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DOMINATION BY POSITIVE DISJOINTLY STRICTLY SINGULAR OPERATORS

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ABSTRACT. We prove that each positive operator from a Banach lattice E to a Banach lattice F with a disjointly strictly singular majorant is itself disjointly strictly singular provided the norm on F is order continuous. We prove as well that if $S : E \to E$ is dominated by a disjointly strictly singular operator, then S^2 is disjointly strictly singular.

1. INTRODUCTION

The classical problem of domination for positive compact operators on Banach lattices was solved by Dodds and Fremlin ([5]) for a pair of positive operators $0 \leq S \leq T$ defined on a Banach lattice E with order continuous dual norm and taking values in a Banach lattice F with order continuous norm: we can guarantee that S is compact if T is so. A full answer to this problem was given by Aliprantis and Burkinshaw in [2], namely if $E = F$ and either the norm on E or the norm on E' is order continuous, then the compactness of T is inherited by the operator S^2. They also show that for an arbitrary Banach lattice, T compact always implies S^3 compact. More recently Wickstead has given in [14] not only sufficient but necessary conditions for the problem of domination for positive compact operators to have a solution.

The problem of domination for weakly compact operators was first considered by Abramovich in [1], giving a positive solution for a Banach lattice E and a KB-space F. Later on, a general result was obtained by Wickstead in [13] where it was shown that the problem has a positive answer if and only if either the norm on E' or F is order continuous. Again Aliprantis and Burkinshaw settled the question by considering the case $E = F$ and showing that T weakly compact implies S^2 weakly compact ([3]).

The aim of this paper is to study the problem of domination for positive disjointly strictly singular operators. We recall that an operator T between a Banach lattice E and a Banach space Y is said to be *disjointly strictly singular* (DSS) if there is no disjoint sequence of non-null vectors $(x_n)_n$ in E such that the restriction of T to the subspace $[x_n]$ spanned by the vectors $(x_n)_n$ is an isomorphism. DSS operators were introduced by Rodríguez-Salinas and the second author in [9]. This class of operators, a generalization of the class of strictly singular (or Kato) operators, is a

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