On the mathematical analysis and numerical approximation of high hydrostatic pressure food

J.I. Díaz , A.M. Ramos

Dpto. Matemática Aplicada,Univ. Complutense de Madrid

L. Otero, A. D. Molina and P. D.Sanz,

Instituto del Frío (CSIC), Madrid, SPAIN

Project AGL2000-1440-C0201

Mathematical and Computer Techniques for Agro-Food Technologies, Barcelona 26-27 November 2001

Part I. Introduction

PRESSURE TREATMENT OF FOODS

- Increasing interest in food processing and preservation
- Importance of knowing the exact pressure/temperature conditions of each process

THERMAL CONTROL IN PRESSURE TREATMENT

➔ Compression

- Temperature increase due to the work of compression
    - Food composition
    - Initial temperature
    - Pressurising fluid
    - Applied pressure

$$\left(\frac{\Delta T}{\Delta P}\right) = \frac{TV\alpha}{c_p}$$

V specific volume

α Thermal expansion coefficient

c_p Specific heat capacity

THERMAL CONTROL IN PRESSURE TREATMENT

Holding time

Heat loss through the wall of the HP vessel

↓

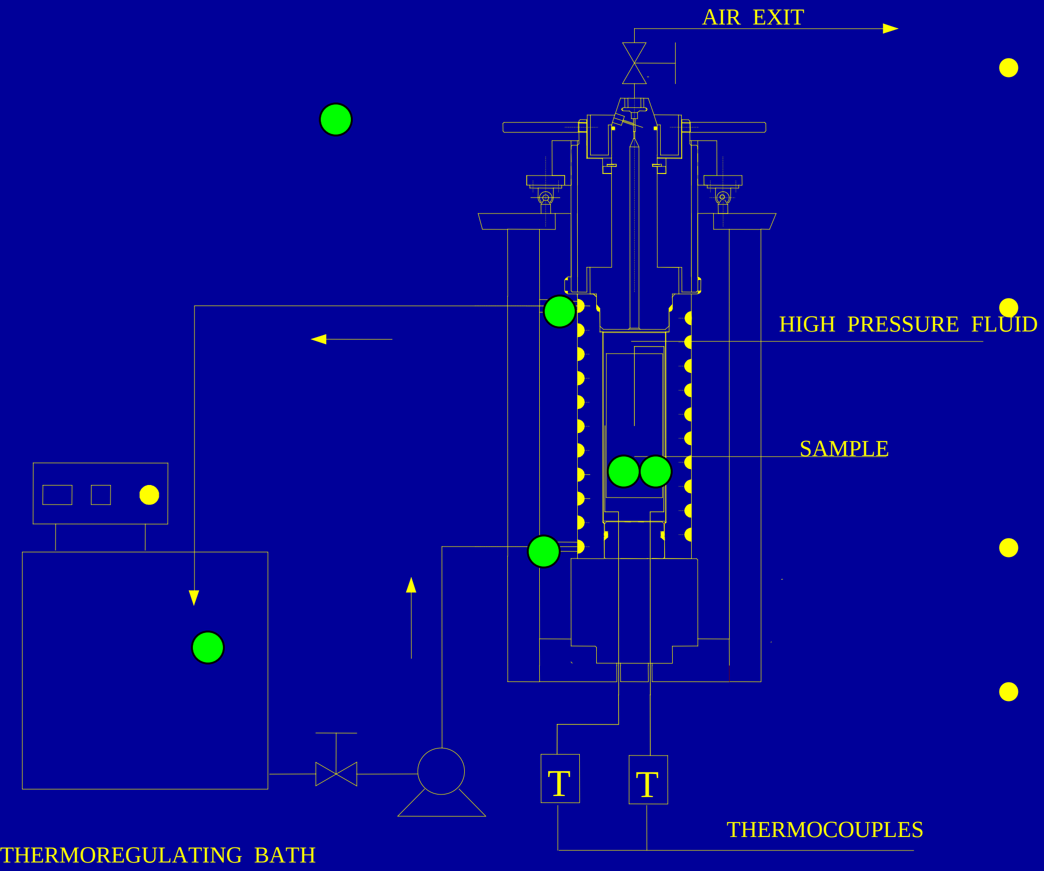
Temperature gradients in the processed food

↓

Non uniform distribution of enzyme and/or microbial inactivation, nutritional and/or sensorial quality degradation,...

Materials and methods

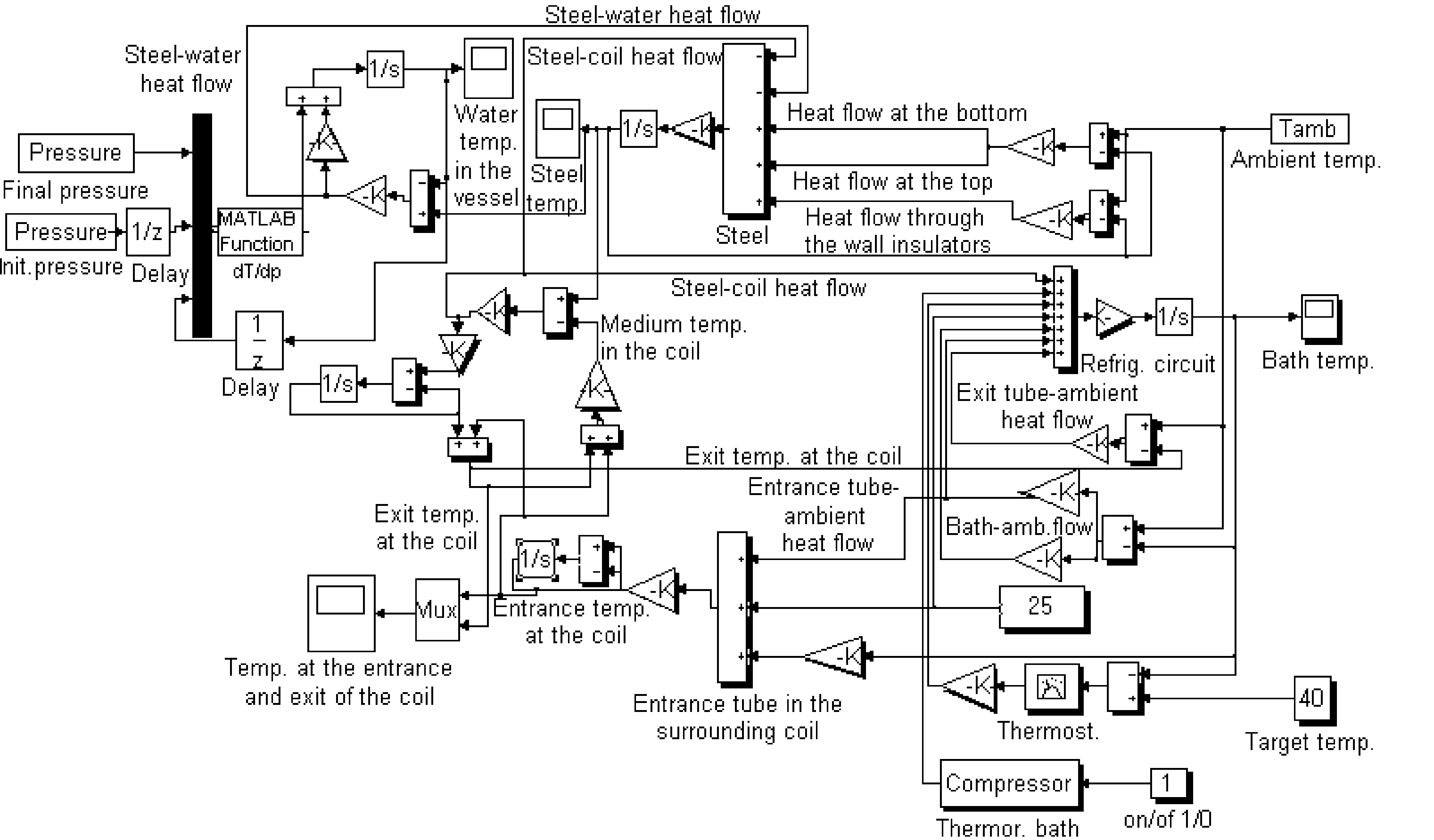
HIGH-PRESSURE EQUIPMENT



● **Position of the thermocouples**

- GEC ALSTHOM ACB High-Pressure equipment
- A cylindrical chamber (2.35 l net volume)
- Thermoregulated at 37ºC
- Assayed pressures: 200, 300 and 400 MPa
- Sample: Liquid water

MODEL TO SIMULATE THE THERMAL EXCHANGE



MODEL DESCRIPTION

The model includes:

All the thermal exchanges that occur

- Thermoregulating bath/ambient
- Pipes/ambient
- Steel of the vessel/thermoregulating fluid
- Steel of the vessel/ambient
- Sample/steel of the vessel

Calculation of the temperature variation during the compression or expansion

Heat transmission ways considered

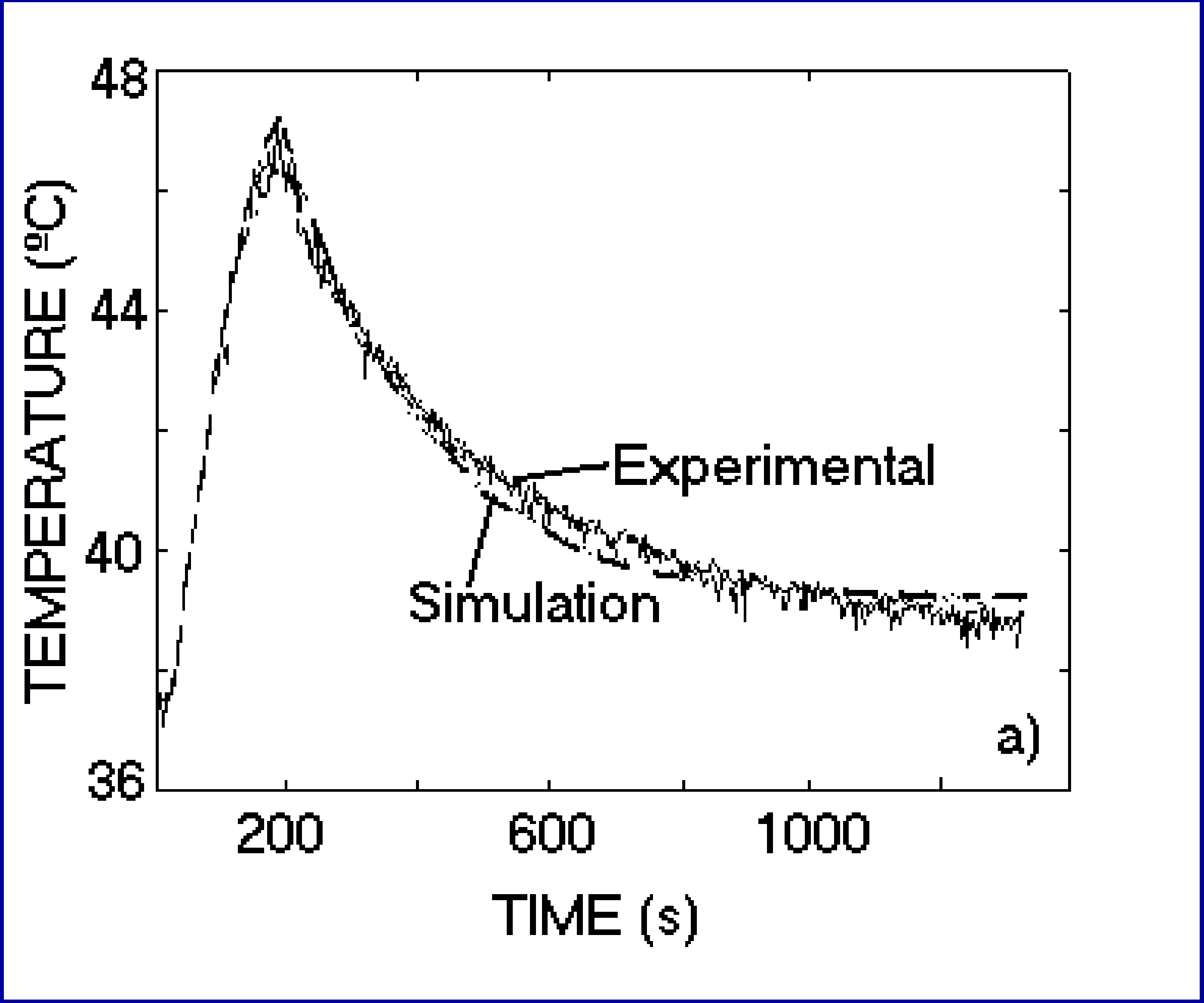
- **Convection**

  (ambient air, fluid inside the bath and fluid inside the vessel or sample)

- **Conduction**

  (through the different solid layers in the vessel)

COMPRESSION UP TO 400 MPa



About 20 minutes were necessary to dissipate heat inside the vessel and re-equilibrate its temperature

Mean absolute errors

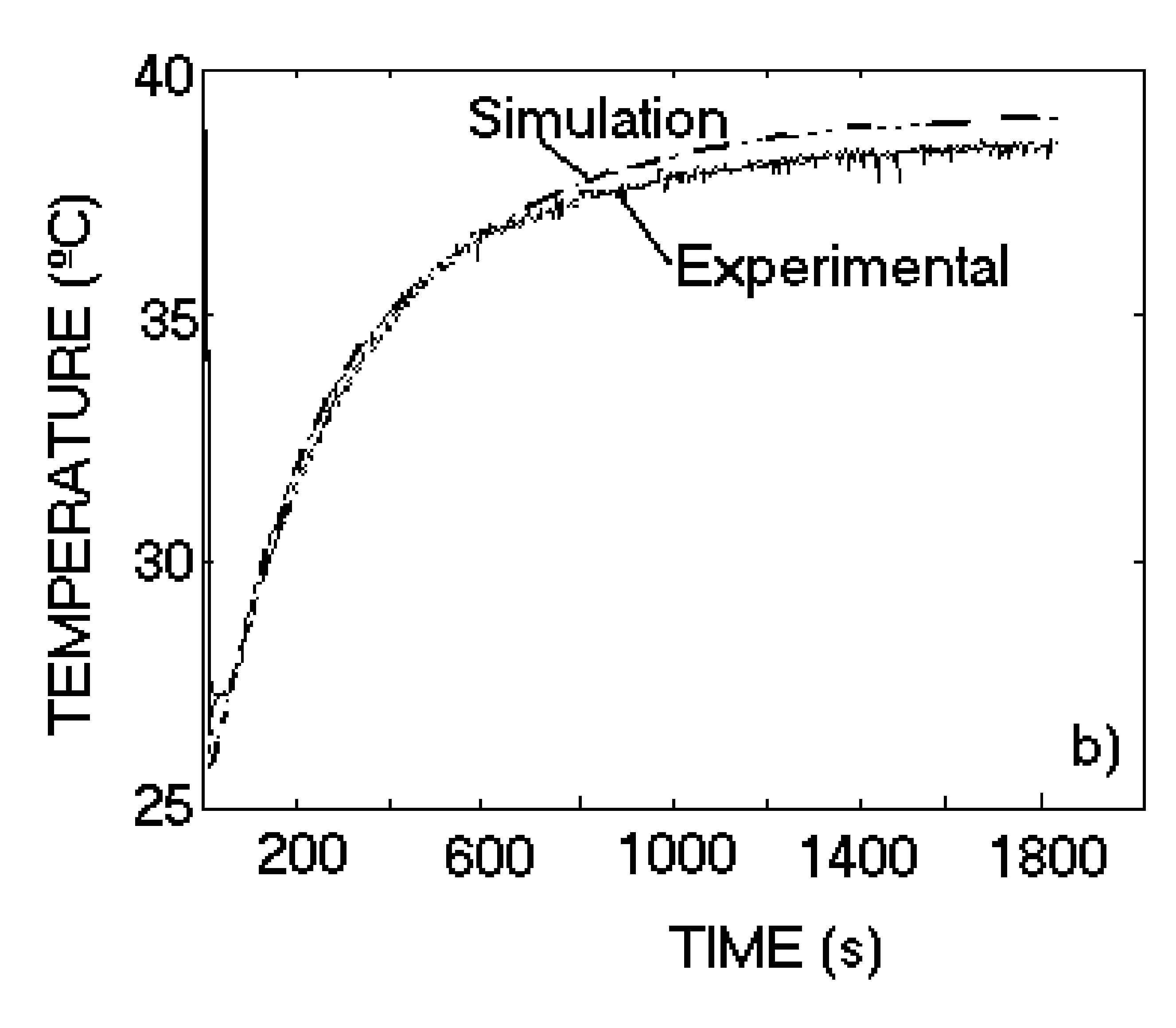
Sample: 0.6%

Entrance of the surrounding coil: 0.2%

Exit of the surrounding coil:0.3%

Thermoregulating bath: 0.1%

EXPANSION FROM 400 Mpa



Good agreement between the simulated and experimental temperature values at different points in the high-pressure equipment

Mean absolute errors

Sample: 1.0%

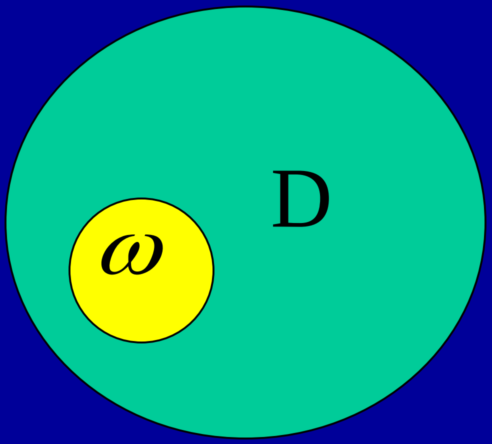
Entrance of the surrounding coil: 0.3%

Exit of the surrounding coil:0.1%

Thermoregulating bath: 0.2%

Part II. A simplified mathematical model: restricted control at the Robin boundary condition

$\Omega = D - \omega$



$$\rho c y_t - k\Delta y = 0 \quad \text{in } \Omega \times (0,T),$$

$$k\frac{\partial y}{\partial n} = h(u(t) - y) \quad \text{on } \partial\omega \times (0,T),$$

$$k\frac{\partial y}{\partial n} = h(w(x,t) - y) \text{ on } \partial D \times (0,T),$$

$$y(x,0) = y^0(x) \qquad x \in \Omega,$$

1. Optimal control: Given k, w and y^0 and y_d find the control $u \in C$ minimizing (over U_{ad})

$$J_k(u) = \frac{1}{2}\| u \|_{L^2(0,T)} + \frac{k}{2}\| y(T,.:u) - y_d \|_{L^2(\Omega)}$$

2. Approximate Controlabilty: Given $\varepsilon > 0$ find

$u \in U_{ad}$ such that $\| y(T,.:u) - y_d \|_{L^2(\Omega)} \leq \varepsilon$

Control constrains (limitations of the thermoregulating device)

$$U_{ad} = \{ u \in L^{\infty}(0,T) : u_1 \leq u(t) \leq u_2 \text{ a.e. } t \in (0,T) \}$$

State constrains (sterilization: Olin Ball &Olson (1957))

$$C = \left\{ u \in U_{\text{ad}} : e \leq \int_0^T F(y(x,t))dt, \text{ a.e.} x \in \Omega \right\} \text{ for some } F \in C^1(\mathbf{R})$$

Mathematical results:

i) Existence of the optimal control (it uses the Lions compactness theorem)

ii) Optimality conditions

$$\rho c y_t - k\Delta y = 0 \quad \text{in } \Omega \times (0,T),$$

$$k\frac{\partial y}{\partial n} = h(\tilde{u}(t) - y) \quad \text{on } \partial\omega \times (0,T),$$

$$k\frac{\partial y}{\partial n} = h(w(x,t) - y) \text{ on } \partial D \times (0,T),$$

$$y(x,0) = y^0(x) \quad x \in \Omega$$

$$-\rho c p_t - k\Delta p = 0 \quad \text{in } \Omega \times (0,T),$$

$$k\frac{\partial p}{\partial n} = -hp \quad \text{on } \partial\omega \times (0,T),$$

$$k\frac{\partial p}{\partial n} = -hp \quad \text{on } \partial D \times (0,T),$$

$$p(x,T) = k(y(x,T) - y_d(x)) \quad x \in \Omega$$

$$\int_0^T \left[\int_{\partial\omega} g\,d\sigma + \gamma\left(\tilde{u} + \int_{\partial\omega} p\,d\sigma\right) \right] (u - \tilde{u})\,dt \geq 0, \ \forall u \in U_{\mathrm{ad}}$$

for some $g = g(\tilde{u})$ and some $\gamma \geq 0$

(the proof uses a result due to Bonnans and Casas (1984))

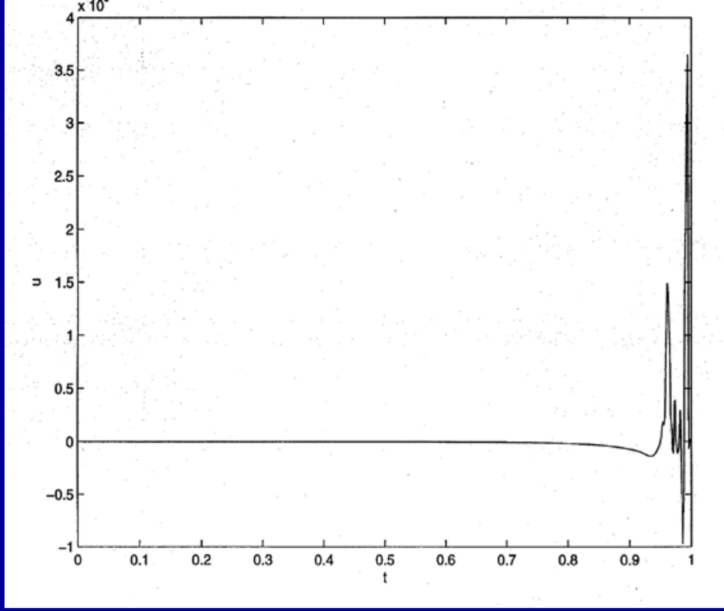
iii) Approximate controllability with constraints

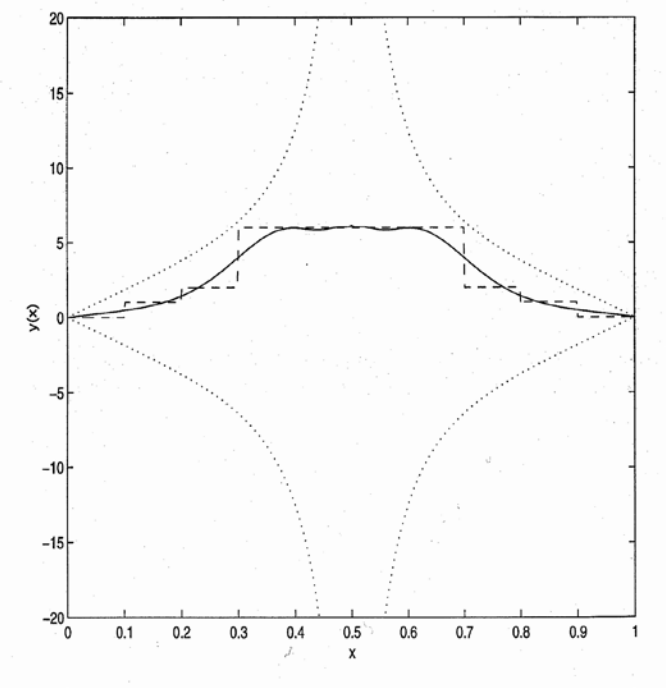
By passing to the limit, in some *a priori* estimates obtained from the optimality conditions, when $k \to \infty$, it is possible to show the approximate controllability once we assume

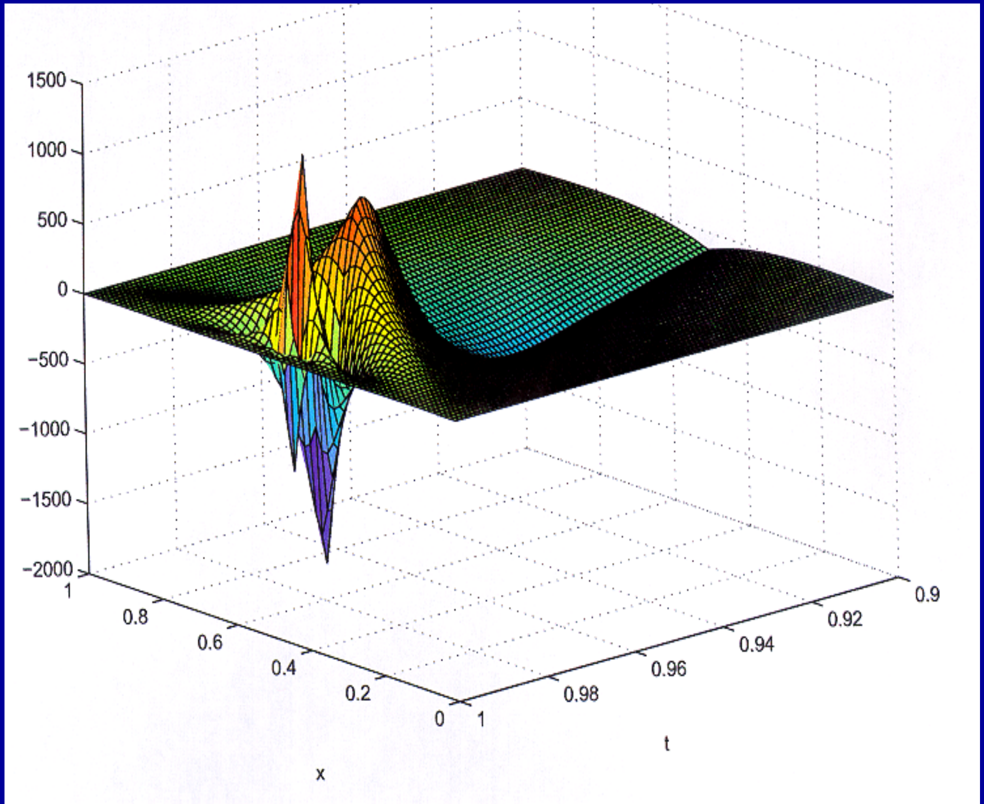
$$y(T,.:u_1) \leq y_d \leq y(T,.:u_2)$$

Numerical experiences

for a related problem: Díaz&Ramos (2000)







CONCLUSIONS

- The model reproduces the thermal behaviour of the high-pressure system in different points satisfactory
- It allows to study the effect of the different variables implied in the system (thermoregulating fluid, flow, target temperature, heating/cooling power of the bath...)
- Non trivial mathematical problems in Control Theory