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Équations aux dérivées partielles/*Partial Differential Equations*

Space localization and uniqueness of solutions of a quasilinear parabolic system arising in semiconductor theory

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Abstract. A degenerate parabolic system consisting in two continuity equations for densities of charged particles and in the Poisson equation for an electric potential is considered. We show the finite speed of propagation, a waiting time property for the vacuum (null) sets and a property of formation of vacuum. The proofs are based on energy methods. Furthermore, some results on the uniqueness of solutions are proved by using different duality methods.

Localisation spatiale et unicité des solutions d'un système parabolique quasi-linéaire dans la théorie des semi-conducteurs

Résumé. *On considère un système parabolique dégénéré consistant en deux équations de continuité pour les densités des particules chargées et en l'équation de Poisson pour le potentiel électrique. On démontre que la vitesse de propagation est finie. On établit une propriété du temps d'attente pour les ensembles du vide et une propriété de formation du vide. Les démonstrations sont basées sur des méthodes d'énergie. En outre, on montre des résultats d'unicité des solutions en utilisant différentes méthodes de dualité.*

Version française abrégée

On considère le problème (1) modélisant le transport des particules chargées dans un semi-conducteur influencé par un champ électrique. Le modèle est très voisin de ce qui apparaît dans la théorie de la filtration biphasique (*voir* [1]). Si la fonction r satisfait (2), le problème est de type dégénéré. Les solutions qui satisfont $n = 0$ ou $p = 0$ localement sont appelées *solutions de vide*. Le modèle est dérivé

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d'un modèle de dérive-diffusion pour les semi-conducteurs, basé sur la statistique de Fermi-Dirac (*voir* [6]). L'existence des solutions satisfaisant (3) a été montrée dans [5].

La première partie de la Note est consacrée à l'étude du comportement des solutions de vide. Si $n(0) = 0$ dans une boule $B_{\rho_o}(x_o) \subset\subset \Omega$, et si l'énergie locale de n et p dans le cylindre $B_{\rho_o}(x_o) \times (0,T)$ est suffisamment petite, alors $n = 0$ et $p = 0$ pour $x \in B_{\rho(t)}(x_o)$, $t \in (0,T_1)$, avec $T_1 > 0$ et $\rho(t)$ une certaine fonction décroissante positive. Si on fait de plus l'hypothèse (5) (et on suppose $p > 0$ localement), alors $n = 0$ dans le cylindre $B_{\rho_o}(x_o) \times (0,T_2)$ avec $T_2 > 0$. Le troisième résultat donne l'existence de solutions de vide sous l'hypothèse (5) même si $n(0)$ est globalement positif.

Pour les démonstrations on utilise une méthode d'énergie présentée dans [2]. On commence avec une formule locale d'intégration par parties (7), où P est le domaine d'intégration défini par (6). La nouveauté de nos estimations réside dans le traitement des termes convectifs dans (1). Les estimations de I_1, I_2 et I_3 sont obtenues comme dans [2]. On emploie un lemme d'interpolation et l'inégalité de Hölder. L'estimation de l'intégrale I_4 est faite de plusieurs manières. Pour la démonstration du Théorème 1 on prend comme P un cône tronqué. Alors la deuxième intégrale sur la surface latérale de P à droite de (10) est négative pour $-\sigma > 0$ suffisamment grand. On prend un cylindre P dans la démonstration du Théorème 2 et un paraboloïde dans le Théorème 3. Dans ces deux cas le terme convectif dans I_4 peut être estimé en utilisant le terme $R(n,p)n^\alpha$. On arrive à certaines inégalités différentielles (*voir*, par exemple, (8)) ce qui permet de conclure.

Des résultats d'unicité pour le problème plus général $(\mathcal{P})$ sont présentés dans la deuxième partie de la Note. Le Théorème 4 montre l'unicité sous trois hypothèses distinctes à l'aide de techniques de dualité. Dans le cas de la partie (i), on adapte la technique de [1], dans (ii) le résultat est valide pour r non strictement croissante mais sous la condition (12) (on utilise un argument de [7]). Finalement, dans la partie (iii) on prend une technique inspirée de [3] pour des solutions fortes telles que $n, p \in L^\infty(0,T; W^{1,q}(\Omega))$ ($q = 1$ si $N = 1$ et $q > N$ si $N > 1$) et pour $r \in C^2(0,\infty)$ convexe strictement croissante.

1. The model

The following initial boundary value problem describes the transport of charged particles in a semiconductor subject to an exterior electric field:

$$
(1)\qquad \begin{cases} n_t - \nabla\cdot(\nabla r(n) - n\nabla V) = -R(n,p) & \text{in } Q_T, \\ p_t - \nabla\cdot(\nabla r(p) + p\nabla V) = -R(n,p) & \text{in } Q_T, \\ \Delta V = n - p - C & \text{in } Q_T, \\ r(n) = r(n_D), \quad r(p) = r(p_D), \quad V = V_D & \text{on } \Sigma_{DT}, \\ \nabla r(n)\cdot\nu = \nabla r(p)\cdot\nu = \nabla V\cdot\nu = 0 & \text{on } \Sigma_{NT}, \\ n(x,0) = n_I(x), \quad p(x,0) = p_I(x) & \text{in } \Omega. \end{cases}
$$

Here, n is the electron density, p the (positively charged) hole density, V the electric potential, $C = C(x)$ the doping profile of the semiconductor device, and ν the exterior normal. The recombination and generation of electrons and holes is modelled by $R(n,p)$. The function r satisfies:

$$
(2)\qquad r \in C^1(\mathbb{R}), \quad r'(0) = 0, \quad \text{and} \quad r'(s) \geq 0 \text{ for any s} \in \mathbb{R},
$$

and can be interpreted as the pressure of the particles. Usually, $r(s) := |s|^\alpha \text{ sign } (s)$ with $\alpha \in (1, 5/2)$ (*see* [6]).

The (bounded) semiconductor domain is denoted by $\Omega \subset \mathbb{R}^d$, $d \leq 3$. Its (regular) boundary consists of two disjoint subsets Γ_D and Γ_N. In [5] and [4] it is proved that there exists a weak solution (n, p, v) to (1) such that

$$
\begin{aligned}
&0 \leq n, p \in L^\infty(Q_T) \cap H^1(0,T;\mathcal{V}') \cap \mathcal{C}([0,T];L^1(\Omega)), \\
&\nabla r(n), \nabla r(p) \in L^2(Q_T), \quad V \in L^\infty(0,T;W^{2,p}(\Omega)) \quad \forall p < \infty,
\end{aligned} \tag{3}
$$

where $\mathcal{V}'$ is the dual space of $\mathcal{V} = \{u \in H^1(\Omega) \,:\, u = 0 \text{ on } \Gamma_D\}$ and where it is assumed that the function R satisfies conditions of monotonicity (*see* [5]) or Lipschitz continuity (*see* [4]), and $C \in L^\infty(\Omega)$, $0 \leq n_I, p_I \in L^\infty(\Omega)$, $0 \leq n_D, p_D \in L^\infty(Q_T) \cap H^1(Q_T)$, $V_D \in L^\infty(0,T;W^{2,\infty}(\Omega))$. In this Note we first perform an analysis of the occurrence of free boundaries and their properties by means of some energy methods (*see* [2]) and afterwards we describe three uniqueness results obtained under suitable assumptions by using different techniques (*see* [1], [7], and [3]).

2. Space localization results

We assume throughout this section that $r(s) := |s|^\alpha \text{sign}\,(s)$ with $\alpha \in (1,\infty)$. We define the local energy $D_n(P)$ of the density n for a domain $P \subset \Omega \times (0,T)$ by:

$$
D_n(P) = \frac{1}{\alpha+1} \sup_{(x,\tau)\in P} |n(x,\tau)|^{\alpha+1} + \int_P |\nabla n^\alpha|^2 dx d\tau. \tag{4}
$$

THEOREM 1. – (Finite speed of propagation) *Let $x_o \in \Omega$, $0 < \rho_o <$ dist $(x_o, \partial\Omega)$, and $T > 0$. Assume that $n_I = 0$, $p_I = 0$ in $B_{\rho_o}(x_o)$, and that*

$$
R(u,v)(u^\alpha + v^\alpha) \geq -\kappa_R(u^{\alpha+1} + v^{\alpha+1}) \qquad \textit{for all } u, v \geq 0,
$$

with $\kappa_R \geq 0$. Then, there exist $M > 0$ and $T_1 \in (0,T)$ such that, if $D_n(B_{\rho_o}(x_o) \times (0,T)) + D_p(B_{\rho_o}(x_o) \times (0,T)) \leq M$, then $n(x,t) = 0$, $p(x,t) = 0$ for a.e. $x \in B_{\rho(t)}(x_o)$, $t \in (0,T_1)$, where $\rho(t) = \rho_o - ct$ and $c > 0$ is some constant independent of t.

For the next theorems we need a stronger condition on R, namely that for $\omega \subset \Omega$ and $b > 0$, we have

$$
R(n(x,t),p(x,t)) \geq b n(x,t)^\delta \quad \text{a.e. } (x,t) \in \omega \times (0,T), \qquad \alpha + \delta < 2. \tag{5}
$$

THEOREM 2. – (Waiting time) *Let $x_o \in \Omega$, $0 < \rho_o < \rho_1 <$ dist$(x_o, \partial\Omega)$, and $T > 0$. Assume that* (5) *holds for $\omega = B_{\rho_o}(x_o)$. Suppose furthermore that $\|n_I\|^{\alpha+1}_{\alpha+1,B_\rho(x_o)} \leq \varepsilon_o(\rho - \rho_o)^\beta_+$, where $\varepsilon_o > 0$ and $\beta = (2d(\alpha-1) + 4(\alpha+1))/(2(\alpha-1)) > 1$. Then there exist $M > 0$, $\varepsilon_1 > 0$, and $T_2 \in (0,T)$ such that, if $\varepsilon_o \leq \varepsilon_1$ and $D_n(B_{\rho_o}(x_o) \times (0,T)) \leq M$, then $n(x,t) = 0$ for a.e. $x \in B_{\rho_o}(x_o)$, $t \in (0,T_2)$.*

THEOREM 3. – (Formation of vacuum) *Let $\omega \subset\subset \Omega$ be a domain, $x_o \in \omega$, and $T > 0$. Assume that* (5) *holds. Then there exist $M > 0$, $T_3 \in (0,T)$, $\sigma, \mu \in (0,1)$ such that if $D_n(\omega \times (0,T)) \leq M$, then $n(x,t) = 0$ for a.e. $x \in B_{\rho(t)}(x_o)$, $t \in (T_3, T)$, where $\rho(t) = \sigma(t - T_3)^\mu$.*

The proofs of these theorems are based on local energy methods. For this we define the domain

$$
P = P(t) = \{(x,\tau) \in Q_T \,:\, |x - x_o| \leq \rho(\tau) = \rho + \sigma(\tau - t)^\mu,\ \tau \in (t,T)\}. \tag{6}
$$

We choose parameters ρ, σ, t, and μ as follows: Theorem 1: P is a truncated cone with $\rho = \rho_o > 0$, $\sigma = -c < 0$, $t = 0$, $\mu = 1$; Theorem 2: P is a cylinder $B_\rho(x_o) \times (0,\tau)$, $0 < \rho \leq \rho_o$, $0 < \tau < T$;

Theorem 3: P is a paraboloid with $\rho = 0$, $\sigma > 0$, $\mu \in (0,1)$. The following integration by parts formula holds (*see* [2]):

$$\begin{aligned}(7)\qquad &\frac{1}{\alpha+1}\int_{P\cap\{\tau=T\}} n(x,T)^{\alpha+1} + \int_P |\nabla n^\alpha|^2 \\ &= \int_{\partial_l P} (\nabla n^\alpha \cdot \nu_x) n^\alpha - \frac{1}{\alpha+1}\int_{\partial_l P} \nu_\tau n^{\alpha+1} + \frac{1}{\alpha+1}\int_{P\cap\{\tau=0\}} n_I^{\alpha+1} \\ &- \int_P \left(\frac{1}{\alpha} n\nabla V\cdot\nabla n^\alpha + n^{\alpha+1}\Delta V + R(n,p)n^\alpha\right) = I_1 + \cdots + I_4,\end{aligned}$$

where (ν_x, ν_τ) is the unitary exterior normal vector to the lateral surface $\partial_l P$.

Proof of Theorem 3. – We only show how to estimate I_4 since the remaining integrals can be estimated as in [2] by using an interpolation-trace lemma and Hölder's inequality. Using hypothesis (5) and Young's inequality, we get

$$I_4 \leq \frac{1}{2}\int_P |\nabla n^\alpha|^2 + \left(c_1 D_n(P)^{2-\alpha-\delta} - b\right)\int_P n^{\alpha+\delta} + c_2(T-t)\sup_{s\in(t,T)}\int_{P\cap\{\tau=s\}} n(x,s)^{\alpha+1},$$

where $c_1, c_2 > 0$ depend on the local L^∞-norms of ∇V, ΔV respectively. Taking $\sigma > 0$ small enough, we can ensure $P \subset \omega$. Choose $M > 0$ small enough such that $c_1 D_n(P)^{2-\alpha-\delta} < b/2$, and let $T^* \in (0,T)$ be such that $c_2(T - T^*) \leq 1/2(\alpha+1)$. Then, for $T^* < t < T$,

$$I_4 \leq \frac{1}{2}\int_P |\nabla n^\alpha|^2 - \frac{b}{2}\int_P n^{\alpha+\delta} + \frac{1}{2(\alpha+1)}\sup_{s\in(t,T)}\int_{P\cap\{\tau=s\}} n(x,s)^{\alpha+1}.$$

The remaining nonnegative terms can be absorbed by the left hand side of (7). Now $I_3 = 0$, and we follow [2] to estimate I_1 and I_2, getting in this way a differential inequality for $E(t) := \int_P |\nabla n^\alpha|^2$:

$$(8)\qquad E^\gamma \leq c\Big(-\frac{dE}{dt}\Big), \qquad E(T) = 0, \quad E(t) \leq D_n(P),$$

where $t \in (T^*, T)$, $c > 0$, and $\gamma \in (0,1)$. For $M > 0$ and $T - T_3 > 0$ small enough, we conclude that $E(t) = 0$ for $t \in (T_3, T)$. This implies the assertion.

Proof of Theorem 2. – The integral I_4 is estimated as in the preceeding proof. Estimating as in [2], if $E(\rho) := \int_P (|\nabla n^\alpha|^2 + |\nabla p^\alpha|^2)$, we get

$$(9)\qquad E^{1-1/\beta} \leq c\frac{dE}{d\rho} + c(\varepsilon_o)(\rho - \rho_o)_+^{\beta-1}, \quad \beta > 1.$$

Then, for global energies and $T - T_2 > 0$ small enough, we get $E(\rho_o) = 0$.

Proof of Theorem 1. – Since we do not assume (5), we can rewrite I_4:

$$(10)\qquad I_2 + I_4 = -\frac{\alpha}{\alpha+1}\int_P n^{\alpha+1}\Delta V - \frac{1}{\alpha+1}\int_{\partial_l P}(\nu_\tau + \nabla V\cdot\nu_x)n^{\alpha+1} - \int_P R(n,p)n^\alpha.$$

Since $\nu_\tau + \nabla V \cdot \nu_x \geq (1+c^2)^{-1/2}(c + \nabla V\cdot e_x) \geq 0$ for sufficiently large $c > 0$, where e_x is the unit vector in the direction of ν_x, we obtain

$$I_2 + I_4 = T^* c_3(\Delta V)\sup_{s\in(0,T^*)}\int_{P\cap\{\tau=s\}} n(x,s)^{\alpha+1} - \int_P R(n,p)n^\alpha.$$

A similar estimate holds for p. By adding the corresponding inequalities, we get

$$-\int_P R(n,p)(n^\alpha + p^\alpha) \leq \kappa_R T^* \sup_{s\in(0,T^*)} \int_P (n^{\alpha+1} + p^{\alpha+1}).$$

Now proceed as in [2] to get a differential inequality similar to (9).

3. Uniqueness of solutions

We consider in this section a system slightly more general than (1) in which we include the case of a nonlinear transport term:

$$(11) \qquad (\mathcal{P})\begin{cases} n_t - \text{div}\,(\nabla r(n) + K(n)\nabla V) = F(n,p) & \text{in } Q_T, \\ p_t - \text{div}\,(\nabla r(p) - K(p)\nabla V) = F(n,p) & \text{in } Q_T, \\ -\Delta V = p - n + C & \text{in } Q_T, \end{cases}$$

where $K \in C^1(\mathbb{R})$. A similar system arises in two-phase filtrations (*see* [1]).

THEOREM 4. – *Under one of the following set of conditions, the solution of* (P) *is unique:*

(i) $(K'(s))^2 \leq Mr'(s)$ *and* $\left(\frac{\partial}{\partial s_i}F(s_1,s_2)\right)^2 \leq Mr'(s_i)$, $i = 1,2$, *for some* $M > 0$ *with* $s \in \mathbb{R}_+$.

(ii) $V \in L^\infty(0,T;W^{2,\infty}(\Omega))$, $K(s)$ *is linear*, $|F(s_1,\sigma_1) - F(s_2,\sigma_2)| \leq c(|r(s_1) - r(s_2)| + |r(\sigma_1) - r(\sigma_2)|)$ *and*

$$(12) \qquad \nabla V \cdot \nu = 0 \quad \textit{on} \quad \Sigma_{DT} \quad \textit{for any third component } V \textit{ of the solution.}$$

(iii) $n, p \in L^1(0,T;W^{1,q}(\Omega))$ *(with* $q = 1$ *if* $N = 1$ *and* $q > N$ *if* $N > 1$*) and* $r \in C^2(0,\infty)$ *is convex and strictly increasing.*

Proof. – The proof of (i) follows arguments similar to [1]. For the proof of (ii), let (n_1, p_1, V_1) and (n_2, p_2, V_2) be two solutions and define $n := n_1 - n_2$, $p := p_1 - p_2$, and $V := V_1 - V_2$. Multiplying the two first equations satisfied by (n,p,V) by appropriate test functions ϕ, ψ, integrating by parts in Q_τ for $\tau \in (0,T)$, and adding the resulting integral identities, we obtain

$$(13) \qquad \int_0^\tau (\langle n_t, \phi\rangle + \langle p_t, \psi\rangle) + \int_{Q_\tau} (\nabla(r(n_1) - r(n_2)) \cdot \nabla\phi + \nabla(r(p_1) - r(p_2)) \cdot \nabla\psi)$$
$$= \int_{Q_\tau} (\nabla V \cdot (n_1\nabla\phi - p_1\nabla\psi) + \nabla V_2 \cdot (n\nabla\phi - p\nabla\psi)) - \int_{Q_\tau} (R_1 - R_2)(\phi + \psi),$$

where $R_i = R(n_i, p_i)$. We choose ϕ, ψ as the unique solutions of

$$(14) \qquad -\Delta\phi = n(x,t), \quad -\Delta\psi = p(x,t) \qquad \text{in } \Omega,$$

with homogeneous mixed boundary conditions and for a.e. $t \in (0,T)$. Note that (14) implies that $\Delta V = \Delta(\psi - \phi)$. The term $\nabla V_2 \cdot (n\nabla\phi) = -(\nabla V_2 \cdot \nabla\phi)\Delta\phi$ can be estimated using (12) as in [7] to get

$$-\int_\Omega (\nabla V_2 \cdot \nabla\phi)\Delta\phi \leq c\|V_2\|_{W^{2,\infty}} \int_\Omega |\nabla\phi|^2,$$

and an analogous expression holds for $\int_\Omega \nabla V_2 \cdot (p\nabla\psi)$. Since by (14), $2\langle n_t(\tau), \phi(\tau)\rangle = \|\nabla\phi(\tau)\|_2^2$, we obtain from (13) after standard estimations that

$$\int_\Omega (|\nabla\phi(\tau)|^2 + |\nabla\psi(\tau)|^2) \leq \int_{Q_\tau} (|\nabla\phi|^2 + |\nabla\psi|^2).$$

From Gronwall's inequality and (14) we conclude the proof. The proof of (iii) uses a technique introduced in [3]. It is known (*see* [6] and [4]) that there exists a solution $(n_\varepsilon, p_\varepsilon, V_\varepsilon)$ to (11) with initial and boundary data $r(n_{D\varepsilon}) = r(n_D + \varepsilon e^{-\lambda_1 t})$, $r(p_{D\varepsilon}) = r(p_D + \varepsilon e^{-\lambda_1 t})$, $n_{I\varepsilon} = n_I + \varepsilon$, $p_{I\varepsilon} = p_I + \varepsilon$, with $\lambda_1, \varepsilon > 0$ arbitrarily chosen, satisfying $n(x,t), p(x,t) \geq c\varepsilon e^{-\lambda t} > 0$ in Q_T for some $c > 0$ and $\lambda > 0$ independent of r and ε. Moreover, such a solution converges strongly in $(L^2(Q_T))^3$ (as $\varepsilon \to 0$) to a solution (n_1, p_1, V_1) of (11). Let (n_2, p_2, V_2) be another solution of (1) and set $N_\varepsilon := n_\varepsilon - n_2$, $P_\varepsilon := p_\varepsilon - p_2$, $v_\varepsilon =: V_\varepsilon - V_2$. Define

$$A_n^\varepsilon := \frac{r(n_\varepsilon) - r(n_2)}{N_\varepsilon}, \quad \mathbf{B}_n^\varepsilon := \frac{K(n_\varepsilon) - K(n_2)}{N_\varepsilon} \nabla w_\varepsilon, \quad R_n^\varepsilon := \frac{R(n_\varepsilon, p_\varepsilon) - R(n_2, p_\varepsilon)}{N_\varepsilon},$$

with similar definitions for terms involving p. Let $A_n^{\varepsilon,j}$, $\mathbf{B}_n^{\varepsilon,j}$, and $F_n^{\varepsilon,n}$ be smooth approximations of these functions. By the convexity of r and the positivity of n, we have $A_n^{\varepsilon,j} > c(\varepsilon)$, with $\lim_{\varepsilon \to 0} c(\varepsilon) = 0$. Also $B_n^{\varepsilon,j}$ and R_n^ε are uniformly bounded in ε. Then, if ψ, ξ, η satisfy

$$\text{(15)} \qquad \begin{cases} \psi_t + A_n^{\varepsilon,j} \Delta \psi + \mathbf{B}_n^{\varepsilon,j} \cdot \nabla \psi + R_n^{\varepsilon,j}(\psi + \xi) = 0 & \text{in } Q_T, \\ \xi_t + A_p^{\varepsilon,j} \Delta \xi - \mathbf{B}_p^{\varepsilon,j} \cdot \nabla \xi + R_p^{\varepsilon,j}(\psi + \xi) = 0 & \text{in } Q_T, \\ \Delta \eta = \text{div } (p_2 \nabla \xi - n_2 \nabla \psi) & \text{in } Q_T, \\ \psi(x,T) = \chi_n^\delta(x), \quad \xi(x,T) = \chi_p^\delta(x) & \text{on } \Omega\ , \end{cases}$$

with homogeneous mixed boundary conditions, we obtain

$$\text{(16)} \qquad \begin{aligned} \int_\Omega \big(\chi_n^\delta(x) N_\varepsilon(x,T) + \chi_p^\delta P_\varepsilon(x,T)\big) &= \varepsilon \int_\Omega (\psi(x,0) + \xi(x,0)) \\ &- \int_{\Sigma_{DT}} [(r(n_\varepsilon) - r(n_2)) \nabla \psi + (r(p_\varepsilon) - r(p_2)) \nabla \xi] \cdot \nu. \end{aligned}$$

As a consequence of the Alexandrov Maximum Principle and using the regularity $n, p \in L^1(0,T; W^{1,p}(\Omega))$, we obtain L^∞-uniform (in ε) estimates of ψ and ξ. We now pass to the limit as $\varepsilon \to 0$ using a uniform boundary estimate (*see* [4], Lemma 2.10) to conclude that $\int_\Omega |N_\varepsilon(x,\tau)| + |P_\varepsilon(x,\tau)| \leq 0$.

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