Connections between the solvability of the Dirichlet problem and flatness of the boundary for PDE in sets without connectivity

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- Let us give a common answer.

Contents

Section 1

Introduction

Boundedness of SIOs

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- Given K, we associate a Singular Integral Operator (SIO):

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- $K \in \mathscr{C}^\infty(\mathbb{R}^n \setminus \{0\})$,
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do we have that

$$T : L^2(E) \longrightarrow L^2(E) \text{ is bounded?}$$

SIOs and UR sets

- How wild can E be so that

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Theorem (Nazarov, Tolsa, Volberg - 2014)

E is ADR:

the Riesz transforms are bounded in $L^2(E)$ $\iff$ E is UR.

UR sets are quantitatively flat

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- Big Pieces of Lipschitz Images:

$$\exists \epsilon > 0, M > 0 \quad \text{s.t.} \quad \forall x \in E, r > 0 \quad \exists f_{x,r} : B(0,r) \longrightarrow \mathbb{R}^n$$

$$\text{s.t.} \quad \|f_{x,r}\|_{\text{Lip}} \leq M \quad \text{s.t.} \quad \frac{|E \cap B(x,r) \cap f_{x,r}(B(0,r))|}{r^n} \geq \epsilon.$$

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Theorem (David, Semmes - 1991)

E is ADR:

E has Very Big Pieces of Bilipchitz Images $\iff$ *E is UR.*

- Very Big Pieces of Bilipschitz Images: same as before, with bilipschitz maps and densities $\geq 1 - \epsilon$.

Section 2

Connections with elliptic PDE

Elliptic measure

- Let $\Omega \subset \mathbb{R}^n$ be open and $L = -\operatorname{div}(A\nabla)$ with
  - $A = A(\cdot) \in L^\infty(\Omega)$,
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$$\begin{cases} Lu &= 0 \quad \text{in } \Omega \\ u &= f \quad \text{on } \partial\Omega. \end{cases} \tag{1}$$

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Theorem (Application of Riesz Rep. Thm.)

The elliptic measure $\{\omega_L^X\}_{X\in\Omega}$ is a family of probabilities in $\partial\Omega$ s.t.

$$u(X) = \int_{\partial\Omega} f(y)\,d\omega_L^X(y), \qquad X \in \Omega,$$

is the solution of (1) if $f \in \mathscr{C}(\partial\Omega)$.

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$$\left(⨍_Q \left(\frac{d\omega_L}{d\sigma} \right)^p d\sigma \right)^{1/p} \lesssim ⨍_Q \frac{d\omega_L}{d\sigma} d\sigma.$$

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$\partial\Omega$ is Lipschitz $\implies \omega_{-\Delta} \in RH_2(\sigma)$.

- However...

Theorem (Caffarelli, Fabes, Kenig - 1981)

$\exists L$ *s.t.* $\omega_L \perp \sigma$ *in the unit ball.*

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$\partial\Omega$ Lipschitz + L symmetric:

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- L^p-solvability for L: solvability, with interior estimates, of

$$\begin{cases} Lu = 0 & \text{in } \Omega \\ u = f \in L^p(\partial\Omega) & \text{on } \partial\Omega. \end{cases}$$

Section 3

In sets with good connectivity

Definitions: ADR and Corkscrews

- $\partial\Omega$ is **ADR** (Ahlfors-David regular): for any $x \in \partial\Omega$,

$0 < r \lesssim \mathrm{diam}(\partial\Omega)$:

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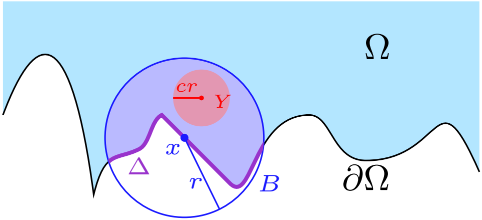

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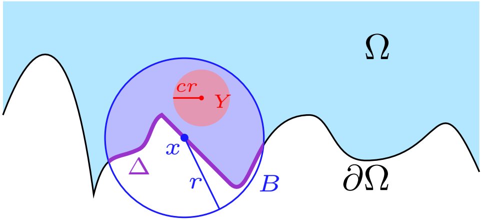

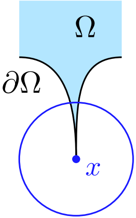

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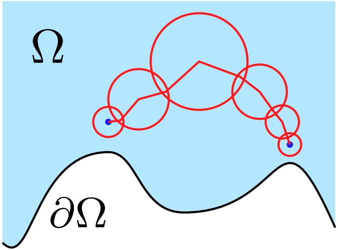

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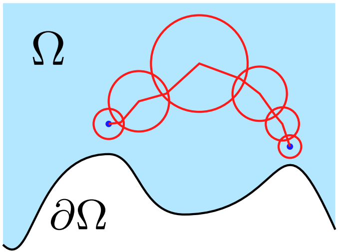

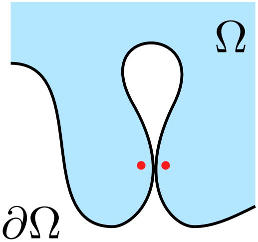

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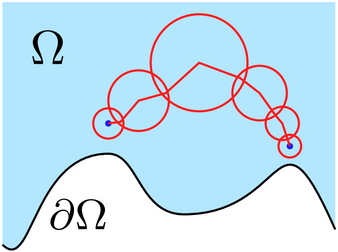

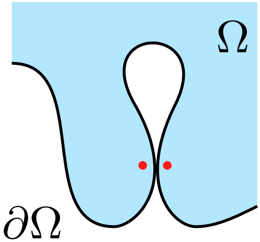

- Ω is **1-sided CAD** (chorded-arc domain) [Jerison, Kenig - 1982]: $\partial\Omega$ is ADR + Ω has interior corkscrews + Ω has Harnack chains.

In 1-sided CAD

Theorem (Compendium of papers)

If Ω is 1-sided CAD
(i.e. $\partial\Omega$ is ADR + Ω has Corkscrews + Ω has Harnack chains):

$$\omega_L \in A_\infty(\sigma) \Longleftrightarrow L \text{ sat. CME}$$

For general L: [Cavero, Hofmann, Martell, Toro - 2020].

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For $-\Delta$: [Hofmann, Martell, Uriarte-Tuero - 2014], [Azzam, Hofmann, Martell, Nyström, Toro - 2017], David, Jerison, Semmes...
For nice L: [Hofmann, Martell, Mayboroda, Toro, Zhao - 2021].
Precedents: [Kenig, Pipher - 2001], [Hofmann, Martell, Toro - 2017].
For general L: [Cavero, Hofmann, Martell, Toro - 2020].

Nice operators

- "Nice" operators $\rightsquigarrow$ Fefferman-Kenig-Pipher operators L:
  - $A \in \mathrm{Lip}_{\mathrm{loc}}(\Omega)$,
  - $|\nabla A| \operatorname{dist}(\cdot, \partial\Omega) \in L^\infty$ and
  - $$\sup_{\substack{x\in\partial\Omega \\ 0<r<\operatorname{diam}(\partial\Omega)}} \frac{1}{r^n} \iint_{B(x,r)\cap\Omega} |\nabla A|^2 \operatorname{dist}(\cdot, \partial\Omega) dX < \infty$$

Section 4

Removing connectivity

Problems without connectivity and general operators

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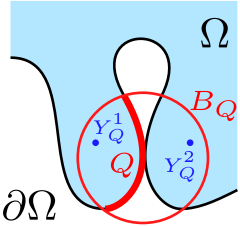

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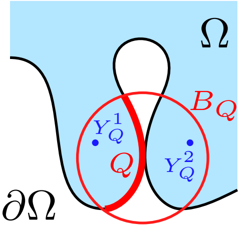

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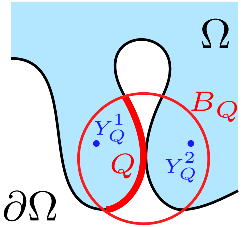

- L behaves well there?
- We do not have the "full" CFMS estimate. Only

$$\frac{G_L(X, Y_Q^i)}{\mathrm{dist}(Y_Q^i, \partial\Omega)} \lesssim \frac{\omega_L^X(Q)}{\sigma(Q)}$$

What was known without connectivity

Theorem (Compendium of papers)

$\partial\Omega$ is ADR + Ω has Corkscrews:

$$L^p\text{-solvability} \overset{L=-\Delta}{\iff} \omega_L \in A_\infty^{\text{weak}}(\sigma) \underset{\text{connectivity}}{\overset{\text{always}}{\rightleftarrows}} L \text{ sat. } CME \underset{L \text{ symm.}}{\overset{L \text{ good}}{\rightleftarrows}} \partial\Omega \text{ UR}$$

For $-\Delta$: [Hofmann, Martell, Mayboroda - 2016], [Hofmann, Le, Martell, Nyström - 2017], [Azzam, Hofmann, Martell, Mourgoglou, Tolsa - 2020].
For "nice" and "symmetric" L: [Azzam, Garnett, Mourgoglou, Tolsa - 2021].

Our contribution

Theorem (Cao, H., Martell - 2022)

$\partial\Omega$ is ADR + Ω has Corkscrews. TFAE:

1. *ω_L admits a corona decomposition.*
2. *G_L admits a corona decomposition.*
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- Corona for G_L: $\mathbb{D} = \sqcup \mathbf{S}$ and

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Perturbation result

Theorem (Cao, H., Martell - 2022)

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Corona decomposition for ω_L, for G_L and CME for L are stable under some perturbations of L.

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- Perturbations in the sense of Fefferman-Kenig-Pipher:

$$\sup_{\substack{x\in\partial\Omega \\ 0<r<\operatorname{diam}(\partial\Omega)}} \frac{1}{\sigma(B(x,r)\cap\partial\Omega)} \iint_{B(x,r)\cap\Omega} \sup_{Y\in B(X,\frac{\delta(X)}{2})} \frac{|A_0(Y)-A_1(Y)|^2}{\operatorname{dist}(X,\partial\Omega)} dX < \infty.$$

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Corollary (Cao, H., Martell - 2022)

$\partial\Omega$ is ADR + Ω has Corkscrews + L close to $-\Delta$:

L satisfies Partial CME $\Longrightarrow$ $\partial\Omega$ is UR.

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$\Updownarrow$

Corona for G_L

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Partial CME

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$$\omega_L \in A_\infty^{\text{weak}}(\sigma) \overset{\text{always}}{\Longrightarrow} \begin{array}{c} \text{Corona for } \omega_L \\ \Updownarrow \\ \text{Corona for } G_L \\ \Updownarrow \\ \text{Partial CME} \end{array}$$

Overview

- $\partial\Omega$ is ADR + Ω has Corkscrews:

$$\omega_L \in A_\infty^{\text{weak}}(\sigma) \overset{\text{always}}{\Longrightarrow} \begin{array}{c} \text{Corona for } \omega_L \\ \Updownarrow \\ \text{Corona for } G_L \\ \Updownarrow \\ \text{Partial CME} \end{array} \overset{L \text{ good}}{\Longleftrightarrow} \partial\Omega \text{ UR}$$

Overview

- $\partial\Omega$ is ADR + Ω has Corkscrews:

$$L^p\text{-solvability for } L \overset{??}{\leftrightsquigarrow} \omega_L \in A_\infty^{\text{weak}}(\sigma) \underset{??}{\overset{\text{always}}{\rightleftarrows}} \begin{matrix}\text{Corona for } \omega_L \\ \Updownarrow \\ \text{Corona for } G_L \\ \Updownarrow \\ \text{Partial CME}\end{matrix} \overset{L \text{ good}}{\Longleftrightarrow} \partial\Omega \text{ UR}$$

Connections between the solvability of the Dirichlet problem and flatness of the boundary for PDE in sets without connectivity

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XX Encuentro de Análisis Real y Complejo

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