J.evol.equ. 3 (2003) 01 – 09
1424–3199/03/010001 – 09

Journal of Evolution Equations

Introduction

The present Volume 3 (2003) of JEE is devoted to Philippe Bénilan.

It consists entirely of invited contributions. Several of them were presented at the conference *"Journées d'Analyse Non-Linéaire"* which was organized in the French Jura in October 2000 on the occasion of his 60th birthday.

Philippe Bénilan died on February 17, 2001. He was a most original and charismatic mathematician who had a deep and decisive impact on the theory of Evolution Equations. He was one of the founders of this journal.

Here we describe some of his essential contributions. They reflect an important part of the development of Nonlinear Analysis and the Theory of Evolution Equations.

At the beginning of his œuvre stands a work which became a bestseller in the domain: His Thèse d'Etat at the University of Paris Orsay in 1972 under the supervision of Haïm Brézis and Jacques Deny with the title: "Equations d'évolution dans un espace de Banach quelconque et applications". Here he proved his famous theorem on the uniqueness of mild solutions (*"bonnes solutions" in French*) for the evolution equation governed by a (non-linear multivalued) accretive operator in an arbitrary Banach space. In the early seventies, it was a great challenge to understand how the well-known theory of strongly continuous semigroups of linear operators in Banach spaces (e.g. the Hille-Yosida theorem) could be extended to nonlinear operators. In the case of Hilbert spaces there was a complete theory (of maximal monotone operators) which could even be extended to uniformly convex spaces. However, nearly everything went wrong in general Banach spaces, even in spaces such as L^1 and L^∞, which are natural for many models. In particular, no notion of solution was known which was suitable for the Cauchy problem $u'(t) + Au(t) = f(t)$ governed by a "good" (i.e. *accretive*) operator on such spaces. The notion of *mild solution* is "simply" defined as the uniform limit of the implicit Euler approximation scheme of the differential equation. The uniqueness result of Philippe Bénilan shows that this limit is independent of the process of approximation. Existence of such solutions was provided by the (equally famous) Crandall-Liggett Theorem under an additional surjectivity hypothesis (*m-accretivity*) which asserts that the exponential formula $(I + t/nA)^{-n}$ converges strongly on the closure of the domain of the operator A.

His Thèse d'Etat contains further very interesting abstract results, for example another deep extension of the Hille-Yosida Theorem to the nonlinear framework. Whereas in Hilbert space there is a perfect bijection between nonlinear contraction semigroups and m-accretive operators, a whole bunch of counterexamples were produced in that period to show that such results are not true in general Banach spaces. Now Philippe Bénilan showed that there is a bijective correspondance between the set of all m-accretive operators and the solutions of the *complete* evolution equation $u'(t) + Au(t) = y$ where y is an arbitrary element of the Banach space.

Concrete applications were also presented in his thesis. He shows in particular how the abstract framework can be applied to two popular nonlinear equations, namely the conservation law

$$u + div\phi(u) = \psi(u),$$

and the porous medium equation

$$u_t - \Delta\Phi(u) = v \text{ on }]0, T[\times\Omega, \ \partial\Phi(u)/\partial\nu = \beta(u) \text{ on }]0, T[\times\partial\Omega.$$

The right m-accretive operators governing these two equations are described in detail. It is absolutely remarkable that their definition is based on the notion of entropic solution in the sense of Kruzhkov.

Already his thesis gives the flavour of his favorite way of doing mathematics: to go back and forth from an abstract theory to concrete applications and models. Behind this is his deep need of understanding the basic ideas and the generality of an approach leading to the solution of a particular problem. As a leitmotiv he would typically ask: *"What is the right abstract framework?"*

Philippe Bénilan devoted much of his scientific activity to the problem of finding a good definition of solutions for nonlinear partial differential equations, which turned out to be surprisingly delicate. He introduced the notion of *entropic solution* for elliptic equations of the type

$$diva(\cdot, u, Du) = f.$$

One knows, that even in the linear case, if the coefficients are merely bounded, the distributional solutions are not unique. For the p-Laplacian uniqueness of the solutions with initial value in L^1 is still an open problem. The framework of entropic solutions, though, leads to a well-posed problem (it is related to the notion of renormalized solution introduced independently by P. L. Lions and F. Murat in the spirit of the type of solutions considered by R. Di Perna and P. L. Lions for the Boltzmann equation). Meanwhile Bénilan's results have been extended to diverse versions of the very nonlinear parabolic equation

$$b(u)_t - diva(\cdot, u, Du) = f.$$

It was most surprising and illuminating that the entropic solutions coincide with the abstract mild solutions for accretive operators in Banach spaces.

Going back in time, we have to mention the famous article of Aronson-Bénilan [13] from 1979 where it is shown that the positive solutions of the porous medium equation $u_t = \Delta u^m$ in $\mathbb{R}^N$ satisfy the pointwise inequality

$$u_t \geq -cu/t, \tag{1}$$

for a certain constant c. This very precise estimate was a breakthrough at the time. In fact, it signifies a regularization effect. For example, it implies that

$$\|u_t(t)\|_{L^1} \leq \frac{\hat{c}}{t}\|u(0)\|_{L^1}, \tag{2}$$

an estimate which in the linear case is equivalent to the analyticity of the semigroup. The Aronson-Bénilan paper was the starting point for numerous works on such regularizing effects for nonlinear equations, and also for the investigation of the regularity of solutions. A direct consequence of the pointwise estimate (1) is that u_t is a measure, this leads to many other nice regularity properties. Surprisingly, these pointwise inequalities remain valid for equations governed by an arbitrary homogeneous m-accretive operator A (i.e. A satisfies $A(\lambda u) = \lambda^m Au$ with $m \neq 1$). This was shown later by Bénilan and Crandall [17]. The result yields very strong regularization effects if the underlying space is L^∞, for example.

We have seen above, that L^1 is the right space for many examples. Philippe Bénilan particularly liked and mastered this space, which shows at the same time good and bad behaviour in the class of general Banach spaces. And indeed, he showed us the important role the space L^1 plays in the theory of non-linear partial differential equations. Most popular is his article [8] jointly with H. Brézis and M. G. Crandall on this subject. In the same spirit he solved a large number of open questions for solutions of nonlinear equations with a measure as initial value. One has also to mention his recent papers joint with S. N. Kruzhkov [73], [79], and Andreianov and Kruzhkov [92] on the conservation law with merely continuous non-linearity. Finally, he gave essential contributions to understanding the role of the space L^∞ exploiting the maximum principle for nonlinear equations.

It is indeed difficult to give an account on an œuvre going deeply into so many different areas of Nonlinear Analysis. Let us add a (non-exhaustive) list of key words and equations corresponding to contributions of Philippe Bénilan not mentioned so far: the Lie-Trotter product formula [22], [31], the equation $u'' + Au = f$ [72], completely m-accretive operators [53], T-accretivity [44], Kato's inequality [55], the equation $u_t + A\beta u = 0$ [10], the equation $u_t + \beta Au = 0$ [18], the equation $u_t + \max A_i u = f$ [19], Thomas-Fermi equations [58], reaction diffusion systems [57], [61], [82], asymptotic behaviour of the equation $u_t = \Delta_p u^m$ as m tends to ∞ [51], [68], [74], [91], the equation $u_t = a(\cdot, u, \varphi(\cdot, u)_x)_x$ [70], [71], the equation $u_t = \alpha(u)\Delta u + |Du|^2 + F(u)$ [76], Schrödinger semigroups [54], and many more.

Philippe Bénilan wrote important contributions to the fundamental treatise edited by Dautray-Lions [26], [27], [28], [39], [40], [41], [50]. In particular, his contribution [25],

[50]: *"The Laplacian"*, is a very successful modern treatment of Potential Analysis via the systematic use of distributions.

Let us come back to the initial subject, *nonlinear semigroups on Banach spaces*. One can say that under the essential influence of Philippe Bénilan it became a fascinating and very elegant theory which is best documented in his treatise *"Nonlinear Evolution Equations Governed by Accretive Operators"* jointly with M. G. Crandall and A. Pazy [93]. Bénilan's book is famous and widely quoted in the community *"Evolution Equations"*, an apparent contradiction since it is not published and not even finished. We greatly hope that this will be done very soon.

The high esteem Philippe Bénilan obtained all over the mathematical community is not only due to his outstanding mathematical oeuvre, but also to his extraordinary personality. He gave the highest value to human relations and he immersed himself completely and without any reserve in all his activities. Characteristic for him is the abundance of international contacts and the popularity which he acquired all over the world.

The community of Evolution Equations and Nonlinear Analysis lost a great mathematician and a wonderful colleague.

The scientific committee of this special volume,

Haïm Brézis (President), Wolfgang Arendt, Louise Barthélemy, Michael Crandall,
Jose Carillo, J. Ildefonso Diaz, Jerome A. Goldstein, Michel Pierre,
Juan Luis Vazquez.

PUBLICATIONS OF PHILIPPE BÉNILAN

[1] BÉNILAN, PH., *Opérateurs localement lipschitziens*. C. R. Acad. Sci. Paris Série A-B *269* (1969), A1125–A1127.

[2] BÉNILAN, PH., *Une remarque sur la convergence des semi-groupes non linéaires*. C. R. Acad. Sci. Paris Série A-B *272* (1971), A1182–A1184.

[3] BÉNILAN, PH., *Solutions intégrales d'équations d'évolution dans un espace de Banach*. C. R. Acad. Sci. Paris Série A-B *274* (1972), A47–A50.

[4] BÉNILAN, PH., and BRÉZIS, H., *Solutions faibles d'équations d'évolution dans les espaces de Hilbert*. Ann. Inst. Fourier *22* (1972), 311–329.

[5] ATTOUCH, H., BÉNILAN, PH., DAMLAMIAN, A., and PICARD, C., *Equations d'évolution avec conditions unilatérale*. C. R. Acad. Sci. Paris Série A *279* (1974), 607–609.

[6] BÉNILAN, PH., *Semi-groupes invariants par un cône de fonctions convexes*. Lecture Notes in Econom. and Math. Systems *102* (1974), 49–65.

[7] BÉNILAN, PH., *Opérateurs m-accrétifs hémicontinus dans un espace de Banach quelconque*. C. R. Acad. Sci. Paris Série A *278* (1974), 1029–1032.

[8] BÉNILAN, PH., BRÉZIS, H., and CRANDALL, M. G., *A semilinear equation in* $L^1(\mathbb{R}^N)$. Ann. Scuola Norm. Sup. Pisa *2* (1975), 523–555.

[9] BÉNILAN, PH., DENY, J., and HIRSCH, F., *Séminaire sur les semi-groupes et les équations d'évolution*. Années 1972–1973/1974–1975, Publication Mathématique d'Orsay, Univ. Paris XI, n° 166–7547, 1975.

[10] BÉNILAN, PH., and HA, K., *Equations d'évolution du type* $(du/dt) + \beta\upsilon\phi(u) \ni 0$ *dans* $L^\infty(U)$. C. R. Acad. Sci. Paris Série A–B Math. n°22, *281* (1975), A947–A950.

[11] BÉNILAN, PH., Existence de solutions fortes pour l'équation des milieux poreux. C. R. Acad. Sci. Paris Série A-B Math. n°16, *285* (1977), A1029–A1031.

[12] BÉNILAN, PH. (ed.), ROBERT, J., (ed.), *Journées d'analyse non linéaire*. Proceedings d'une conférence tenue à Besançon, Juin 1977, Lecture Notes in Mathematics *665* (1978), 256.

[13] ARONSON, D.G., and BÉNILAN, PH., *Régularité des solutions de l'équation des milieux poreux dans* $\mathbb{R}^N$. C. R. Acad. Sci. Paris Série I Math. n°2, *288* (1979), A103–A105.

[14] BÉNILAN, PH., and CATTÉ, F., *Sur le traitement d'une équation de la théorie des plasmas par des techniques d'accrétivité dans* L^∞. Publications Mathématiques de Besançon, Analyse non linéaire, 1979, 1–8.

[15] BÉNILAN, PH., and PICARD, C., *Quelques aspects non-linéaires du principe du maximum*. Séminaire de Théorie du Potentiel, N°4 (Paris, 1977/1978), *Lecture Notes in Mathematics 713* (1979), 1–37.

[16] BÉNILAN, PH., and PIERRE, M., *Inéquations différentielles ordinaires avec obstacles irréguliers*. Ann. Fac. Sci. Toulouse Math. n°1, *1*(5) (1979), 1–8.

[17] BÉNILAN, PH., and CRANDALL, M. G., *Regularizing effects of homogeneous evolution equations*. Contributions to analysis and geometry (Baltimore, Md., 1980), Johns Hopkins Univ. Press 1981, 23–39.

[18] BÉNILAN, PH., and CRANDALL, M. G., *The continuous dependence on* φ *of solutions of* $u^t - \Delta\varphi(u) = 0$. Indiana Univ. Math. J. *30* (1981), 161–177.

[19] BÉNILAN, PH., and CATTÉ, F., *Equation d'évolution du type* $(du/dt) + \max_i A_i u = f$ *par la théorie des semi-groupes non linéaires dans* L^∞. C. R. Acad. Sci. Paris Série I Math. *295* (1982), 447–450.

[20] BÉNILAN, PH., and DÍAZ, J. I., *Comparison of solutions of nonlinear evolution problems with different nonlinear terms*. Israel J. Math. *42* (1982), 241–257.

[21] BÉNILAN, PH., *A strong regularity* L^p *for solution of the porous media equation*. Contributions to nonlinear partial differential equations (Madrid, 1981), Res. Notes in Math. *89* (1983), 39–58.

[22] BÉNILAN, PH., and ISMAIL, S., *Une généralisation d'un résultat de Kato-Masuda sur la formule de Trotter*. Publications Mathématiques de Besançon, Analyse non linéaire n° 7, 1983, 17.

[23] BÉNILAN, PH., CRANDALL, M.G., and PIERRE, M., *Solutions of the porous medium equation in* $\mathbb{R}^N$ *under optimal conditions on initial values*. Indiana Univ. Math. J. *33* (1984), 51–87.

[24] BÉNILAN, PH., and TOURÉ, H., *Sur l'équation générale* $u_t = \varphi(u)_{xx} - \psi(u)_x + v$. C. R. Acad. Sci. Paris Série I Math. *299* (1984), 919–922.

[25] BÉNILAN, PH., *L'opérateur de Laplace*. Chapitre II, p. 255–769, in: Dautray, R., Lions, J. -L., *Analyse mathématique et calcul numérique pour les sciences et les techniques*, Tome I, Masson, Paris, 1984.

[26] BÉNILAN, PH., *Opérateurs differentiels linéaires*. Chapitre V, p. 947–1086, in: Dautray, R., Lions, J.-L., Analyse mathématique et calcul numérique pour les sciences et les techniques, Tome I, Masson, Paris, 1984.

[27] AUTHIER, M., ARTOLA, M., BÉNILAN, PH., CESSENAT, H. LANCHON, M., and MERCIER, B., *Problèmes variationnelles linéaires. Régularité.* Chapitre VII, p 1215–1306, in: Dautray, R., Lions, J.-L., Analyse mathématique et calcul numérique pour les sciences et les techniques, Tome I, Masson, Paris, 1984.

[28] BÉNILAN, PH., *Intégrales singulières*. Annexe du Chapitre XIII, p. 969–1040, in: Dautray, R., Lions, J. -L., Analyse mathématique et calcul numérique pour les sciences et les techniques, *Tome II,* Masson, Paris, 1985.

[29] BÉNILAN, PH., and PIERRE, M., *Quelques remarques sur la "localité" dans L^1 d'opérateurs differentiels*. Semesterberichte Funktionalanalysis Tübingen, Wintersemester 1987/88, p. 23–29.

[30] BÉNILAN, PH., and BERGER, J., *Estimation uniforme de la solution de $u_t = \Delta\phi(u)$ et caractérisation de l'effet régularisant*. C. R. Acad. Sci. Paris Série I Math. *300* (1985) 573–576.

[31] BÉNILAN, PH., and ISMAIL, S., *Générateur des semi-groupes non linéaires et la formule de Lie-Trotter*. Ann. Fac. Sci. Toulouse Math. n°2, *7*(5) (1985), 151–160.

[32] BÉNILAN, PH., *An introduction to partial differential equations*. Semigroups, theory and applications, Vol. II (Trieste, 1984), Pitman Research Notes in Mathematics Series *152* (1986), 33–92.

[33] BÉNILAN, PH., and SPITÉRI, P., *Discrétisation par schéma implicite d'un problème abstrait $d^2u/dt^2 \in Au$ sur* $[0, 1], u(0) = u_0, u(1) = u_1$. Publications Mathématiques de Besançon, Analyse non linéaire *10* (1987), 93–100.

[34] BÉNILAN, PH., and VÁZQUEZ, J. L., *Concavity of solutions of the porous medium equation*. Trans. Am. Math. Soc. *299* (1987), 81–93.

[35] BARTHÉLEMY, L., and BÉNILAN, PH., *Phénomène de couche limite pour la pénalisation d'équations quasi-variationnelles elliptiques avec conditions de Dirichlet au bord.* Nonlinear partial differential equations and their applications. Collège de France Seminar, Vol. VIII (Paris, 1984–1985), Pitman Research Notes in Mathematics Series *166* (1988), 47–68.

[36] BARTHÉLEMY, L., and BÉNILAN, PH., *Sous-potentiels d'un opérateur non-linéaire*. Israel J. Math. *61* (1988), 85–112.

[37] BÉNILAN, PH., CRANDALL, M.G., and PAZY, A., *"Bonnes solutions" d'un problème d'évolution semi-linéaire*. C. R. Acad. Sci. Paris Série I Math. *306* (1988), 527–530.

[38] BÉNILAN, PH., CRANDALL, M. G., and SACKS, P., *Some L^1 existence and dependence results for semi-linear elliptic equations under nonlinear boundary conditions*. Appl. Math. Optimization *17* (1988), 203–224.

[39] BÉNILAN, PH., *Linear differential operators*. Chapter V, p. 148–268, in: Dautray, R., Lions, J.-L., Mathematical analysis and numerical methods for science and technology, Vol. 2, Functional and Variational Methods, Springer-Verlag, Berlin, 1988.

[40] AUTHIER, M., ARTOLA, M., BÉNILAN, PH., CESSENAT, LANCHON, H., and MERCIER, B., *Linear variational problems. Regularity.* Chapitre VII, p. 375–456, in: Dautray, R., Lions, J. -L., Mathematical analysis and numerical methods for science and technology, vol. 2, Functional and Variational Methods, Springer-Verlag, Berlin, 1988.

[41] BÉNILAN, PH. (ed.), CHIPOT, M. (ed.), EVANS, L.C. (ed.), and PIERRE, M. (ed.), *Recent advances in nonlinear elliptic and parabolic problems*. Proceedings of an International Conference held in Nancy, March 1988, Pitman Research Notes in Mathematics Series *208* (1989), 340.

[42] BÉNILAN, PH., and LABANI, H., *Existence of attractors for the Brusselator*. Recent advances in nonlinear elliptic and parabolic problems (Nancy, 1988), Pitman Research Notes in Mathematics Series *208* (1989), 139–151.

[43] BÉNILAN, PH., and RIEDER, G. R., *Introduction to multivalued evolution equations*. Semesterbericht Funktionalanalysis, eds. G. Greiner, R. Nagel, F. Räbiger, U. Schlotterbeck, Tübingen, 1989, 3–14.

[44] BÉNILAN, PH., and DE LEÓN, S. S., *Accretivity, T-accretivity: the uniqueness-comparison theorem for strong solutions*. Semesterbericht Funktionalanalysis, eds. G. Greiner, R. Nagel, F. Räbiger, U. Schlotterbeck, Tübingen, 1989, 15–24.

[45] BÉNILAN, PH., and EGBERTS, P., *Mild solutions*. Semesterbericht Funktionalanalysis, eds. G. Greiner, R. Nagel, F. Räbiger, U. Schlotterbeck, Tübingen, 1989, 25–36.

[46] BÉNILAN, PH., *An introduction to concrete accretive operators in L^p*. Semesterbericht Funktionalanalysis, eds. G. Greiner, R. Nagel, F. Räbiger, U. Schlotterbeck, Tübingen, 1989, 37–48.

[47] BÉNILAN, PH., and KÉYANTUO, V., *The main theorems for mild solutions*. Semesterbericht Funktionalanalysis, eds. G. Greiner, R. Nagel, F. Räbiger, U. Schlotterbeck, Tübingen, 1989, 49–58.

[48] BÉNILAN, PH., BLANCHARD, D., and GHIDOUCHE, H., *On a nonlinear system for shape memory alloys*. Contin. Mech. Thermodyn. *2* (1990), 65–76.

[49] BÉNILAN, PH., *Singular integrals*. Appendix of Chapter XIII, p. 371–426, in: Dautray, R., Lions, J. -L.: Mathematical analysis and numerical methods for science and technology. vol 2, Volume 4, Spectral Theory and Applications, Springer-Verlag, Berlin, 1990.

[50] BÉNILAN, PH., *The Laplace operator*. Chapter II, p. 220–690, in: Dautray, R., Lions, J.-L.: Mathematical analysis and numerical methods for science and technology. Vol 1, Physical Origins and Classical Methods, Springer-Verlag, Berlin, 1990.

[51] BÉNILAN, PH., BOCCARDO, L., and HERRERO, M. A., *On the limit of solutions of $u_t = \Delta u^m$ as $m \to \infty$. Some topics in nonlinear PDEs*. Rend. Sem. Mat. Univ. Politec. Torino *1989* (1991), 1–13.

[52] BÉNILAN, PH., CORTÁZAR, C., and ELGUETA M., M., *Uniqueness and non uniqueness of the solutions of a mixed boundary value problem for the porous medium equation*. Rev. Union Mat. Argentina *37* (1991), 10–15.

[53] BÉNILAN, PH., and CRANDALL, M. G., *Completely accretive operators*. Semigroup theory and evolution equations (Delft, 1989), Lecture Notes in Pure and Appl. Math. *135* (1991), 41–75.

[54] ARENDT, W., BATTY, C. J. K., and BÉNILAN, PH., *Asymptotic stability of Schrödinger semigroups on $L^1(\mathbb{R}^N)$*. Math. Z. *209* (1992), 511–518.

[55] ARENDT, W., and BÉNILAN, PH., *Inégalités de Kato et semigroupes sous-markoviens*. Rev. Mat. Univ. Complutense Madrid *5* (1992), 279–308.

[56] BARTHÉLEMY, L., and BÉNILAN, PH., *Subsolutions for abstract evolution equations*. Potential Anal. *1* (1992), 93–113.

[57] BÉNILAN, PH., GOLDSTEIN, J. A., and RIEDER, G. R., *A nonlinear elliptic system arising in electron density theory*. Commmun. Partial Differential Equations *17* (1992), 2079–2092.

[58] BÉNILAN, PH., GOLDSTEIN, J. A., and RIEDER, G. R., *The Fermi-Amaldi correction in spinpolarized Thomas-Fermi theory*. Differential equations and mathematical physics (Birmingham, AL, 1990), Math. Sci. Eng. *186* (1992), 25–37.

[59] BÉNILAN, PH., and WITTBOLD, P., *Absorptions non linéaires*. J. Funct. Anal. *114* (1993), 59–96.

[60] BÉNILAN, PH., *Solutions d'équations d'évolution non linéaires: quelques problèmes ouverts*. Publications Mathématiques de Besançon, Analyse non linéaire n° 14, 1994, 10.

[61] BÉNILAN, PH., and LABANI, H., *Systèmes de réaction-diffusion abstraits*. Publications Mathématiques de Besançon, Analyse non linéaire n° 14, 1994, 18.

[62] BÉNILAN, PH., and LACROIX, M.-TH., *Estimation L^∞ du gradient d'un problème de Dirichlet dans un coin arrondi*. Publications Mathématiques de Besançon, Analyse non linéaire n° 14, 1994, 10.

[63] BÉNILAN, PH., Touré, H., *Solution entropique pour une équation parabolique-hyperbolique non linéaire*. Ann. Fac. Sci. Toulouse Math. *3* (1994), 63–80.

[64] BÉNILAN, PH., and WITTBOLD, P., *Nonlinear evolution equations in Banach spaces: basic results and open problems*. Funktionalanalysis (Essen, 1991), Lecture Notes in Pure and Appl. Math. *150* (1994), 1–32.

[65] ALAARABIOU, E. H., and BÉNILAN, PH., *Une version abstraite du héorème de Crandall-Liggett*. C. R. Acad. Sci. Paris Série I Math. *320* (1995), 923–928.

[66] BÉNILAN, PH., BOCCARDO, L., GALLOUËT, TH., GARIEPY, R., PIERRE, M., and VÁZQUEZ, J.L., *An L^1-theory of existence and uniqueness of solutions of nonlinear elliptic equations*. Ann. Scuola Norm. Sup. Pisa *22* (1995), 241–273.

[67] BÉNILAN, PH., and GARIEPY, R., *Strong solutions in L^1 of degenerate parabolic equations*. J. Differential Equations *119* (1995), 473–502.

[68] BÉNILAN, PH., and IGBIDA, N., *La limite de la solution de $u_t = \Delta_p u^m$ lorsque $m \to \infty$*. C. R. Acad. Sci. Paris Série I Math. *321* (1995), 1323–1328.

[69] BÉNILAN, PH., and KRUZHKOV, S. N., *First-order quasilinear equations with continuous nonlinearities*. Russian Acad. Sci. Dokl. Math. *50* (1995), 391–396.

[70] BÉNILAN, PH., and TOURÉ, H., *Sur l'équation générale $u_t = a(., u, \varphi(., u)_x)_x + v$ dans L^1. I. Etude du problème stationnaire*. Evolution equations (Baton Rouge, LA, 1992) Lecture Notes in Pure and Appl. Math. *168* (1995), 35–62.

[71] BÉNILAN, PH., and TOURÉ, H., *Sur l'équation générale $u_t = a(., u, \varphi(., u)_x)_x + v$ dans L^1 : II. Le problème d'évolution*. Ann. I. H. P. Nonlinear An. *12* (1995), 727–761.

[72] ALAARABIOU, E. H., and BÉNILAN, PH., *Sur le carré d'un opérateur non linéaire*. Archiv Math. *66* (1996), 335–343.

[73] BÉNILAN, PH., and BOUILLET, J. E., *On a parabolic equation with slow and fast diffusions*. J. Nonlinear Anal. *26* (1996), 813–822.

[74] BÉNILAN, PH., and IGBIDA, N., *Singular limit of perturbed nonlinear semigroups*. Communications in Appl. Nonlinear Anal. *3* (1996), 23–42.

[75] BÉNILAN, PH., and KRUZHKOV, S.N., *Conservation laws with continuous flux functions*. Nonlinear Differential Equations and Applications *3* (1996), 395–419.

[76] BÉNILAN, PH., and MALIKI, M., *Approximation de la solution de viscosité d'un problème d'Hamilton-Jacobi*. Rev. Mat. Univ. Complutense Madrid *9* (1996), 369–383.

[77] BÉNILAN, PH., and WITTBOLD, P., *On mild and weak solutions of elliptic-parabolic problems*. Advances in Differential Equations *1* (1996), 1053–1073.

[78] BÉNILAN, PH., *Quelques remarques sur la convergence singulière des semi-groupes linéaires*. Publications Mathématiques de Besançon, Analyse non linéaire n° 15, 1997, 1–11.

[79] BÉNILAN, PH., and BOUHSISS, F., *Un contre-exemple dans le cadre des équations hyperboliques paraboliques dégénérées*. Publications Mathématiques de Besançon, Analyse non linéaire n° 15, 1997, 123–126.

[80] BÉNILAN, PH., and BOUHSISS, F., *Une remarque sur l'unicité des solutions pour l'opérateur de Serrin*. C. R. Acad. Sci. Paris Série I Math. *325* (1997), 611–616.

[81] BÉNILAN, PH., MOUGEOT, M., and NAEGELEN, F., *Enchères asymétriques: contribution à la détermination numérique des stratégies d'équilibre Bayésien*. Ann. d'Economie et de Statistiques, n°46 1997, 225–251.

[82] BÉNILAN, PH., and WRZOSEK, D., *On an infinite system of reaction-diffusion equations*. Advances in Mathematical Sciences and Applications *7* (1997), 351–366.

[83] ABOURJAILY, C., and BÉNILAN, PH., *Symmetrization of quasilinear parabolic problems*. Dedicated to the memory of Julio E. Bouillet, Revista Union Mat. Argentina *41* (1998), 1–13.

[84] BÉNILAN, PH., and LAMI DOZO, E., *In memoriam : Julio E. Bouillet (1942–1994)*. Revista Union Mat. Argentina *41* (1998), i–iii.

[85] ARENDT, W., and BÉNILAN, PH., *Wiener regularity and heat semigroups on spaces of continuous functions*. Topics in nonlinear analysis, Progress in Nonlinear Differential Equations Appl. *35*, Birkhäuser, Basel, 1999, 29–49.

[86] BÉNILAN, PH., and BOULAAMAYEL, B., *Sous-solutions d'équations elliptiques dans L^1*. Potential Anal. *10* (1999), 215–241.

[87] BÉNILAN, PH., and WITTBOLD, P., *Sur un problème parabolique-elliptique*. M2AN. Mathematical Modelling Numerical Analysis *33* (1999), 121–127.

[88] ANDREIANOV, B., BÉNILAN, PH., and KRUZHKOV, S. N., *L^1-theory of scalar conservation law with continuous flux function*. J. Funct. Anal. *171* (2000), 15–33.

[89] AUSCHER, P., BARTHÉLEMY, L., BÉNILAN, PH., and OUHABAZ, E.M., *Absence de la L^∞-contractivité pour les semi-groupes associés aux opérateurs elliptiques complexes sous forme divergence*. Potential Anal. *12* (2000), 169–189.

[90] BÉNILAN, PH., CARRILLO, J., and WITTBOLD, P., *Renormalized entropy solutions of scalar conservation laws*. Ann. Scuola Norm. Sup. Pisa *29* (2000), 313–327.

[91] BÉNILAN, PH., and IGBIDA, N., *Limite de la solution de $u_t - \Delta u^m + \operatorname{div} F(u) = 0$ lorsque $m \to \infty$*. Rev. Mat. Complut. *13* (2000), 195–205.

[92] BÉNILAN, PH., and KAMYNIN, V.L., *A priori estimates for and the asymptotic behavior of solutions of semilinear integrodifferential parabolic equations on a plane*. Mathematical physics, mathematical modeling and approximate methods, (Obninsk, 2000), Mat. Model. *13* (2001), 10–22.

[93] BÉNILAN, PH., CRANDALL, M., and PAZY, A., *Nonlinear Evolution Equations Governed by Accretive Operators*, Book Manuscript, Besançon, 2001.

[94] BÉNILAN, PH., EVANS, L. C., and GARIEPY, R. F., *On singular limits of homogeneous semigroups*, J. evol. equ. *3* (2003).

[95] BÉNILAN, PH., and IGBIDA, N., *Singular limits of changing sign solutions of the porous medium equation*, J. evol. equ. *3* (2003).

[96] BÉNILAN, PH., and DIAZ, J. I., *Pointwise gradient estimates to one-dimensional nonlinear parabolic equations*, J. evol. equ. *3* (2003).

[97] BÉNILAN, PH., and BRÉZIS, H., *Nonlinear problems to the Thomas-Fermi equation*, J. evol. equ. *3* (2003).