AVR PRÁCTICA-11

Nombre y apellidos...

1.- Resuelve las siguientes integrales:

- $\int 4x^6 + 3x^2 + 1 dx =$ $4\int x^6 dx + 3\int x^2 dx + \int dx = 4\frac{x^7}{7} + x^3 + x + K$
- $\int (3x-2)^2 dx =$ $\frac{(3x-2)^3}{9}$
- $\int \frac{1}{\sqrt{1-x^2}} dx =$ $\text{Arc Sen } x$
- $\int \cosh x \cosh(\operatorname{senh} x) dx =$ $\text{Senh}(\text{senh } x)$ OBSERVAR QUE $(\text{senh } x)' = \text{cosh } x$
- $\int \frac{dx}{\sqrt{x-1}+\sqrt{x+1}} =$ $\int \frac{\sqrt{x-1} - \sqrt{x+1}}{(x-1)-(x+1)} = \frac{1}{2}\int \sqrt{x-1} - \frac{1}{2}\int \sqrt{x+1} = \frac{1}{2}\left(\frac{2(x-1)^{3/2}}{3} - 2\frac{(x+1)^{3/2}}{3}\right)$
- $\int \frac{e^x + e^{2x} + e^{3x}}{e^{4x}} dx =$ $\int e^{-3x} + e^{-2x} + e^{-x} dx = \frac{e^{-3x}}{-3} + \frac{e^{-2x}}{-2} - e^{x}$
- $\int \tan^2 x \, dx =$ $\int 1 + tn^2 x - 1 \, dx = tn\, x - x$
- $\int \frac{dx}{1+\operatorname{sen} x} =$ $\int \frac{1 - \text{sen}\, x}{1 - \text{sen}^2 x} = \int \frac{1}{\cos^2 x} - \frac{\text{sen}\, x}{\cos^2 x} dx = tn\, x - \frac{1}{\cos x}$

2.- Resuelve por partes: $\int x^3 e^{x^2} dx$ y $\int \frac{\ln(\ln x)}{x} dx$.

- $\int x^3 e^{x^2} dx = x^2 \frac{e^{x^2}}{2} - \frac{1}{2}\int 2x\, e^{x^2} dx =$

PARTES: $\frac{2xe^{x^2}}{2} = \left(\frac{e^{x^2}}{2}\right)'$

$= x^2 \frac{e^{x^2}}{2} - \frac{1}{2} e^{x^2} = \frac{1}{2} e^{x^2}\left[x^2 - 1\right]$

- $\int \frac{1}{x} \ln(\ln x) dx = \ln x \; \ln(\ln x) - \int \cancel{\ln x} \frac{1}{\cancel{\ln x}} \cdot \frac{1}{x} dx$

PARTES: $(\ln x)' = \frac{1}{x}$

$= \ln x \; \ln(\ln x) - \ln x = \ln x\left[\ln(\ln x) - 1\right].$

3.- Demuestra las siguientes fórmulas de reducción:

*) $\displaystyle\int \cos^n x dx = \frac{1}{n}\cos^{n-1}\operatorname{sen} x + \frac{n-1}{n}\int \cos^{n-2} x dx$, para $n > 2$ y par.

$$\int \cos x \cos^{n-1} x \, dx \underset{\text{PARTES}}{=} \operatorname{sen} x \cos^{n-1} x - \int \operatorname{sen} x \left((n-1)\cos^{n-2} x\,(-\operatorname{sen} x)\right) dx$$

$$= \operatorname{sen} x \cos^{n-1} x + (n-1)\int (1-\cos^2 x)\cos^{n-2} x \, dx$$

DESPEJANDO $\quad n\int \cos^n x \, dx = \operatorname{sen} x \cos^{n-1} x + (n-1)\int \cos^{n-2} x \, dx$ y ASÍ

$$\int \cos^n x \, dx = \frac{1}{n}\cos^{n-1} x \operatorname{sen} x + \frac{n-1}{n}\int \cos^{n-2} x \, dx$$

) $\displaystyle\int \frac{dx}{(x^2+1)^n} = \frac{1}{2n-2}\frac{x}{(x^2+1)^{n-1}} + \frac{2n-3}{2n-2}\int \frac{dx}{(x^2+1)^{n-1}}$ (Indicación:** usa que $\frac{1}{(x^2+1)^n} = \frac{1}{(x^2+1)^{n-1}} - \frac{x^2}{(x^2+1)^n}$).

$$\int \frac{dx}{(x^2+1)^n} = \int \frac{1}{(x^2+1)^{n-1}} dx - \int \frac{x^2}{(1+x^2)^n} dx \underset{\text{PARTES EN LA ÚLTIMA INTEGRAL}}{=} \int \frac{1}{(x^2+1)^{n-1}} dx$$

$$- \left[-\frac{1}{2n-2}\frac{x}{(1+x^2)^{n-1}} + \frac{1}{2n-2}\int \frac{1}{(1+x^2)^{n-1}} dx \right] =$$

$$= \frac{1}{2n-2}\frac{x}{(1+x^2)^{n-1}} + \left(1 - \frac{1}{2n-2}\right)\int \frac{1}{(1+x^2)^{n-1}} dx =$$

$$= \frac{1}{2n-2}\frac{x}{(1+x^2)^{n-1}} + \frac{2n-3}{2n-2}\int \frac{1}{(x^2+1)^{n-1}} dx$$

4.- Calcula $\int \cos^2 x dx$ y $\int \operatorname{sen}^2 x dx$ (**Indicación:** usa que $\cos 2x = \cos^2 x - \operatorname{sen}^2 x$).

$$\cos 2x = \cos^2 x - \operatorname{sen}^2 x = \cos^2 x - (1-\cos^2 x) = 2\cos^2 x - 1$$

DESPEJANDO $\quad \cos^2 x = \dfrac{\cos 2x + 1}{2}$

•) $$\int \cos^2 x \, dx = \int \frac{\cos 2x + 1}{2} dx = \frac{1}{2}\int \cos 2x \, dx + \frac{1}{2}\int dx =$$

$$= \frac{1}{2}\frac{\operatorname{sen} 2x}{2} + \frac{1}{2}x = \frac{\operatorname{sen} 2x + 2x}{4}$$

••) $$\int \operatorname{sen}^2 x \, dx = \int 1 - \cos^2 x \, dx = \int 1 \, dx - \int \cos^2 x \, dx =$$

$$= x - \left[\frac{\operatorname{sen} 2x + 2x}{4}\right] = \frac{2x - \operatorname{sen} 2x}{4}$$