ESPACIOS DE ORLICZ

(Retículos de Banach de funciones. En rojo espacios simétricos)

$f : (\Omega, \mu) \to \mathbb{R} \qquad (\Omega, \mu)$ espacio de medida; $\; f$ medible.

$$L^p(\Omega) = \{ f : \int_\Omega |f(t)|^p d\mu(t) < \infty \}$$

$\mu(\Omega) < \infty$	$\mu(\Omega) = \infty$	$\Omega = \mathbb{N},\ \mu(n) = 1$	$\Omega = \mathbb{N},\ \mu(n) = w_n > 0$
$L^p(0,1)$	$L^p(0,\infty)$	l_p	$l_p(w)$

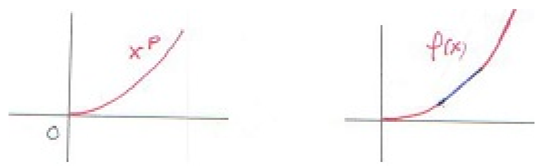

Figura 1. Funciones convexas.

Sea $\varphi : [0,\infty) \to [0,\infty)$, creciente, convexa, $\varphi(0) = 0$ y $\lim_{x\to\infty} \varphi(x) = \infty$ (**función de Orlicz**).

$$L^\varphi(\Omega) = \{ f : \ \exists r > 0 \quad \text{de modo que} \quad \int_\Omega \varphi(r|f(t)|) d\mu(t) < \infty \}.$$

$\mu(\Omega) < \infty$	$\mu(\Omega) = \infty$	$\Omega = \mathbb{N},\ \mu(n) = 1$	$\Omega = \mathbb{N},\ \mu(n) = w_n > 0$
$L^\varphi(0,1)$	$L^\varphi(0,\infty)$	l_φ	$l_\varphi(w)$

Sea $M : \Omega \times [0,\infty) \to [0,\infty)$, $M(t,\cdot)$ función de Orlicz y $M(\cdot, x)$ función medible (**función de Musielak-Orlicz**).

$$L^M(\Omega) = \{ f : \ \exists r > 0 \quad \text{de modo que} \quad \int_\Omega M(t, r|f(t)|) d\mu(t) < \infty \}.$$

$\mu(\Omega) < \infty$	$\mu(\Omega) = \infty$	$\Omega = \mathbb{N},\ \mu(n) = 1$	$\Omega = \mathbb{N},\ \mu(n) = w_n > 0$
$L^M(0,1)$	$L^M(0,\infty)$	l_{M_n}	$l_{M_n}(w)$

Sea $M(t,x) = x^{p(t)}$ (**función exponente variable**).

$$L^{p(\cdot)}(\Omega) = \{ f : \ \exists r > 0 \quad \text{de modo que} \quad \int_\Omega (r|f(t)|)^{p(\cdot)} d\mu(t) < \infty \}.$$

$\mu(\Omega) < \infty$	$\mu(\Omega) = \infty$	$\Omega = \mathbb{N},\ \mu(n) = 1$	$\Omega = \mathbb{N},\ \mu(n) = w_n > 0$
$L^{p(\cdot)}(0,1)$	$L^{p(\cdot)}(0,\infty)$	l_{p_n}	$l_{p_n}(w)$

Si $N(t,x) \le M(t,x)$ y $\mu(\Omega) < \infty$ sin átomos, entonces

$$L^\infty(\Omega) \hookrightarrow L^M(\Omega) \hookrightarrow L^N(\Omega) \hookrightarrow L^1(\Omega)$$

inclusiones continuas.