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Disjointly strictly singular operators and interpolation*

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Dedicated to Professor Rodriguez-Salinas on the occasion of his 70th birthday

Interpolation properties of the class of disjointly strictly singular operators on Banach lattices are studied. We also give some applications to compare the lattice structure of two rearrangement invariant function spaces. In particular, we obtain suitable analytic characterisations of when the inclusion map between two Orlicz function spaces is disjointly strictly singular.

1. Introduction

An operator T between two Banach spaces X and Y is called *strictly singular* (or *Kato*) if it fails to be an isomorphism on any infinite-dimensional subspace. The class of strictly singular operators has been extensively studied. In particular, this class is a closed operator ideal (in the sense of Pietsch [**21**]) and its behaviour under the real interpolation method has been studied by Beucher [**3**] and Heinrich [**9**].

Disjointly strictly singular operators have been introduced in [**11**] as a natural extension of the concept of strict-singularity for operators defined on Banach lattices. Thus, this class of operators is quite useful in the study of the lattice structure of Banach lattices. For instance, in getting 'non-natural' projections: If there exists a Riesz operator from a Banach lattice X to L^p, $1 \leqq p < \infty$, which is not disjointly strictly singular, then X has an l^p-complemented *sublattice*.

An operator T from a Banach (or quasi-Banach) lattice X to a Banach (or quasi-Banach) space Y is called *disjointly strictly singular* (DSS) if there is no disjoint sequence of non-null vectors (x_n) in X such that the restriction of T to the subspace $[x_n]$ spanned by the vectors (x_n) is an isomorphism. Clearly, every strictly singular operator is DSS. However, the converse is not true in general (e.g. the inclusion map $L^p(0, 1) \hookrightarrow L^q(0, 1)$, for $0 < q < p < \infty$, is DSS but it is not strictly singular). On the other hand, in the case when X has a Schauder basis of disjoint vectors, the class of DSS operators defined on X coincides with the class of stringly singular operators.

One of the aims of this paper is to study the interpolation properties of the class

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