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DISJOINTLY STRICTLY-SINGULAR INCLUSIONS BETWEEN REARRANGEMENT INVARIANT SPACES

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1. *Introduction*

A linear operator between two Banach spaces X and Y is *strictly-singular* (or *Kato*) if it fails to be an isomorphism on any infinite dimensional subspace. A weaker notion for Banach lattices introduced in **[8]** is the following one: an operator T from a Banach lattice X to a Banach space Y is said to be *disjointly strictly-singular* if there is no disjoint sequence of non-null vectors $(x_n)_{n\in\mathbb{N}}$ in X such that the restriction of T to the subspace $[(x_n)_{n=1}^{\infty}]$ spanned by the vectors $(x_n)_{n\in\mathbb{N}}$ is an isomorphism. Clearly every strictly-singular operator is disjointly strictly-singular but the converse is not true in general (consider for example the canonic inclusion $L^q[0,1] \hookrightarrow L^p[0,1]$ for $1 \leqslant p < q < \infty$). In the special case of considering Banach lattices X with a Schauder basis of disjoint vectors both concepts coincide. The notion of disjointly strictly-singular has turned out to be a useful tool in the study of lattice structure of function spaces (cf. **[7–9]**). In general the class of all disjointly strictly-singular operators is not an operator ideal since it fails to be stable with respect to the composition on the right.

The aim of this paper is to study when the inclusion operators between arbitrary rearrangement invariant function spaces $E[0,1] \equiv E$ on the probability space $[0,1]$ are disjointly strictly-singular operators.

In the context of Orlicz spaces $L^{\varphi}(\mu)$ characterizations of when the inclusion operator $L^{\varphi}(\mu) \hookrightarrow L^{\psi}(\mu)$ is disjointly strictly-singular have been given by Kalton in **[10]** for sequence spaces and in **[6, 8]** for function spaces. The case of Lorentz spaces $\Lambda(\phi)$ and Marcinkiewicz spaces $M(\phi)$ has been recently studied by Astashkin in [**1**]. In [**8**, Theorem 3.5] it is proved that for any L^p-space on $[0,1]$, $1 < p < \infty$, there exists an Orlicz space $L^F[0,1] = L^F$ with $L^F \hookrightarrow L^p$ such that the inclusion operator $L^F \hookrightarrow L^p$ is not a disjointly strictly-singular operator. One of the purposes of this paper is to extend these results to the general setting of rearrangement invariant spaces.

A well known result of Grothendieck states that the inclusion operator $L^{\infty} \hookrightarrow L^p$ is strictly-singular for $1 \leqslant p < \infty$ (cf. [**18**, p. 111]). More generally it holds that for any rearrangement invariant space $E \neq L^{\infty}$ the canonical inclusion operator $L^{\infty} \hookrightarrow E$ is always strictly-singular and hence disjointly strictly-singular (cf. [**15**]). On the other hand the inclusion operator $E \hookrightarrow L^1$ is always disjointly strictly-

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