LINEAR COMBINATIONS OF ITERATES OF BLASCHKE PRODUCTS AND PEANO CURVES

JUAN J. DONAIRE

ABSTRACT. Given an inner function f on the unit disk $\mathbb{D}$, let f^n denote the n-th iterate of f. In some sense, the funtions (f^n) behave as a sequence of independent random variables on the boundary of the unit disk. This phenomenon is similar to the context of lacunary series, that is power series of the form $\sum a_k z^{n_k}$, with $n_{k+1}/n_k \geq q > 1$.

Recently, A. Nicolau and O. Soler have obtained several results which exhibit the parallelism existing between the behaviour of lacunary series and the behaviour of linear combinations of the iterates of an inner function, that is, the behaviour of $\sum a_n f^n$.

As in the case of sums of independent random variables, the asymptotic behaviour of $\sum a_n f^n$ is governed by the convergence of $\sum |a_n|^2$. If last series is divergent, then $\sum a_n f^n$ has radial limit almost nowhere, but if it is finite, then $\sum a_n f^n$ belongs to BMOA, so it has nontangential limit almost everywhere and this limit coincides with $\sum_{n=1}^{\infty} a_n f^n(\zeta)$ a.e. $\zeta \in \partial\mathbb{D}$.

The purpose of this talk is to explore a little bit more the case $\sum |a_n|^2 < \infty$, following the ideas of M. Weiss.

We will show that in this case, if $\sum |a_n| = \infty$, any $w \in \mathbb{C}$ is the nontangential limit of $\sum a_n f^n$ at some point $\zeta \in \partial\mathbb{D}$.

On the other hand we will see that if $\sum |a_n| < \infty$, but the convergence is slow enough (in some sense), then the image of the unit circle under $\sum a_n f^n$ is a Peano curve.

This work is a joint collaboration with Artur Nicolau.

UNIVERSITAT AUTÒNOMA DE BARCELONA
E-mail address: donaire@mat.uab.cat