Program

4th Spanish Young Topologists Meeting

Madrid, June 29th-July 2nd, 2015

Minicourse I: Branched Coverings

José María Montesinos Amilibia (UCM)

1. Examples of branched coverings. 2-bridge knots and lens spaces.
2. 3-Manifolds as branched coverings over knots.
3. Universal groups and 3-manifolds: hyperbolic manifolds.
4. Open manifolds, wild knots and branched coverings.

Minicourse II: Some Applications of Algebraic Topology

Aniceto Murillo Mas (UMA)

1. Topological complexity.
2. Rational topological complexity.
3. Persistent homology.

Realization of manifolds as leaves

Ramón Barral Lijó (USC)

A foliated space or lamination X is a metric space which is partitioned into manifolds ("leaves") that locally glue like a bundle of hyperplanes in Euclidean space. A lot of geometrical constructions like tangent bundles or Riemannian metrics can be generalized to foliated spaces. If X is compact and has a Riemannian metric then all of its leaves have uniformly bounded geometry. The question that inspires our work is "Can every manifold of bounded geometry be realized as a leaf of some compact foliated space?" We show this conjecture to be true using embeddings into some universal foliated space.

Non metrizable topologies having countable dual group

Daniel de la Barrera Mayoral (UCM)

The study of duality in the realm of locally convex space has been an inspiring issue for many mathematicians in the twentieth century. In the 40s, Vilenkin introduced the notion of quasi-convex subsets of an abelian topological group. This brought the opportunity of studying duality in the context of locally quasi-convex topological groups, and obtain counterparts of important results of locally convex spaces for locally quasi-convex groups.

In particular, one interesting problem is the existence of a maximum among all the locally quasi-convex topologies on a group G, having as a dual group a fixed subgroup $H \leq Hom(G, T)$. Such a topology could be denominated a Mackey topology in the duality (G, H). So far this is an open problem.

For the particular case of the group $\mathbb{Z}$ of the integers we describe a family of topologies which have no non-trivial convergent sequences and having a countable dual group.

Cohomology: Massey products and transferred structures

Francisco Belchí (UMA)

In this talk I am going to relate the Massey products (a generalization of the cup product that can prove that the Borromean rings cannot be pulled apart) with certain structures that can be transferred to cohomology up to homotopy.

On homological stability for configuration spaces in compact manifolds

Federico Cantero Morán (WWU Münster)

In 1975, McDuff found that the ith homology group of the configuration space of k points in a non-compact manifold is independent of the number of points provided that i is small relative to k (for instance, $k \geq 2i$). In this talk I will address this question when the manifold is compact. This is a joint work with Martin Palmer (arXiv:1406.4916).

On rational sectional category

José Gabriel Carrasquel Vera (UC Louvain)

We will generalise many of the classical results for rational Lusternik-Schnirelmann category to the more general context of sectional category. These results will be applied to the study of M. Farber's topological complexity of rational spaces.

Legendrian knots: A Tale of Two Cusps

Roger Casals Gutiérrez (ICMAT-CSIC)

In this talk we introduce Legendrian knots and prove two results in the higher dimensional theory of Legendrian knots. We discuss their role as attaching spheres in the construction of symplectic handlebodies and deduce the existence of certain symplectic cobordisms between contact manifolds. The necessary definitions of contact and symplectic structures will be introduced via simple examples.

From utility to multi-utility: a generalized Debreu's Open Gap Lemma is needed

Asier Estevan Muguerza (UPNA-NUP)

In the talk we generalize the Debreu's Open Gap Lemma. Given any subset of the real line, this lemma guarantees the existence of a strictly increasing real function such that all the gaps of the image of the subset are open. Now we extend it to the case of n subsets of the real line and study the existence of a strictly increasing real function such that all the gaps of the image of each set are open. This function does not exist in general so, we characterize the cases in which it exists. This new generalization is used in order to obtain new results on continuous representations of biorders, for example.

Operads and enveloping algebras

Xabier García Martinez (USC)

We solve the problem of extending the universal enveloping algebra of Lie algebras to the category of n-Lie algebras. We regard the solution from the point of view operad theory.

Homotopy idempotent functors on infinite loop spaces and Postnikov pieces

Alberto Gavira-Romero (UDC)

Given a homology theory h_*, A. K. Bousfield introduced homological localization as an idempotent endofunctor in the category of topological spaces isolating in the space the information detected by h_*. Given a pointed connected space A, A. K. Bousfield and E. Dror-Farjourn formalized the concept of A-homotopy of a pointed space X as the study of $\operatorname{map}_*(A, X)$. They introduced the A-nullification and A-cellularization as idempotent endofunctors in the category of pointed topological spaces, the first one kills all the information from A in X, and the second one isolates the essential information from A to X.

In this talk we compare the effect of nullification and cellularization functors in A-homotopy combined with homological localizations. Using this comparison, we obtain a description of the cellularization of infinite loop spaces and Postnikov pieces with respect to torsion spaces.

Dynamical zeta functions and symmetric products

Luis Hernández Corbato (IMPA)

In the same fashion the local degree of a map measures the multiplicity of a zero of a map, the fixed point index, $i(f,p) \in \mathbb{Z}$ measures the multiplicity of a fixed point p of a map $f : X \to X$. For a given fixed point p, we can study the sequence of integers $i(f^n, p)$, $n \geq 1$, encoded in an exponential generating function, which in the literature is often referred to as Lefschetz zeta function. Via a uniqueness result for "dynamical" zeta functions, this object is linked with the symmetric products of f and X. For $n \geq 1$, the n–symmetric product of X is the quotient of X^n under the action of the permutation group and the original map f naturally induces a map in the symmetric product.

Gauge theory on 8-manifolds

Juan José Madrigal Martínez (UCM)

Since the seminal paper of S. Donaldson and R. Thomas, there has been huge work for extending the results of anti-self-duality in 4 dimensions to manifolds of higher dimensions. Among those, the theory of Spin (7)-instantons on 8-manifolds is the one that translates more closely the philosophy behind moduli spaces of 4-dimensional instantons. In this talk we will explain this whole scenario and some of the ideas behind (Yang-Mills functional, moduli spaces. . .).

Highly connected inflexible manifolds

David Méndez Martínez (UDC)

The existence of spaces with a trivial group of self-homotopy equivalences, the so-called homotopically-rigid spaces, was thought to be quite rare [3]. In [1] infinitely many non homotopically rigid spaces were constructed. They all have in common their connectivity as they are based on an example of Arkowitz-Lupton. In this talk we will show how to generalize this example in order to obtain infinitely many rigid spaces that are highly connected.

Those spaces are proven to be the rationalization of simply-connected inflexible manifolds. Our main motivation is to develop tools to prove the existence of strongly inflexible manifolds, which is related with a question raised by Gromov [2].

[1] C. Costoya, A. Viruel, Every finite group is the group of self-homotopy equivalences of an elliptic space, *Acta Math.* **213** (2014), no. 1, 4962.
[2] M. Gromov, Metric structures for Riemannian and non-Riemannian spaces, *Progress in Mathematics* **152** (1999), Birkhäuser.

[3] D.W. Kahn, Realization problems for the group of homotopy classes of self-equivalences. *Math. Ann.*, **220** (1976), 3746.

Embeddings in hyperspaces

Diego Mondéjar Ruiz (UCM)

For every topological space X, we can consider the hyperspace 2^X, the set of non-empty closed sets of X. It can be endowed with several topologies. For example, if X is a metric space, the Hausdorff extended metric is the most common. In this talk, we review the upper semifinite topology for hyperspaces and we establish some embedding results for Alexandrov spaces concerning it. Given a set X, we propose a very simple hyperspace which turns out to be universal for the Čech homology of every possible metric making X compact.

Topological triviality of families of map germs from $\mathbb{R}^2 \to \mathbb{R}^2$

Juan Antonio Moya Pérez (Aix-Marseille University)

We consider a 1-parameter unfolding of f, that is, a map germ $F : (\mathbb{R}^2 \times \mathbb{R}, 0) \to (\mathbb{R}^2 \times \mathbb{R}, 0)$ of the form $F(x,t) = (f_t(x), t)$ and such that $f_0 = f$. We are interested in the topological triviality of F, which means that it is topologically equivalent as an unfolding to the constant unfolding. We prove that F is topologically trivial if it is excellent in the sense of Gaffney [1] and moreover, the family of the discriminant curves $\Delta(F)$ is a topologically trivial deformation of $\Delta(f)$. This can be seen as a real version of the same result obtained by Gaffney for complex analytic map germs [Th. 9.9,1]

[1] T. Gaffney, Polar multiplicities and equisingularity of map germs, *Topology* **32** (1993) 185–223.
[2] J.A. Moya-Pérez and J.J. Nuño-Ballesteros, The link of a finitely determined map germ from $\mathbb{R}^2$ to $\mathbb{R}^2$, *J. Math. Soc. Japan* **62** (2010) No. 4, 1069–1092.
[3] J.A. Moya-Pérez and J.J. Nuño-Ballesteros, Topological triviality of families of map germs from $\mathbb{R}^2$ to $\mathbb{R}^2$, *Journal of Singularities* **6** (2012) 112–123.

Blowing up Double Points

Guillermo Peñafort Sanchís (UV)

If f is a curvilinear map (i.e. its differential matrix is not too degenerate), then f is stable if and only if its multiple point spaces are smooth manifolds. Otherwise, they are singular spaces. We introduce the Blowing-Up double points and show that they are regular for all stable (possibly non curvilinear) maps.

LS category and eigenvalues

María José Pereira Sáez (UDC)

Given a topological space X, its LS category $\text{cat}(X) + 1$, is the least number of open sets contractible in X covering the whole space. In T. Ganea's list of problems on *numerical* *homotopy invariants* the first one is "to compute the category of the familiar manifolds: Stiefel manifolds, Lie groups, etc. In the case of symplectic groups, it is only known that cat $Sp(2) = 3$ [2] and cat $Sp(3) = 5$ [3]. N. Iwase and M. Mimura also proved that cat $Sp(n) \geq n+2$ when $n \geq 3$ but none of these proofs is easy.

A useful technique in the case of the unitary group, used by Singhof in [3] and by Mimura in [1], is to obtain contractible open sets associated to eigenvalues. We will prove that it is not possible to extend this method in the quaternionic setting. Indeed, even when cat $Sp(2) = 3$, given any four unitary quaternions, we are always able to find a matrix in $Sp(2)$ such that it has these four quaternions as left eigenvalues.

[1] M. Mimura, K. Sugata, On the Lusternik-Schnirelmann category of symmetric spaces of classical type. *Geometry & Topology Monographs* **13** (2008) 323–334.
[2] P. Schweitzer, Secondary Cohomology Operations Induced by the Diagonal Mapping. *Topology* **3** (1964-1965) 143–148.
[3] W. Singhof, On the Lusternik-Schnirelmann Category of Lie Groups I. *Math. Z.* **145** (1975) 11–116.
[4] L. Fernández-Suárez,; A. Gómez-Tato, J. Strom, D. Tanré, The Lyusternik-Schnirelmann category of $Sp(3)$. *Proc. Am. Math. Soc.* **132**, No. 2 (2004) 587–595.

What is an Engel structure?

Álvaro del Pino Gómez (ICMAT-CSIC)

Engel structures are maximally non-integrable 2-distributions in 4 manifolds. They are very related to contact structures in 3-folds and even contact structures in 4-folds. It is known by work of Gromov and Eliashberg that while the latter are fully flexible - and therefore can be classified by their algebraic topology - the former present rigidity and therefore constitute interesting objects of study.

A natural question then is whether Engel structures are flexible or not. Is there a meaningful notion of "Engel topology"? The aim of the talk will be to introduce all these notions from the perspective of h-principle.

Alternative and pseudoalternating links: a conjecture by Louis Kauffman

Marithania Silvero Casanova (US)

In 1983 Louis Kauffman introduced the family of alternative links. He conjectured that this class was identical to the class of pseudoalternating links, introduced by Mayland and Murasugi in 1976.

In this talk we show that both families are equal in the particular case of knots of genus one. However, Kauffman's Conjecture does not hold in general, as we also show by finding two counterexamples. In the way we work with the family of homogeneous links, introduced by Peter Cromwell; the techniques used here allow us to give an explicit characterization of homogeneous knots of genus 1.

On the centralizer of generic braids

Dolores Valladares García (US)

In order to solve the conjugacy problem in braid groups one computes a finite set of elements, called ultra summit set, which is a complete invariant of the conjugacy class of the braid involved. We will show that the ultra summit set of a generic braid exhibits a simple structure. That is, the conjugating elements involved in the ultra summit set of a braid X will correspond to the simple factors of the normal form of X. In this case, we will be able to describe an explicit minimal set of generators for the centralizer of X. Hence we can determine the centralizer of a generic braid and we also give an algorithm which computes this set.

The triple point spectrum of closed orientable 3-manifolds

Rubén Vigara Benito (Centro Universitario de la Defensa)

A natural trend in the study of closed 3-manifolds is to sort them using some kind of complexity measure. Classical examples of these complexity measures are the Heegaard genus and the Matveev complexity. Recently, other measures have been introduced using filling Dehn surfaces, as the surface complexity proposed by G. Amendola or the Montesinos complexity and the triple point spectrum introduced in recent works of the author with Á. Lozano Rojo.