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DOMINATION BY POSITIVE STRICTLY SINGULAR OPERATORS

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1. *Introduction*

The problem of domination for positive compact operators between Banach lattices was solved by Dodds and Fremlin in **[6]**: given a Banach lattice E with order continuous dual norm, an order continuous Banach lattice F and two positive operators $0 \leqslant S \leqslant T : E \longrightarrow F$, the operator S is compact if T is. A similar problem has been considered in the class of weakly compact operators by Abramovich **[1]** and in a general form by Wickstead **[26]**. Precisely, Wickstead's result shows that the operator S is weakly compact if T is whenever one of the following two conditions holds: either E' is order continuous or F is order continuous. When it comes to Dunford–Pettis operators, Kalton and Saab **[15]** have proved that the operator S is Dunford–Pettis if T is, provided that the Banach lattice F is order continuous. On the other hand, Aliprantis and Burkinshaw settled the problem of domination for compact **[2]** and weakly compact **[3]** endomorphisms (that is, the case when $E = F$). For example, they proved that if either the norm on E or the norm on E' is order continuous, then the compactness of T is inherited by the power operator S^2. Also, they showed that, for E an arbitrary Banach lattice, T being compact always implies that S^3 is compact. More recently, Wickstead studied converses for the Dodds–Fremlin and Kalton–Saab theorems in **[27]** and for the Aliprantis–Burkinshaw theorems in **[28]**.

The main purpose of this paper is to study the domination problem for positive strictly singular operators between Banach lattices. Recall that a bounded operator T between two Banach spaces X and Y is said to be *strictly singular* (or Kato) if the restriction of T to any infinite-dimensional (closed) subspace of X is not an isomorphism. The class of all strictly singular operators is a closed operator ideal (in the sense of Pietsch **[20]**), which contains the ideal of all compact operators and is not stable by duality **[25]**. A well known characterization of strict singularity is the following: an operator $T : X \longrightarrow Y$ is strictly singular if and only if every infinite-dimensional subspace M of X contains an infinite-dimensional subspace N of M such that the restriction of T to N is compact (cf. **[11, 16]**). The class of all strictly singular operators between X and Y is denoted by $\mathscr{S}\mathscr{S}(X, Y)$.

In general, the problem of domination in the class of strictly singular operators has a negative answer, as Examples 3.12 and 3.14 show. We present here natural sufficient conditions on the Banach lattices E and F that yield positive results on domination. One of the difficulties we run into is that we cannot take the natural

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