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DIFFERENTIABLE EXTENSIONS WITH UNIFORMLY CONTINUOUS DERIVATIVES

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Abstract. Let X be a Hilbert space, $E \subset X$ an arbitrary subset, ω a modulus of continuity, and $f : E \to \mathbb{R}$, $G : E \to X$ two functions. What are the optimal conditions on (f, G) for the existence of a function $F \in C^{1,\omega}(X)$ such that $(F, \nabla F)$ agrees with (f, G) on E?

In this talk, we give a full solution to this problem. By means of explicit formulas, we construct an extension operator $(f, G) \mapsto (F, \nabla F)$ whose norm has almost sharp bounds, which are dimension-free in the particular case $X = \mathbb{R}^n$. In addition, if f is bounded (resp. G is bounded), then so is F (resp. F is Lipschitz). Moreover, $(F, \nabla F)$ depends continuously on the given data (f, G). Similar results are true on superreflexive Banach spaces.

Also, we will see how our solution to the problem in the $C^{1,1}$ case leads us to a constructive proof of Kirszbraun's theorem via simple and explicit formulas.

This is joint work with Daniel Azagra.

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