AVR PRÁCTICA-13

Nombre y apellidos..

1.- Calcula $\int \frac{x^3+3x^2-4x+4}{x^3+x^2-5x+3}dx.$ $= \int \frac{x^3+x^2-5x+3 \; +2x^2+x+1}{x^3+x^2-5x+3}\,dx$

$= \int 1 + \frac{2x^2+x+1}{x^3+x^2-5x+3}\,dx = \int 1\,dx + \int \frac{2x^2+x+1}{(x+3)(x-1)^2}\,dx$

X=1 ES RAIZ
Y X=-3 ES RAIZ
DE x^3+x^2-5x+3

$\int 1\,dx = x$

$\int \frac{2x^2+x+1}{(x+3)(x-1)^2}\,dx = \int \frac{A}{x+3} + \frac{B}{x-1} + \frac{C}{(x-1)^2}\,dx =$

DESCOMPOSICIÓN EN FRACCIONES SIMPLES

$\frac{A}{x+3} + \frac{B}{x-1} + \frac{C}{(x-1)^2} = \frac{A(x-1)^2 + B(x+3)(x-1) + C(x+3)}{(x+3)(x-1)^2} =$

$= \frac{Ax^2 - 2Ax + A + Bx^2 + 2Bx - 3B + Cx + 3C}{(x+3)(x-1)^2} = \frac{(A+B)x^2 + (-2A+2B+C) + A - 3B + C}{(x+3)(x-1)^2}$

$\Rightarrow \begin{cases} 2 = A+B \\ 1 = -2A+2B+C \\ 1 = A-3B+3C \end{cases} \Rightarrow \begin{matrix} 2 = A+B \\ 1 = -4+4B+C \\ 1 = 2-4B+3C \end{matrix} \Rightarrow \begin{matrix} 2 = A+B \\ 5 = 4B+C \\ 4 = 4C \end{matrix} \Rightarrow \begin{matrix} A=1 \\ B=1 \\ C=1 \end{matrix}$

$= \int \frac{1}{x+3} + \frac{1}{x-1} + \frac{1}{(x-1)^2}\,dx = \ln(x+3) + \ln(x-1) - \frac{1}{x-1}$

ASÍ $x + \ln(x+3) + \ln(x-1) - \frac{1}{x-1}$ ES LA PRIMITIVA QUE BUSCAMOS.

2.- Calcula $\int \frac{1}{\operatorname{sen}^2 x}dx.$ $= \int \frac{1}{\left(\frac{2u}{1+u^2}\right)^2} \cdot \frac{2}{1+u^2}\,du =$

$u = \tan\frac{x}{2}$

$\operatorname{sen} x = \frac{2u}{1+u^2}$

$dx = \frac{2}{1+u^2}\,du$

$= \int \frac{(1+u^2)^{\cancel{2}}}{4u^2} \cdot \frac{2}{\cancel{1+u^2}}\,du = \int \frac{1}{2u^2} + \frac{1}{2}\,du = -\frac{1}{2u} + \frac{1}{2}u =$

$= -\frac{1}{2}\frac{\cos\frac{x}{2}}{\operatorname{sen}\frac{x}{2}} + \frac{1}{2}\frac{\operatorname{sen}\frac{x}{2}}{\cos\frac{x}{2}} = \frac{1}{2}\left[\frac{\operatorname{sen}\frac{x}{2}}{\cos\frac{x}{2}} - \frac{\cos\frac{x}{2}}{\operatorname{sen}\frac{x}{2}}\right] = \frac{\operatorname{sen}^2\frac{x}{2} - \cos^2\frac{x}{2}}{2\cos\frac{x}{2}\operatorname{sen}\frac{x}{2}} =$

$u = \tan\frac{x}{2}$

$= \frac{-\cos x}{\operatorname{sen} x}$

3.- Calcula $\int \frac{1}{x\sqrt{x^2-1}}dx.$ $=$ $\int \operatorname{sen} u \frac{1}{\sqrt{\frac{1}{\operatorname{sen}^2 u}-1}} \cdot \frac{-\cos u}{\operatorname{sen}^2 u}\, du =$

$x = \frac{1}{\operatorname{sen} u}$

$dx = \frac{-\cos u}{\operatorname{sen}^2 u}\, du.$

$= \int \cancel{\operatorname{sen}^2 u} \frac{1}{\sqrt{1-\operatorname{sen}^2 u}} \cdot \frac{-\cos u}{\cancel{\operatorname{sen}^2 u}}\, du = -u =$

$\frac{1}{x} = \operatorname{sen} u.$

$= -\operatorname{Arc\,sen} \frac{1}{x}.$

4.- Calcula $\int \operatorname{arc\,sen} \sqrt{x}dx.$ $\underset{\text{PARTES}}{=}$ $x \operatorname{Arc\,sen} \sqrt{x} - \int x \frac{1}{\sqrt{1-x}} \cdot \frac{1}{2\sqrt{x}}\, dx$

$= x \operatorname{Arc\,Sen} \sqrt{x} - \frac{1}{2}\int \sqrt{\frac{x}{1-x}}\, dx$

VEAMOS $\int \sqrt{\frac{x}{1-x}}\, dx =$ $\int \sqrt{\frac{\cos^2 u}{1-\cos^2 u}} (-2\cos u \operatorname{sen} u)\, du =$

$x = \cos^2 u.$

$dx = -2\cos u \operatorname{sen} u$

$= -2\int \frac{\cos u}{\cancel{\operatorname{sen} u}} \cdot \cos u \cancel{\operatorname{sen} u}\, du = -2\int \cos^2 u\, du =$

$= -2\int \frac{1+\cos 2u}{2}\, du = -2\left[\frac{u}{2} + \frac{\operatorname{sen} 2u}{4}\right] = -u - \frac{\operatorname{sen} 2u}{2}$

$= -\operatorname{Arc\,cos} \sqrt{x} - \frac{1}{2}\operatorname{sen}(2\operatorname{Arc\,cos}(\sqrt{x})) =$

$u = \operatorname{Arc\,cos} \sqrt{x}$

$= -\operatorname{Arc\,cos} \sqrt{x} - \cos(\operatorname{Arc\,cos} \sqrt{x}) \operatorname{sen}(\operatorname{Arc\,cos} \sqrt{x}) =$

$= -\operatorname{Arc\,cos} \sqrt{x} - \sqrt{x} \cdot \sqrt{1-\cos^2(\operatorname{Arc\,cos} \sqrt{x})} = -\operatorname{Arc\,cos} \sqrt{x} - \sqrt{x-x^2}$

$\int \operatorname{arc\,sen} \sqrt{x}\, dx = x \operatorname{Arc\,sen} \sqrt{x} + \frac{1}{2}\operatorname{Arc\,cos} \sqrt{x} + \frac{1}{2}\sqrt{x-x^2}.$