AVR PRÁCTICA-12

Nombre y apellidos..

1.- Mediante un cambio de variable resuelve:

$\int e^x \operatorname{sen} e^x \, dx \underset{\substack{u=e^x \\ du=e^x dx}}{=} \int \operatorname{sen} u \, du = -\cos u \underset{u=e^x}{=} -\cos e^x$

$\int \frac{e^x}{e^{2x}+2e^x+1} \, dx \underset{\substack{u=e^x \\ du=e^x dx}}{=} \int \frac{du}{u^2+2u+1} = \int \frac{1}{(u+1)^2} \, du = -\frac{1}{u+1} \underset{u=e^x}{=}$

$= -\frac{1}{e^x+1}$

$\int \frac{x}{\sqrt{1-x^4}} \, dx \underset{\substack{u=x^2 \\ du=2x\,dx}}{=} \frac{1}{2} \int \frac{1}{\sqrt{1-u^2}} \, du = \frac{1}{2} \operatorname{Arcsen} u \underset{u=x^2}{=} \frac{1}{2} \operatorname{Arcsen} x^2$

$\int x\sqrt{1-x^2} \, dx \underset{\substack{u=x^2 \\ du=2x\,dx}}{=} \frac{1}{2} \int \sqrt{1-u} \, du = -\frac{1}{2}\, 2\, \frac{(1-u)^{3/2}}{3} = \frac{-\sqrt{(1-u)^3}}{3} \underset{u=x^2}{=}$

$= -\frac{\sqrt{(1-x^2)^3}}{3}$

2.- Calcula las siguientes primitivas usando el cambio de variable que se indica:

$\int \frac{dx}{\sqrt{x^2-2}}$; ($x = \sqrt{2}\cosh u$).

$$\int \frac{dx}{\sqrt{x^2-2}} = \int \frac{\sqrt{2}\,\text{senh}\,u}{\sqrt{2\cosh^2 u - 2}}\,du =$$

$x = \sqrt{2}\cosh u$.

$dx = \sqrt{2}\,\text{senh}\,u\,du$

$$= \int \frac{\text{senh}\,u}{\sqrt{\cosh^2 u - 1}}\,du = \int \frac{\text{senh}\,u}{\text{senh}\,u}\,du = u = \text{Arc}\cosh\frac{x}{\sqrt{2}}$$

$\boxed{\cosh^2 u - \text{senh}^2 u = 1}$ $\quad u = \text{Arc}\cosh\frac{x}{\sqrt{2}}$

$\int \frac{x\,dx}{\sqrt{x+1}}$; ($t = \sqrt{x+1}$).

$$2\int \frac{x}{2\sqrt{x+1}}\,dx = 2\int t^2 - 1\,dt =$$

$dt = \frac{1}{2\sqrt{x+1}}\,dx$

$x = t^2 - 1$

$$= \frac{2}{3}t^3 - 2t = \frac{2}{3}\sqrt{(x+1)^3} - 2\sqrt{x+1}.$$

$t = \sqrt{x+1}$

$\int \sqrt{a^2+x^2}\,dx$; ($x = a\,\text{senh}\,t$, usa las propiedades del $\text{senh}\,x$).

$$\int \sqrt{a^2+x^2}\,dx = \int \sqrt{a^2 + a^2\,\text{senh}^2 t}\cdot a\cosh t\,dt = \int a^2\cosh^2 t\,dt =$$

$x = a\,\text{senh}\,t$

$dx = a\cosh t\,dt$

$\boxed{\cosh^2 t - \text{senh}^2 t = 1}$ $\quad \cosh^2 t = \frac{\cosh 2t + 1}{2}$

$$= a^2\int \frac{\cosh 2t + 1}{2} = a^2\left(\frac{\text{senh}\,2t}{4} + \frac{1}{2}t\right) = a^2\,\frac{\text{senh}(2\,\text{Arcsen}\frac{x}{a})}{4} + \frac{a^2}{2}\,\text{Arcsenh}\frac{x}{a}.$$

$t = \text{Arcsenh}\frac{x}{a}$

3.- Transforma en una integral racional $\int \frac{\sqrt{x^3}\,dx}{\sqrt[3]{x} + \sqrt[5]{x^2}}$ usando el cambio $x = t^{30}$. ¿Qué cambio hay que hacer para transformar en una integral racional $\int \frac{x^{q/p} + x^{s/r}}{x^{u/t} - x^{w/v}}dx$, siendo p, q, r, s, t, u, v y w números naturales? Resuelve $\int \frac{dx}{\sqrt[3]{x} + \sqrt{x}}$.

Ss $x = t^{30}$
$dx = 30t^{29}dt$

$$\int \frac{\sqrt{x^3}}{\sqrt[3]{x} + \sqrt[5]{x^2}}\,dx = \int 30\frac{t^{45}}{t^{10} + t^{12}}\,t^{29}dt = 30\int \frac{t^{64}}{1+t^2}\,dt.$$

Ss $M = \text{mcm}\{p, r, t, v\}$, el cambio $x = t^M$ $\quad dx = Mt^{M-1}$ hace que $(x^{q/p}) = t^{Mq/p}$, ahora $Mq/p \in \mathbb{N}$ ... etc.

$$\int \frac{dx}{\sqrt[3]{x} + \sqrt{x}} = \int \frac{6t^5}{t^2 + t^3}\,dt = 6\int \frac{t^3}{1+t}\,dt = 6\int t^2 - t + 1 - \frac{1}{1+t}\,dt$$

$x = t^6$
$dx = 6t^5$

$t^3 \;|\; t+1$
$-1 \;|\; t^2 - t + 1$

$$= 6\left(\frac{t^3}{3} - \frac{t^2}{2} + t - \ln|1+t|\right) = 2\sqrt{x} - 3\sqrt[3]{x} + 6\sqrt[6]{x} + \ln\frac{1}{(1+\sqrt[6]{x})^6}$$

$t = \sqrt[6]{x}$