tn}^2\frac{x}{2}\right)\frac{1}{2}}{\operatorname{tn} x/2} = \frac{1 + \operatorname{tn}^2\frac{x}{2}}{2 \operatorname{tn} x/2} =$$

$$= \frac{1}{2} \frac{\cos x/2}{\operatorname{sen} x/2} + \frac{\operatorname{sen} x/2}{2 \cos x/2} = \frac{\cos^2 x/2 + \operatorname{sen}^2 x/2}{2 \operatorname{sen} x/2 \cos x/2} = \frac{1}{\operatorname{sen} x}$$

PRIMITIVAS II:

PROBLEMA 4:) a) $\int \frac{dx}{x(1-x)} =$

$x = sen^2 t$

$dx = 2\, sen t \cos t\, dt$

$$= \int \frac{2\, sen t \cos t}{sen^2 t(1-sen^2 t)}\, dt = 2\int \frac{1}{sen t \cos t}\, dt =$$

$$= 2 \int \frac{\cos t}{sen t} + \frac{sen t}{\cos t}\, dt = 2\, Ln|sen t| - 2\, L|\cos t| =$$

$$= 2\, Ln\left|\frac{sen t}{\cos t}\right|$$

c) $\int \frac{dx}{e^x+1} = \int \frac{-1/t}{\frac{1}{t}+1}\, dt = -\int \frac{1}{t+1}\, dt =$

$x = -Ln t$

$dx = -\frac{1}{t}\, dt$

$$= -Ln(t+1) = -Ln(e^{-x}+1).$$

e) $\int \frac{\sqrt{x^2+1}}{x^2}\, dx = \int \frac{\sqrt{1+tn^2 t}}{tn^2 t}(1+tn^2 t)\, dt =$

$x = tn\, t$

$dx = 1+tn^2 t\, dt$

$$= \int \sqrt{1+tn^2 t} + \frac{\sqrt{1+tn^2 t}}{tn^2 t}\, dt = \int \frac{1}{\cos t} + \frac{\frac{1}{\cos t}}{\frac{sen^2 t}{\cos^2 t}}\, dt =$$

$$= \int \frac{1}{\cos t}\, dt + \int \frac{\cos t}{sen^2 t}\, dt = Ln\left|\tan\left(\frac{t}{2}+\frac{\pi}{4}\right)\right| - \frac{1}{sen t}$$

PROBLEMA ANTERIOR

$$= Ln\left|\tan\left(\frac{Arctn\, x}{2}+\frac{\pi}{4}\right)\right| - \frac{1}{sen(Arctn\, x)} =$$

$sen(Arctn x) = \frac{x}{\sqrt{1+x^2}}$

$$= Ln\left|\tan\left(\frac{Arctn\, x}{2}+\frac{\pi}{4}\right)\right| - \frac{\sqrt{1+x^2}}{x}$$

f) $\int \sqrt{a^2+x^2}\, dx = \int \sqrt{a^2+a^2 senh^2 t}\cdot a \cosh t\, dt =$

$x = a\, senh\, t$

$dx = a \cosh t$

$y\ \cosh^2 t = 1 + senh^2 t$

$$= a^2 \int \cosh^2 t\, dt = \frac{1}{2}t + \frac{1}{4} senh\, 2t = \frac{1}{2} Arcsenh\left(\frac{x}{a}\right) +$$

TEORÍA

$$+ \frac{1}{4} senh\left(2\, Arcsenh\left(\frac{x}{a}\right)\right).$$

PRIMITIVAS II

PROBLEMA 5: a) $\displaystyle\int \frac{2x^2+7x-1}{x^3+x^2-x-1}\,dx =$

HAY QUE DESCOMPONER x^3+x^2-x-1. CLARAMENTE $x=1$ ES UNA RAIZ

$$
\begin{array}{rrrr|l}
x^3 & +x^2 & -x & -1 & x-1 \\
\cline{5-5}
-x^3 & x^2 & & & x^2+2x+1 \\
\hline
 & 2x^2 & -x & -1 & \\
 & -2x^2 & +2x & & \\
\hline
 & & x & -1 & \\
\end{array}
$$

$$= \int \frac{2x^2+7x-1}{(x-1)(x^2+2x+1)}\,dx = \int \frac{2x^2+7x-1}{(x-1)(x+1)^2}\,dx =$$

DESCOMPOSICIÓN EN FRACCIONES SIMPLES

$$\frac{2x^2+7x-1}{(x-1)(x+1)^2} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{(x+1)^2} = \frac{A(x+1)^2 + B(x-1)(x+1) + C(x-1)}{(x-1)(x+1)^2}$$

$$= \frac{Ax^2+2Ax+A + Bx^2-B + Cx - C}{(x-1)(x+1)^2} = \frac{(A+B)x^2+(2A+C)x+(A-B-C)}{(x-1)(x+1)^2}$$

IGUALANDO COEFICIENTES

$$
\begin{aligned}
A+B &= 2 & B &= 2-A \\
2A + C &= 7 & C &= 7-2A \\
A-B-C &= -1 & &
\end{aligned}
$$

$A-(2-A)-(7-2A)=-1$

$4A-9=-1 \Rightarrow \boxed{A=2}$

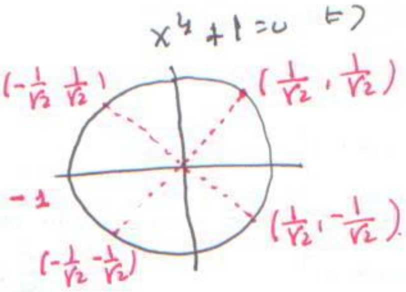
$\boxed{B=0}$ $\boxed{C=3}$

$$= \int \frac{2}{x-1} + \frac{3}{(x+1)^2}\,dx = 2\ln|x-1| - \frac{3}{x+1}.$$

b) $\displaystyle\int \frac{dx}{x^4+1} =$

HAY QUE DESCOMPONER x^4+1. AHORA LAS RAÍCES DE x^4+1 SON LAS 4 RAÍCES COMPLEJAS DE LA UNIDAD DE -1, e.d.

$x^4+1=0 \Leftrightarrow x=\sqrt[4]{-1}$



$$x^4+1 = \left(x-\left(\tfrac{1}{\sqrt2}+\tfrac{1}{\sqrt2}i\right)\right)\left(x-\left(\tfrac{1}{\sqrt2}-\tfrac{1}{\sqrt2}i\right)\right)\left(x-\left(-\tfrac{1}{\sqrt2}+\tfrac{1}{\sqrt2}i\right)\right)\left(x-\left(-\tfrac{1}{\sqrt2}-\tfrac{1}{\sqrt2}i\right)\right)$$

$$= \left(x^2-\tfrac{2}{\sqrt2}x+1\right)\left(x^2+\tfrac{2}{\sqrt2}x+1\right).$$

ASÍ $\displaystyle\int \frac{dx}{x^4+1} = \int \frac{Ax+B}{x^2-\frac{2}{\sqrt2}x+1}\,dx + \int \frac{Cx+D}{x^2+\frac{2}{\sqrt2}x+1}\,dx$

PRIMITIVAS II

PROBLEMA 6: a) $\int \frac{dx}{1+\sqrt{1+x}} =$ CAMBIO DE VARIABLE $y=\sqrt{1+x}$.

b) $\int \frac{dx}{1+e^x}$ PROBLEMA 4.

c) $\int \frac{dx}{\sqrt{x}+\sqrt[3]{x}} = \int \frac{dx}{x^{3/6}+x^{2/6}}$ CAMBIO DE VARIABLE $u=x^{1/6}$

d) $\int \frac{dx}{\sqrt{1+e^x}}$ CAMBIO DE VARIABLE $u=\sqrt{1+e^x}$

e) $\int \frac{dx}{2+\tan x}$ CAMBIO DE VARIABLE $u=\tan x$

$du = 1+\tan^2 x\, dx$

$$= \int \frac{1}{2+\tan x}\cdot\frac{1+\tan^2 x}{1+\tan^2 x}\,dx = \int \frac{1}{2+u}\cdot\frac{1}{1+u^2}\,du =$$

$$= \int \frac{A}{2+u} + \frac{Bu+C}{1+u^2}\,du = \int \frac{1/5}{2+u} + \frac{-\frac{1}{5}u+2/5}{1+u^2}\,du$$

$$= 1/5\ \ln(2+u) - \frac{1}{10}\int \frac{2u}{1+u^2}\,du + \frac{2}{5}\int\frac{du}{1+u^2} =$$

$$= 1/5\ \ln(2+u) - \frac{1}{10}\ln(1+u^2) + \frac{2}{5}\,\mathrm{Arctan}\, u. =$$

$u=\tan x$

$$= 1/5\ \ln(2+\tan x) - \frac{1}{10}\ln(1+\tan^2 x) + \frac{2}{5}x.$$

f) $\int \mathrm{sen}^3 x \cos^4 x\,dx = -\int(-\mathrm{sen}\,x)(1-\cos^2 x)\cos^4 x\,dx =$

$$= -\int(-\mathrm{sen}\,x)\cos^4 x\,dx + \int(-\mathrm{sen}\,x)\cos^6 x\,dx. =$$

$$= -\frac{\cos^5 x}{5} + \frac{\cos^7 x}{7}.$$

g) $\int \cos^5 x\,\mathrm{sen}\,2x\,dx = \int 2\cos^6 x\,\mathrm{sen}\,x\,dx$

$\mathrm{sen}\,2x = 2\cos x\,\mathrm{sen}\,x$

h) $\int \frac{dx}{\sqrt{\sqrt{x}+1}}$ CAMBIO DE VARIABLE $u=\sqrt{\sqrt{x}+1}$.

i) $\int \frac{4^x+1}{2^x+1}\,dx = \int \frac{(2^x)^2+1}{2^x+1}\,dx$ CAMBIO $u=2^x=e^{x\ln 2}$

j) $\int e^{\sqrt{x}}\,dx$ $u=\sqrt{x}$

k) $\int \frac{\sqrt{1-x}}{1-\sqrt{x}}\,dx$ CAMBIO $u=\sqrt{1-x}$ ó CAMBIO $u=1-\sqrt{x}$.

PRIMITIVAS II

PROBLEMA 7:] b) $\int \frac{1}{1+\operatorname{sen}^2 x}\,dx$ CAMBIO DE

VARIABLE $u = \operatorname{tg}\frac{x}{2}$ ASÍ $\operatorname{sen}x = \frac{2u}{1+u^2}$

y $du = (1+\operatorname{tg}^2\frac{x}{2})\frac{1}{2}\,dx$

LUEGO $\int \frac{1}{1+\operatorname{sen}^2 x}\,dx = \int \frac{2}{1+u^2}\cdot\frac{1}{\frac{4u^2}{(1+u^2)^2}}\,du =$

$$= \int \frac{2(1+u^2)}{4u^2} = \int \frac{1}{2u^2} + \frac{1}{2}\,du = -\frac{1}{2u} + \frac{1}{2}u =$$

$$= -\frac{1}{2}\,\frac{1}{2\operatorname{Arctg} x} + \frac{1}{2}\,2\operatorname{Arctg} x = -\frac{1}{4\operatorname{Arctg} x} + \operatorname{Arctg} x.$$

PROBLEMA 8:] SI $u = \operatorname{tg} x$ $\quad x = \operatorname{Arctg} u$

ASÍ $\operatorname{sen} x = \operatorname{sen}(\operatorname{Arctg} u) = \frac{u}{\sqrt{1+u^2}}$

TEORÍA (*)

$\cos x = \cos(\operatorname{Arctg} u) = \frac{1}{\sqrt{1+u^2}}$

(*) VEAMOS EL CASO $\operatorname{sen}x$

$u = \operatorname{tg} x = \frac{\operatorname{sen} x}{\sqrt{1-\operatorname{sen}^2 x}}$ SI $A = \operatorname{sen}x = \operatorname{sen}(\operatorname{Arctg} u)$

EN FUNCIÓN $u = \frac{A}{\sqrt{1-A^2}}$ DESPEJAMOS A EN FUNCIÓN DE u.

$u^2(1-A^2) = A^2 \iff u^2 = (1+u^2)A^2$ ASÍ $A^2 = \frac{u^2}{1+u^2}$

y $A = \frac{u}{\sqrt{1+u^2}} \implies \operatorname{sen} x = \operatorname{sen}(\operatorname{Arctg} u) = \frac{u}{\sqrt{1+u^2}}$

INTEGRALES II

PROBLEMA 9:

a) $\int \frac{1}{\sqrt{1+x^2}}\,dx = (\operatorname{senh})^{-1}(x)$
inmediata

b) $\int \frac{1}{\sqrt{x^2-1}}\,dx = \int \frac{1}{\sqrt{\frac{1}{\operatorname{sen}^2 u}-1}} \cdot \frac{-\cos u}{\operatorname{sen}^2 u}\,du =$

$x = \frac{1}{\operatorname{sen} u}$

$dx = \frac{-\cos u}{\operatorname{sen}^2 u}\,du$

$= \int \frac{\operatorname{sen} u}{\cos u} \cdot \frac{-\cos u}{\operatorname{sen}^2 u}\,du = -\int \frac{1}{\operatorname{sen} u}\,du =$

$= -\ln\left|\tan\frac{x}{2}\right|$

ver problema 3

c) $\int \frac{1}{x\sqrt{x^2-1}}\,dx = \int \operatorname{sen} u \cdot -\frac{1}{\operatorname{sen} u}\,du = -u =$

$x = \frac{1}{\operatorname{sen} u}$

$= -\operatorname{Arcsen}\frac{1}{x}$

$dx = \frac{-\cos u}{\operatorname{sen}^2 u}\,du$

PROBLEMA 10:

a) $\int \sqrt{x^2-1}\,dx =$

$x = \cosh u$

$dx = \operatorname{senh} u\,du$

$\int \sqrt{\cosh^2 u - 1}\;\operatorname{senh} u\,du =$

$= \int \operatorname{senh}^2 u\,du = \int \cosh^2 u - 1\,du =$
teoría

$= \frac{1}{2}u + \frac{1}{4}\operatorname{senh} 2u - u =$

$= \frac{1}{2}\operatorname{Arccosh} x + \frac{1}{4}\operatorname{senh}(2\operatorname{Arccosh} x) =$

$= 1/2\,\operatorname{Arccosh} x + \frac{2}{4}\operatorname{senh}(\operatorname{Arccosh} x)\cosh(\operatorname{Arccosh} x)$

$= 1/2\,\operatorname{Arccosh} x + \frac{1}{2}x\left(\sqrt{\cosh^2(\operatorname{Arccosh} x) - 1}\right) =$

$= 1/2\,\operatorname{Arccosh} x + \frac{1}{2}x\sqrt{x^2-1}.$

PRIMITIVAS II

PROBLEMA 11: a) $\int \sqrt{x^2+1}\, dx$

$x = \operatorname{senh} u$

$du = \cosh u.$

$= \int \sqrt{\operatorname{senh}^2 u + 1}\ \cosh u = \int \cosh^2 u\, du$ = TEORÍA.

b) $\int \frac{dx}{\sqrt{x^2+1}} = (\operatorname{senh})^{-1}(x)$ → TABLAS

o BIEN $\int \frac{dx}{\sqrt{x^2+1}} = \int \frac{\cosh u}{\cosh u}\, du = \int du = u =$

$x = \operatorname{senh} u$

$du = \cosh u\, du$

$= \operatorname{Arc\,senh} x.$

PROBLEMA 12: a) $\int \frac{dx}{1+e^x}$ Problema 4)

b) $\int \frac{dx}{\sqrt{\sqrt{x}+1}}$ PROBLEMA 6)

c) $\int \sqrt{\operatorname{tn} x}\, dx = \int \sqrt{\operatorname{tn} x}\ \frac{2\sqrt{\operatorname{tn} x}}{(1+\operatorname{tn}^2 x)}\ \frac{1}{2\sqrt{\operatorname{tn} x}}\ (1+\operatorname{tn}^2 x)\, dx =$

$u = \sqrt{\operatorname{tn} x}$

$du = \frac{1}{2\sqrt{\operatorname{tn} x}}(1+\operatorname{tn}^2 x)\, dx$

$= \int u\ \frac{2u}{1+u^4}\, du = \int \frac{2u^2}{1+u^4}\, du =$ USAR EJERCICIO 5 b)

d) $\int \frac{\operatorname{Arctn} x}{1+x^2}\, dx = \frac{\operatorname{Arctn}^2 x}{2}$

INMEDIATA

$(\operatorname{Arctn} x)' = \frac{1}{1+x^2}$

e) $\int \frac{x \operatorname{arctn} x}{(1+x^2)^3} = -\frac{1}{4}\ \frac{1}{(1+x^2)^2} \cdot \operatorname{Arctn} x + \frac{1}{4}\underbrace{\int \frac{1}{(1+x^2)^3}\, dx}_{\text{USAR FÓRMULA DE REDUCCIÓN.}}$

PARTES

f) $\int \ln(\sqrt{1+x^2})\, dx = x \ln(\sqrt{1+x^2}) - \int x \left(\frac{2x}{2\sqrt{1+x^2}\sqrt{1+x^2}}\right) dx$

PARTES

$= x \ln(\sqrt{1+x^2}) - \int \frac{x^2}{1+x^2}\, dx = x \ln(\sqrt{1+x^2}) - \int 1\, dx + \int \frac{1}{1+x^2}\, dx$

PRIMITIVAS II

PROBLEMA 12: g) $\int x \ln(\sqrt{1+x^2})\,dx =$ PARTES

$= \frac{x^2}{2} \ln(\sqrt{1+x^2}) - \int \frac{x^2}{2} \frac{\cancel{2x}}{\cancel{2}\sqrt{1+x^2}\sqrt{1+x^2}}\,dx =$

$= \frac{x^2}{2} \ln(\sqrt{1+x^2}) - \frac{1}{2}\int \frac{x^3}{1+x^2}\,dx =$

$\begin{array}{r|l} x^3 & 1+x^2 \\ \hline -x^3-x & x \\ \hline -x & \end{array}$ $\quad \frac{x^2}{2} \ln(\sqrt{1+x^2}) - \frac{1}{2}\int \frac{x(1+x^2)-x}{1+x^2}\,dx$

$= \frac{x^2}{2} \ln(\sqrt{1+x^2}) - 1/2 \int x - \frac{2x}{2(1+x^2)}\,dx =$

$= \frac{x^2}{2} \ln(\sqrt{1+x^2}) - 1/4\, x^2 + \frac{1}{4} \ln(1+x^2)$

h) $\int \text{arc sen}\sqrt{x}\,dx = x \,\text{Arcsen}\sqrt{x} - \int x \frac{1}{\sqrt{1-x}} \cdot \frac{1}{2\sqrt{x}}\,dx =$
PARTES

$= x\,\text{Arcsen}\sqrt{x} - \frac{1}{2}\int \sqrt{\frac{x}{1-x}}\,dx$

VEAMOS $\int \sqrt{\frac{x}{1-x}}\,dx = \int \sqrt{\frac{\cos^2 u}{\sen^2 u}}\,(-2\cos u \,\sen u)\,du =$

$x = \cos^2 u$

$dx = -2\cos u\,\sen u.$

$= -2\int \cos^2 u\,d = -2\left[\int \frac{1+\cos 2u}{2}\,du\right] = -2\left(\frac{u}{2} + \frac{\sen 2u}{4}\right) =$

$= -\text{Arc}\cos\sqrt{x} - \frac{1}{2}\sen(2\,\text{Arc}\cos\sqrt{x})).$

i) $\int \left(\sen x \int_0^x \sen t\,dt\right)dx = \left(\int_0^x \sen t\,dt\right)^2 / 2$

$\left(\int_0^x \sen t\,dt\right)' = \sen x$

j) $\int \left(2x^3 \int_1^{x^2} t\,dt\right) = \left(\int_1^{x^2} t\,dt\right)^2 / 2$

$\left(\int_1^{x^2} t\,dt\right)' = 2x^3$

PRIMITIVAS II

PROBLEMA 12:] k) $\int \frac{x}{1+\operatorname{sen}x}\,dx =$

$$= \int \frac{x}{1+\operatorname{sen}x}\,\frac{1-\operatorname{sen}x}{1-\cos x}\,dx = \underbrace{\int \frac{x}{\cos^2 x}\,dx}_{1} - \underbrace{\int \frac{x\operatorname{sen}x}{\cos^2 x}\,dx}_{2} = \underset{\text{PARTES}}{\downarrow}$$

$$= \underbrace{x\operatorname{tg}x - \int \operatorname{tg}x}_{1} \; \underbrace{-\left[x\frac{1}{\cos x} - \int \frac{1}{\cos x}\,dx\right]}_{2}$$

$$= x\operatorname{tg}x + \ln\cos x - x\frac{1}{\cos x} - \int \frac{1}{\cos x}\,dx$$

$$\int \frac{1}{\cos x}\,dx \underset{u=\operatorname{tg}\frac{x}{2}}{=} \int \frac{2}{1+u^2}\cdot\frac{1}{\frac{1-u^2}{1+u^2}}\,du = \int \frac{2}{(1+u)(1-u)}\,du = \dots \text{ok.}$$

l) $\int \frac{e^x}{e^{5x}+e^x+1}\,dx \underset{\substack{u=e^x\\ du=e^x dx}}{=} \int \frac{du}{u^5+u+1} =$ ↓ PARECE COMPLICADO DESCOMPONER u^5+u+1

m) $\int e^{\operatorname{sen}x}\,\frac{x\cos^3 x - \operatorname{sen}x}{\cos^2 x}\,dx =$

$$= \int \underbrace{e^{\operatorname{sen}x}x\cos x}_{1} - \underbrace{e^{\operatorname{sen}x}\frac{\operatorname{sen}x}{\cos^2 x}}_{2}\,dx \underset{\text{PARTES}}{=} x e^{\operatorname{sen}x} \underbrace{- \cancel{\int e^{\operatorname{sen}x}\,dx}}_{1} -$$

$$\underbrace{- e^{\operatorname{sen}x}\frac{1}{\cos x} + \cancel{\int e^{\operatorname{sen}x}\frac{\cos x}{\cos x}\,dx}}_{2} = e^{\operatorname{sen}x}\left(x - \frac{1}{\cos x}\right)$$

n) $\int \frac{dx}{1+\operatorname{sen}^2 x}$ ESTÁ EN 7:] b)

ñ) $\int \frac{2+\sqrt{1+x}}{(x+1)^2-\sqrt{1+x}}\,dx \underset{\substack{y=\sqrt{1+x}\\ dy=\frac{1}{2\sqrt{1+x}}dx}}{=} \int \frac{2+y}{y^4-y}\cdot 2y\,dy = \int \frac{4+2y}{y^3-1}\,dy =$

$= \dots$ etc.

o) $\int \frac{1}{x}\left(\int_1^x \ln t\,dt\right)dx \underset{\text{PARTES}}{=} \ln x\int_1^x \ln t\,dt - \int \ln^2 x\,dx \underset{\text{PARTES}}{=} \ln x\int_1^x \ln t\,dt - x\ln^2 x + \int 2\ln x\,dx \underset{\text{PARTES}}{=}$

INTEGRALES II

PROBLEMA 13: $\displaystyle\int \frac{\sqrt{x^3}}{\sqrt[3]{x}+\sqrt[5]{x^2}}\,dx = \int \frac{x^{3/2}}{x^{1/3}+x^{2/5}}\,dx =$

$=$ $x = t^{30}$

OBSERVAMOS QUE $30 = 2\cdot 3\cdot 5 = \text{mcm}(2,3,5)$

$dx = 30\,t^{29}\,dt$

$$= \int \frac{t^{30\cdot 3/2}}{t^{30\cdot 1/3}+t^{30\cdot 2/5}}\cdot 30\,t^{29}\,dt = \int \frac{30\,t^{45+29}}{t^{10}+t^{12}}\,dt =$$

$$= 30\int \frac{t^{10}\,t^{64}}{t^{10}(1+t^2)}\,dt = 30\int \frac{t^{64}}{1+t^2}\,dt \quad \ldots\text{etc}$$

PARA RESOLVER $\displaystyle\int \frac{x^{q/r}+x^{s/t}}{x^{u/z}-x^{w/v}}\,dx =$ $x = y^{\text{mcm}(r,t,z,v)}$

AL HACER EL CAMBIO NOS QUEDA UNA INTEGRAL DE UNA FUNCIÓN RACIONAL.

$$\int \frac{dx}{\sqrt[3]{x}+\sqrt{x}} \underset{\substack{x = t^6\\ dx = 6t^5\,dt}}{=} \int \frac{6t^5\,dt}{t^{6/3}+t^{6/2}}\,dt = 6\int \frac{t^5}{t^2+t^3}\,dt =$$

$$= 6\int \frac{t^3}{1+t}\,dt = 6\int \frac{(1+t)(t^2-t+2)-2}{1+t}\,dt$$

$$\begin{array}{rl|l} t^3 & & 1+t \\ \hline -t^3-t^2 & & t^2-t+2 \\ -t^2+t & & \\ +t^2+t & & \\ 2t & & \\ -2t-2 & & \\ -2 & & \end{array}$$

$$= 6\int t^2-t+2\,dt - 12\int \frac{1}{1+t}\,dt = \ldots \text{etc}$$

INTEGRALES II

PROBLEMA 14: a) $\int (1+x)\sqrt{1+x+x^2}\,dx$

$$= \int \frac{1}{2}(1+1+2x)\sqrt{1+x+x^2}\,dx =$$

$$= \frac{1}{2}\underbrace{\int (1+2x)\sqrt{1+x+x^2}\,dx}_{\frac{2}{3}(1+x+x^2)^{3/2}} + \frac{1}{2}\int \sqrt{1+x+x^2}\,dx$$

¿ $\int \sqrt{1+x+x^2}\,dx$? $\int \sqrt{1+x+x^2}\,dx = \int \sqrt{\frac{3}{4}+\frac{1}{4}+x+x^2}\,dx =$

NOS ESTAMOS EN EJER 11 a)

$$= \int \sqrt{\frac{3}{4}+\left(\frac{1}{2}+x\right)^2}\,dx =$$

$$= \int \sqrt{\frac{3}{4}\left(1+\frac{(1/2+x)^2}{(\sqrt{3/4})^2}\right)} = \frac{\sqrt{3}}{2}\int \sqrt{1+\left(\frac{1/2+x}{\sqrt{3}/2}\right)^2}\,dx =$$

$$= \frac{\sqrt{3}}{2}\sqrt{1+\left(\frac{1+2x}{\sqrt{3}}\right)^2}\,dx = \frac{3}{4}\int \sqrt{1+y^2}\,dy =$$

$y = \frac{1+2x}{\sqrt{3}}$

$dy = \frac{2}{\sqrt{3}}\,dx$

b) $\int \sqrt{2ax-x^2}\,dx = \int \sqrt{a^2-a^2+2ax-x^2}\,dx =$

$$= \int \sqrt{a^2-(x-a)^2}\,dx = a\int \sqrt{1-\left(\frac{x-a}{a}\right)^2}\,dx =$$

$y = \frac{x-a}{a}$

$dy = \frac{1}{a}\,dx$

$$= a^2\int \sqrt{1-y^2}\,dy =$$

$y = \operatorname{sen} t$

$dy = \cos t\,dt$

$$= a^2\int \cos^2 t\,dt = a^2\left[\frac{t}{2}+\frac{\operatorname{sen} 2t}{4}\right] = a^2\left(\frac{\operatorname{Arcsen} y}{2}+\frac{\operatorname{sen} 2(\operatorname{Arcsen} y)}{4}\right)$$

TEORÍA

$$\underset{y=\frac{x-a}{a}}{=} a^2\left(\frac{\operatorname{Arcsen}\frac{x-a}{a}}{2}+\frac{\operatorname{sen}\left(2\left(\operatorname{Arcsen}\frac{x-a}{a}\right)\right)}{4}\right)$$

INTEGRALES II

PROBLEMA 15:

$$\int \operatorname{sen} x \, e^{-x} =$$

por partes (2 veces)

$$\int \frac{3x^2+7}{1+x+x^3}\,dx$$

Función racional
¿pero como descompongo $1+x+x^3$?

$$\int \frac{2x \operatorname{Arcsen}(x^2)}{\sqrt{1-x^4}}\,dx =$$

$$(\operatorname{Arcsen} x^2)' = \frac{1}{\sqrt{1-(x^2)^2}} \cdot 2x$$

$$= \frac{\operatorname{Arcsen}^2(x^2)}{2}$$