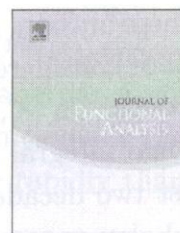




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Weak compactness in variable exponent spaces

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ABSTRACT

This paper shows necessary and sufficient conditions on subsets of variable exponent spaces $L^{p(\cdot)}(\Omega)$ in order to be weakly compact. Useful criteria are given extending Andô results for Orlicz spaces. As application, we prove that all separable variable exponent spaces are weakly Banach-Saks. Also, L -weakly compact and weakly compact inclusions between variable exponent spaces are studied.

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1. Introduction

The classical Riesz-Kolmogorov compactness theorem in L_p -spaces ($1 \leq p < \infty$) has been recently extended to the context of variable exponent Lebesgue spaces $L^{p(\cdot)}(\Omega)$ (or Nakano spaces) by Górká and Macios [12], Górká and Bandaliyev [11] and Dong et al. [8].

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