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COMPACTNESS AND VARIATIONAL ANALYSIS

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ABSTRACT. One of the most important achievements in optimisation in Banach space theory is the James's weak compactness theorem. It says that a weakly closed subset A of a Banach space E is weakly compact if, and only if, every linear form $x^* \in E^*$ attains its supremum over A at some point of A. We will concentrate on recent extensions of James's theorem. Among them we shall study the following one: Let A be a closed, convex, bounded and not weakly compact subset of a Banach space E. Let us fix a convex and weakly compact subset D of E, a functional $z_0^* \in E^*$ and $\epsilon > 0$. Then there is a linear form $x_0^* \in B_{p_W}(z_0^*, \epsilon)$, i.e.

$$|x_0^*(d) - z_0^*(d)| < \epsilon$$

for all $d \in D$, which does not attain its supremum on A. Moreover, if $z_0^*(A) < 0$ the same can be provided for the former non attaining linear form : $x_0^*(A) < 0$ (one sided James's theorem). We shall present a multiset version of James's theorem too.

James's theorem is strongly connected with variational principles and optimisation theory. Indeed we will study unbounded versions of the former results. The first case should be the epigraph of a weakly lower semicontinuous function

$$\alpha : E \longrightarrow (-\infty, +\infty],$$

where we shall see that $\partial\alpha(E)$ has non empty interior for the Mackey topology if, and only if, the level sets $\{\alpha \leq c\}$ are weakly compact. We will see that reflexive spaces are the natural frame to develop variational analysis, we will study robust representation theorem for risk measures $\rho : \mathbb{L}^\infty \longrightarrow \mathbb{R}$, and we characterize when they verify the Lebesgue dominated convergence theorem.

We shall finish our talk with $\sigma(E^*, E)$ versions of the formed results. One-sided and unbounded versions of classical Godefroy's results will be presented and new applications considered. All former results have been obtained in joint works with F. Delbaen.

REFERENCES

[1] F. Delbaen and J. Orihuela: Mackey constraints for James's compactness theorem and risk measures. *Journal of Mathematical Analysis and Applications 2019. 485(1):123764*

[2] F. Delbaen and J. Orihuela: On the range of subdifferential in non reflexive Banach spaces. *J. Functional Analysis 2021.281(2):108915*

[3] F. Delbaen and J. Orihuela: A multiset version of James's Theorem. *J. Functional Analysis 2022.*

[4] F. Delbaen and J. Orihuela: A nonlinear James's w^*-compactness theorem. *Preprint 2022.*

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