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On Musielak-Orlicz spaces isomorphic to Orlicz spaces

Abstract. It is shown that every separable Musielak-Orlicz sequence space l^M (also called a modular sequence space) is isomorphic to some Orlicz space $L^\varphi(\Omega,\Sigma,\mu)$ over a σ-finite purely atomic measure space. Examples of separable Musielak-Orlicz function spaces $L^M(0,1)$ non-isomorphic to any Orlicz space $L^\varphi(\Omega,\Sigma,\mu)$ are given.

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The aim of this note is to study the question of whether every Musielak-Orlicz space L^M is isomorphic to some Orlicz space $L^\varphi(\Omega,\Sigma,\mu)$ over a certain measure space (Ω,Σ,μ). This question is affirmatively answered for the discrete case and negatively for the function case. Namely, we show that every separable Musielak-Orlicz sequence space l^M is isomorphic to a suitable Orlicz space $L^\varphi(\Omega,\Sigma,\mu)$ over a σ-finite purely atomic measure space (Ω,Σ,μ), where the measure of the atoms tends to ∞. On the other hand, we give natural examples of separable Musielak-Orlicz function spaces L^M which are not isomorphic to any Orlicz space $L^\varphi(\Omega,\Sigma,\mu)$ over a measure space (Ω,Σ,μ).

First, let us give some notations and definitions. Let (Ω,Σ,μ) be a measure space and M be a *Musielak-Orlicz function*, i.e. a function $M:\Omega\times\mathbb{R}^+\to\mathbb{R}^+$ satisfying the following two conditions: (i) For every $s\in\Omega$ the function $M(s,\cdot):\mathbb{R}^+\to\mathbb{R}^+$ is non-decreasing, convex, continuous at 0 and $M(s,0)=0$. (ii) For every $r\in\mathbb{R}^+$ the function $M(\cdot,r):\Omega\to\mathbb{R}^+$ is Σ-measurable. The *Musielak-Orlicz space* $L^M(\Omega,\Sigma,\mu)$ is the vector space of the $\mathbb{K}$-scalar Σ-measurable functions verifying that there exists $t>0$ such that

$$I_M\Big(\frac{f}{t}\Big)=\int_\Omega M\Big(s,\frac{|f(s)|}{t}\Big)\,d\mu<\infty,$$

endowed with the norm $\|f\|_M=\inf\{t>0: I_M(\frac{f}{t})\leqslant 1\}$ (c.f. [7], [8]). When

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