

Gaudi, Frei Otto: Geometría y Arquitectura.

J. I. Díaz

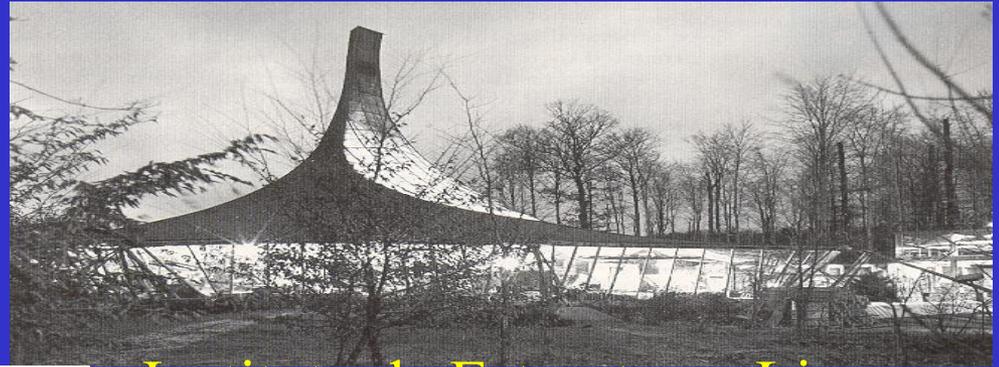
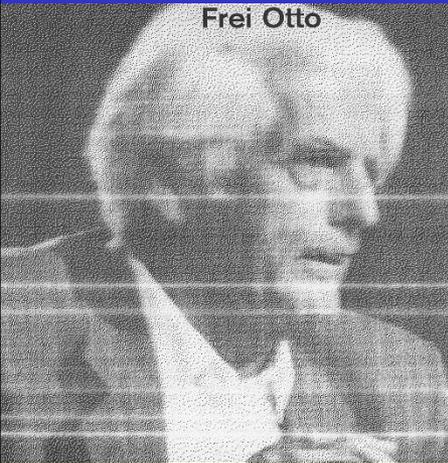


ETS de Arquitectura. CEU.
16 de octubre de 2002

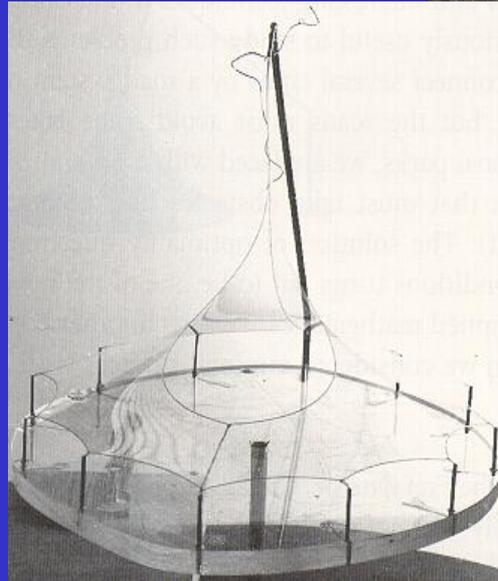
1. Introducción

Dos motivaciones personales

Frei Otto (1925 -)



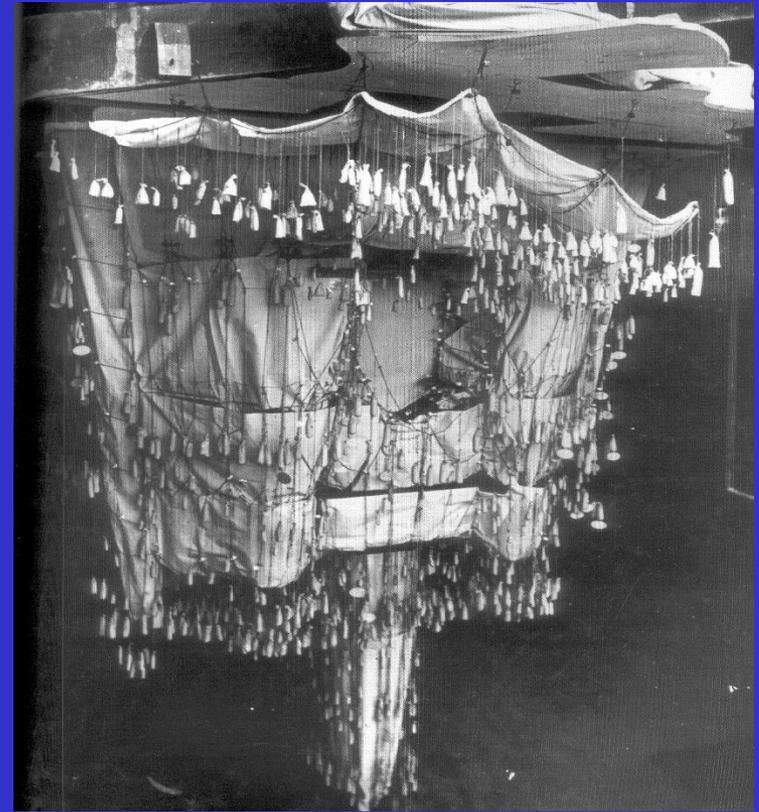
Instituto de Estructuras Ligeras
(Stuttgart)



Iglesia de la Colonia Güell



Antoni Gaudí (1825-1926)



Modelos analógicos/modelos matemáticos

Inversión del camino de las técnicas de construcción: de la maqueta al cálculo, al dibujo y a la construcción



La belleza de lo económico:

Guillermo de Occam (1285-1349) “Principio de la navaja (o de parsimonia)”: *Entia non sunt multiplicanda sine necessitate*

Gaudi:

“Para que un objeto sea extraordinariamente bello es necesario que su forma no contenga nada de superfluo”

“Soy geómetra, es decir ,sintético”



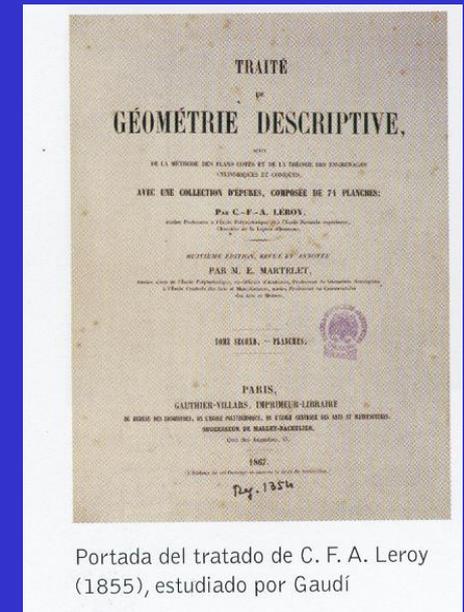
Gaudi:

“Yo lo calculo todo”

“Mis ideas estructurales y estéticas son una lógica *indisputable*. Convencido del perfeccionamiento que representan tengo el deber de aplicarlas”

“Este árbol cercano a mi obrador: éste es mi maestro”

“La geometría en la ejecución de las superficies no complica, sino que simplifica la construcción”



Portada del tratado de C. F. A. Leroy (1855), estudiado por Gaudí

G. Leibniz (1646-1716): “Nuestro mundo ha sido creado como el mejor de los mundos”

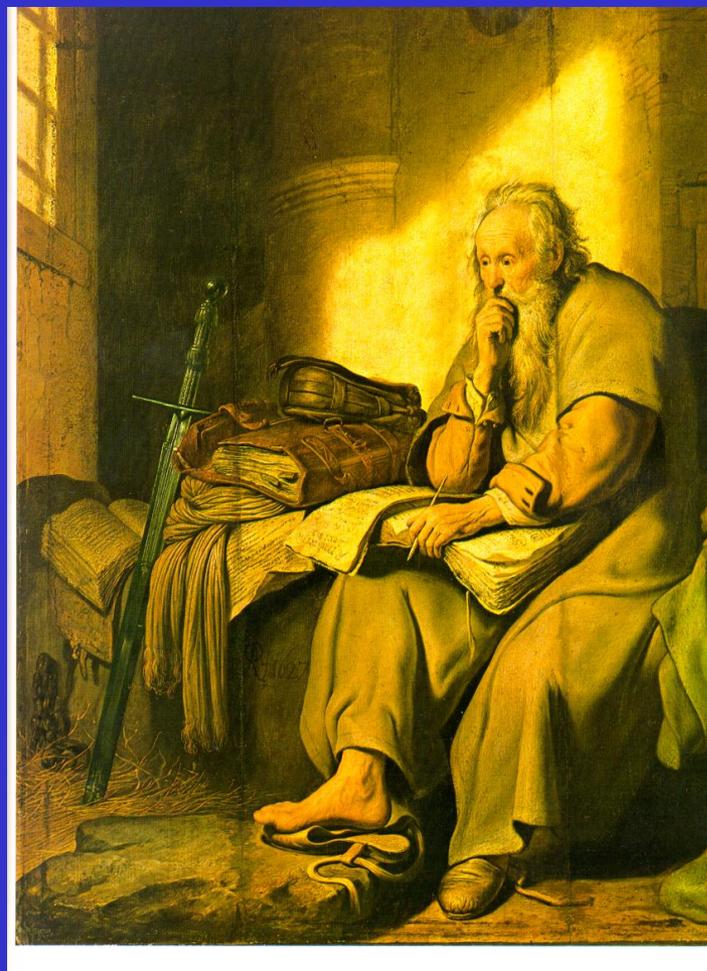


Búsqueda de un esquema filosófico del mundo

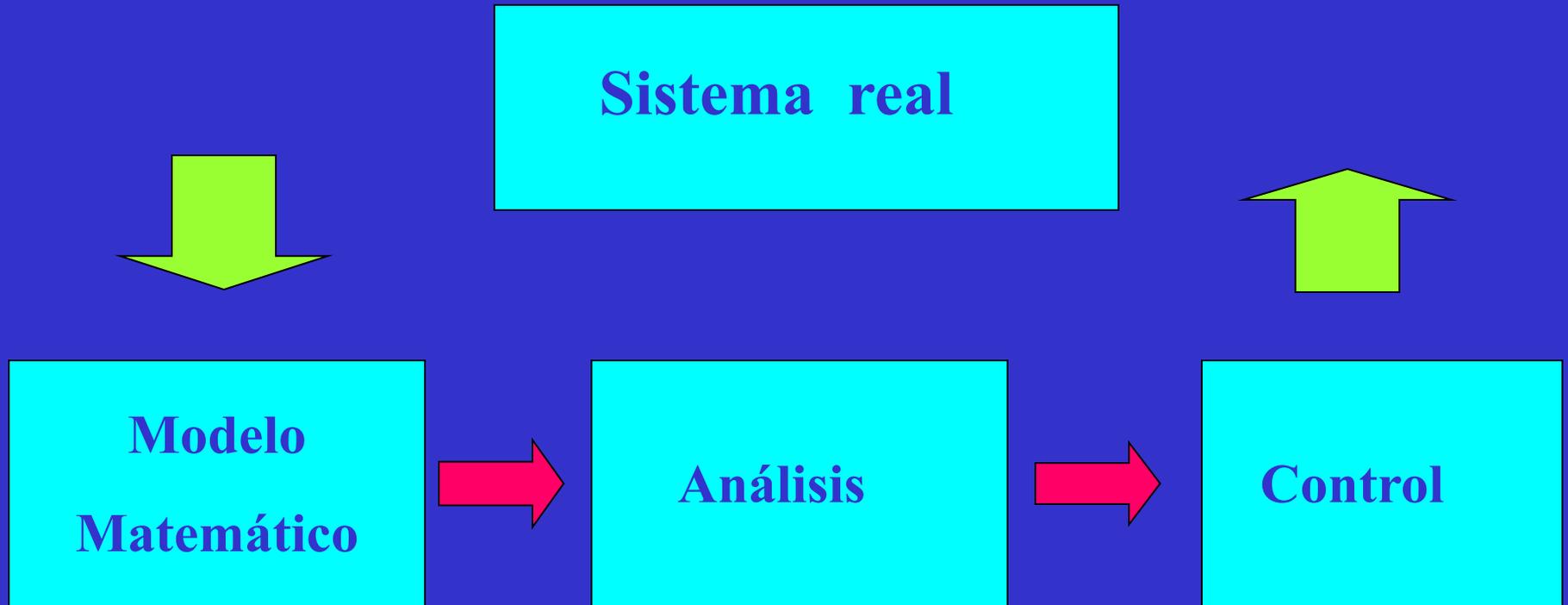


P. L. De Maupertuis (1698-1759): “Si se produce un cambio en la naturaleza la cantidad de acción para llevarla a cabo debe ser mínima”

Presencia de las matemáticas: tramas ocultas



una metodología



2. Gaudi : modelos antifuniculares

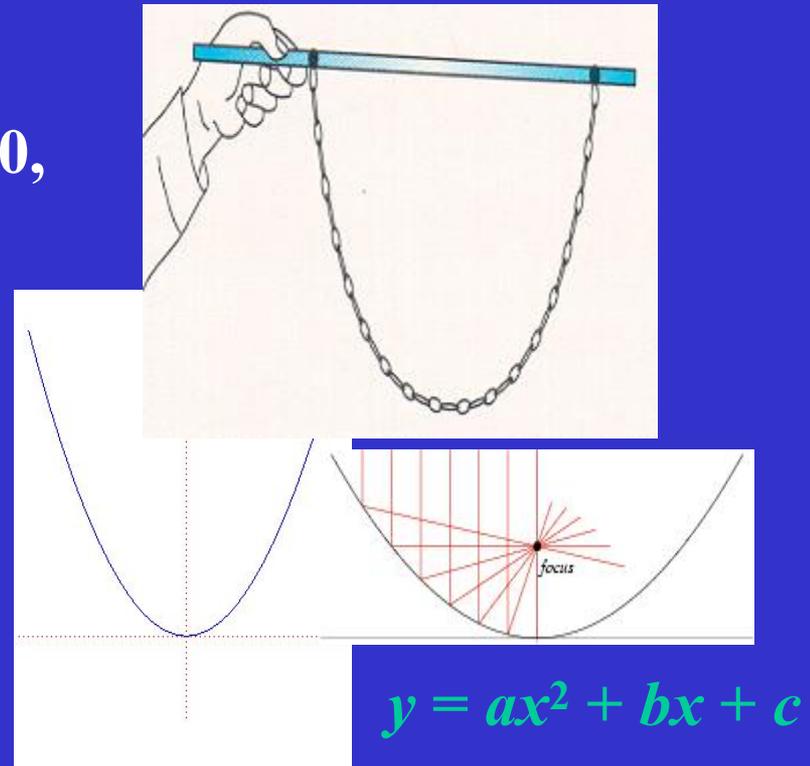
“La catenaria”:

Leonardo da Vinci (1452-1519) 1490,

En 1669, Jungius puso en tela de juicio la afirmación de Galileo Galilei (1562-1642): parábola

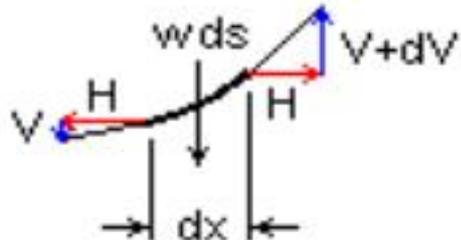
Huygens primero en utilizar el término de catenaria (carta a Leibniz en 1690)

La ecuación fue obtenida por Leibniz, Huygens, y Johann Bernoulli en 1691 en respuesta a un problema de Jakob Bernoulli.



$$y = a \cosh(x/a)$$

Deducción:



$$w ds = H y''$$

$$ds = \sqrt{1 + y'^2} dx$$

$$\int_0^x \frac{dy'}{\sqrt{1 + y'^2}} = \int_0^x \frac{w}{H} dx$$

$$\sinh^{-1} y' = \frac{w}{H} x$$

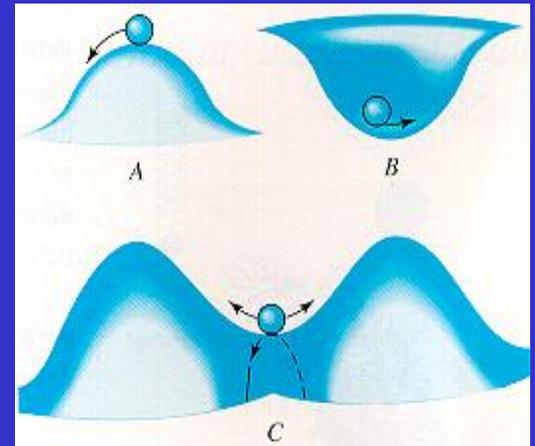
$$y' = \sinh \frac{w}{H} x$$

$$y = \frac{H}{w} \left(\cosh \frac{w}{H} x - 1 \right)$$

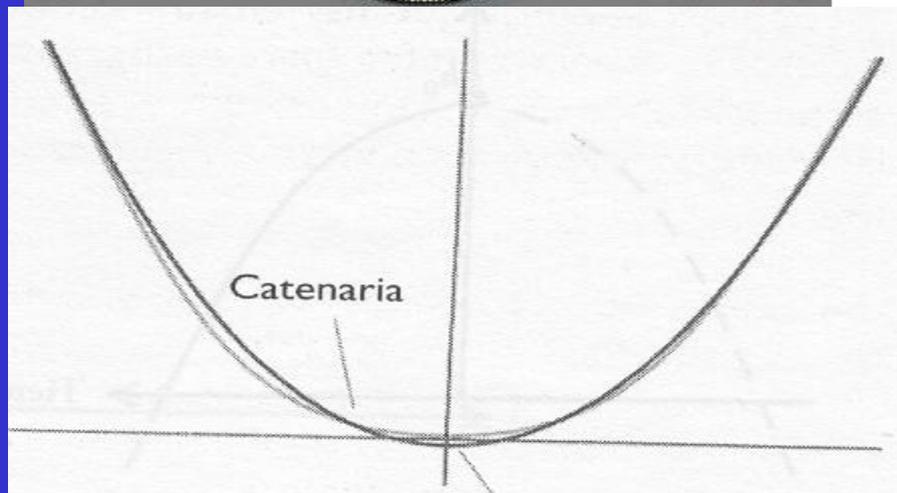
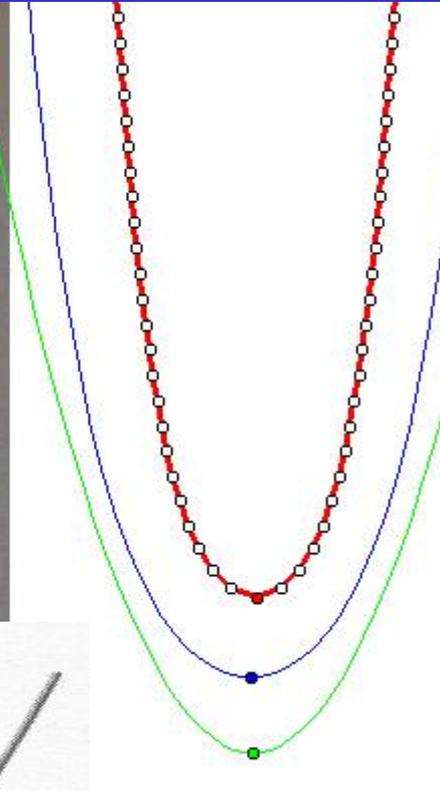
Finding the Catenary



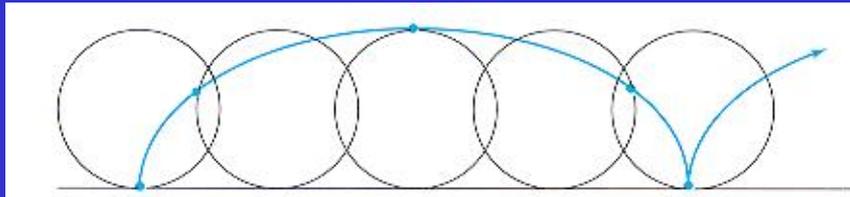
**Estado de equilibrio
de la cuerda vibrante
bajo la acción de la
gravedad**



La sutileza de las matemáticas

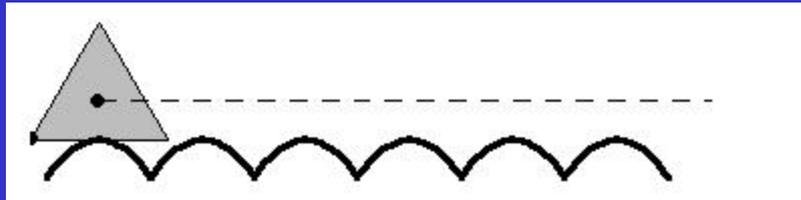
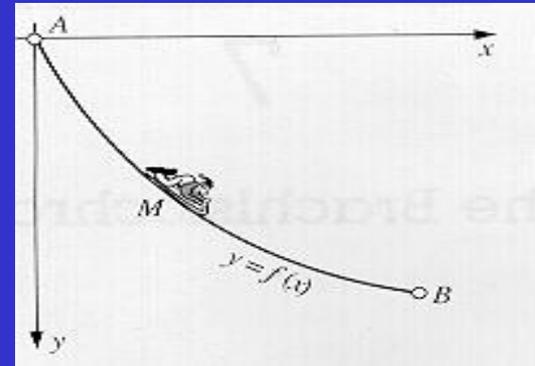


La catenaria como curva de apoyo de una cicloide recta



Galileo Galilei (1562-1642),
Blaise Pascal (1623-1662),...

La braquistócrona



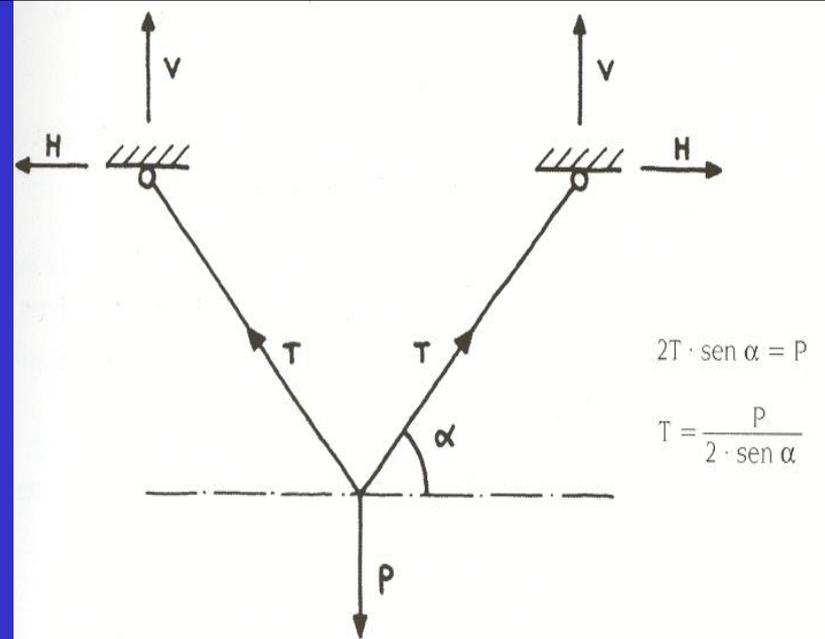
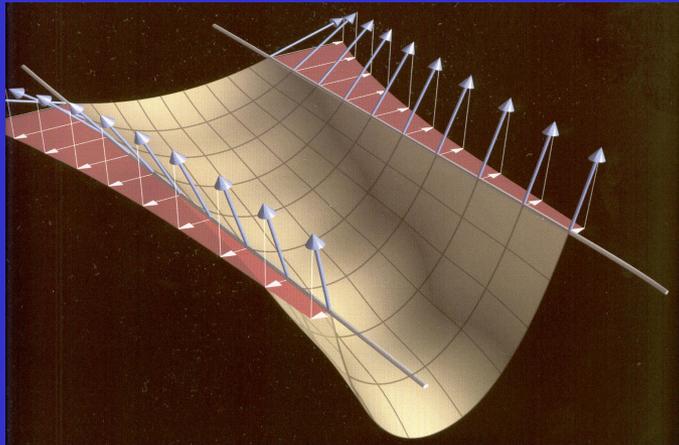
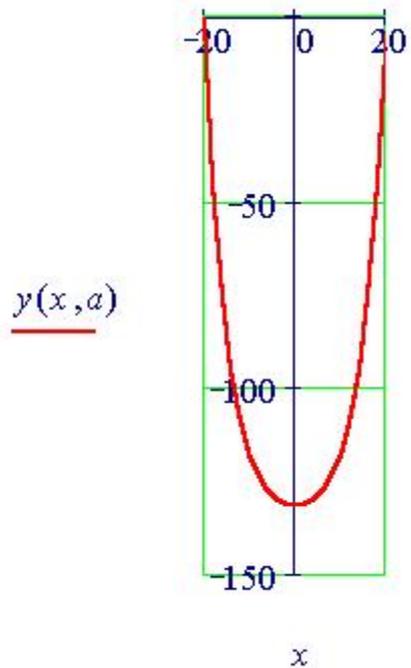


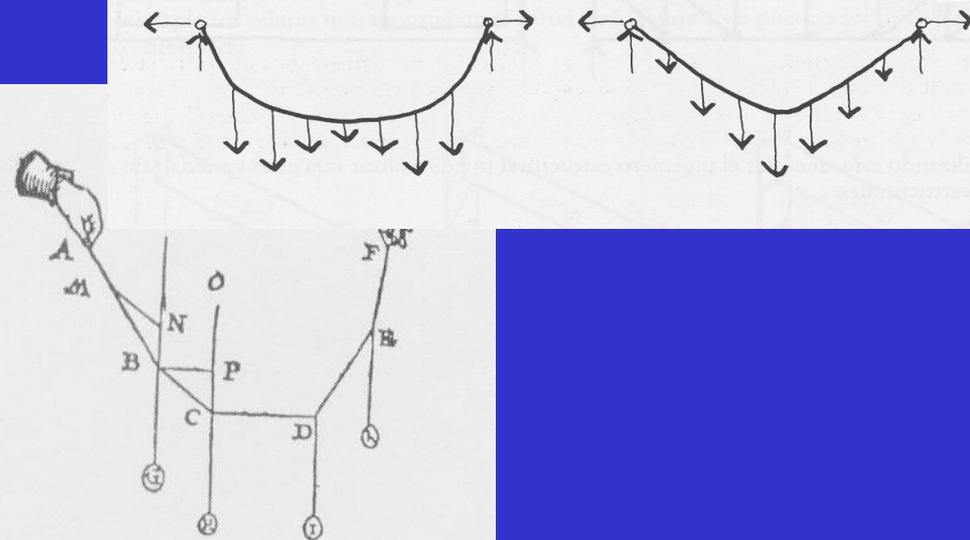
Fig. 1. Mecanismo resistente básico.



$a = 5.0$

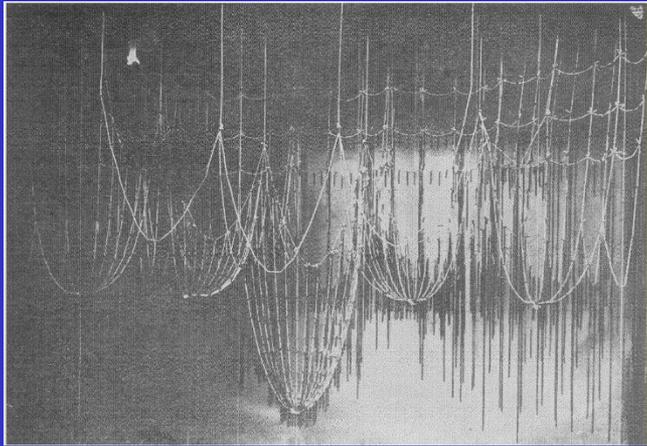
$y(0, a) = -131.5$

$Length = 272.9$

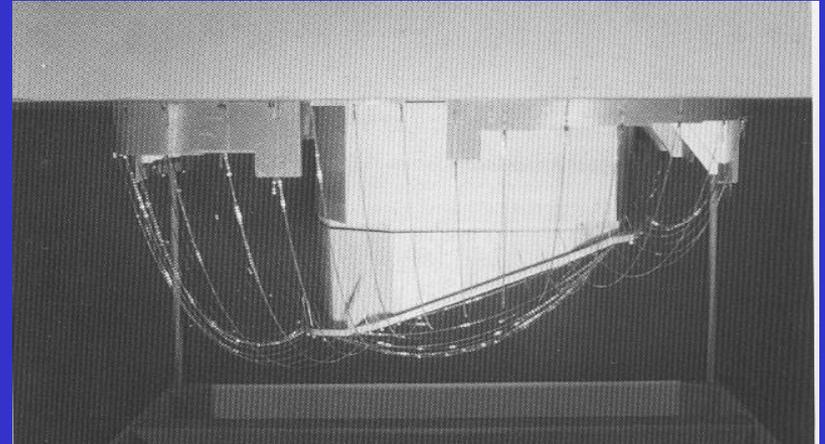


Representación funicular de Sterin (siglo xvii)

Algunas otras maquetas anti-funiculares



Primer estereofunicular
de Gaudi



Suntory Pavilion,
Expo'85, Tsukuba, Japan
Arquitecto: Keizo Sataka



REISER+UMEMOTO

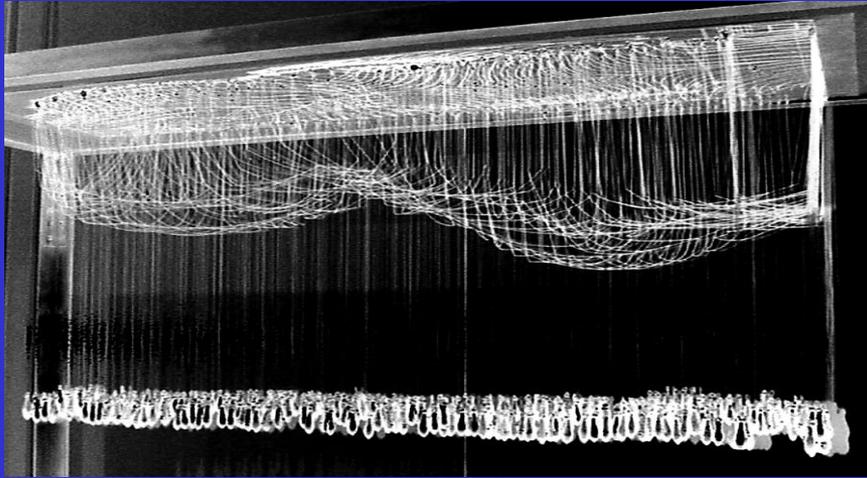
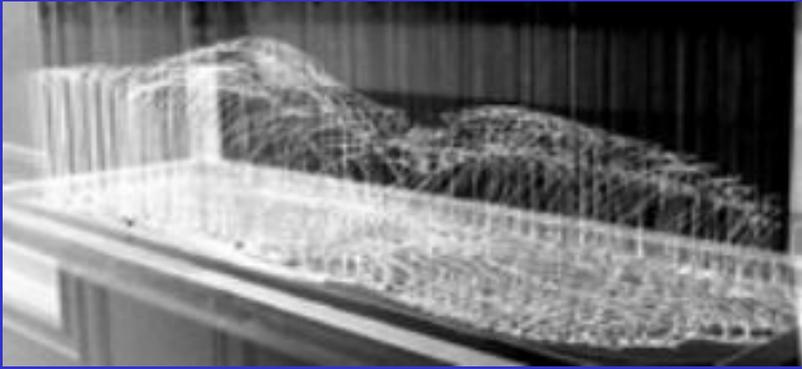




Fig. 8.15. Homenaje al agua (Chillida).

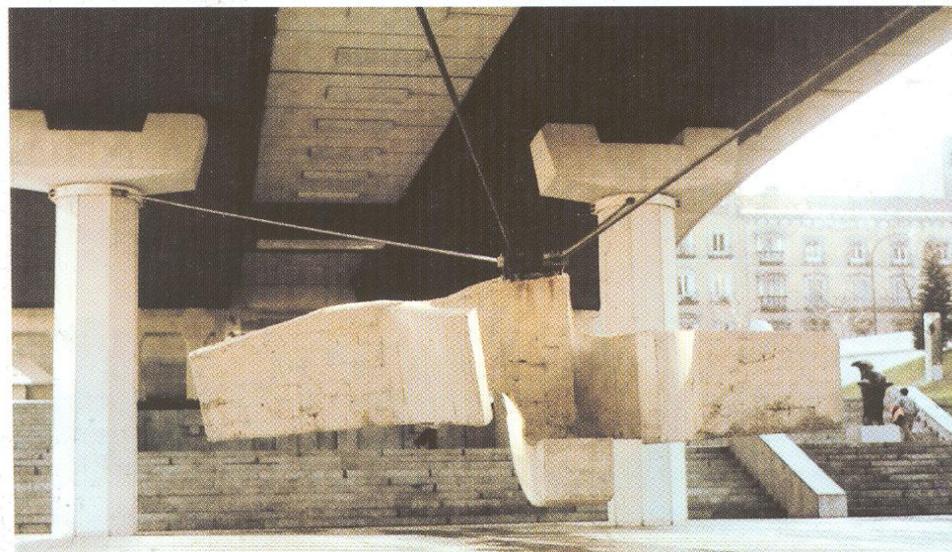


Fig. 8.16. Sirena varada (Chillida).

Puentes colgantes (en suspensión)

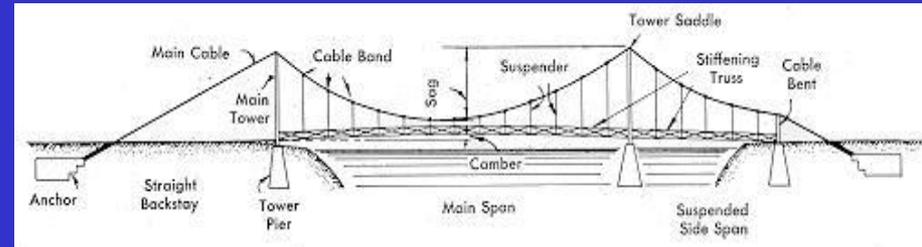
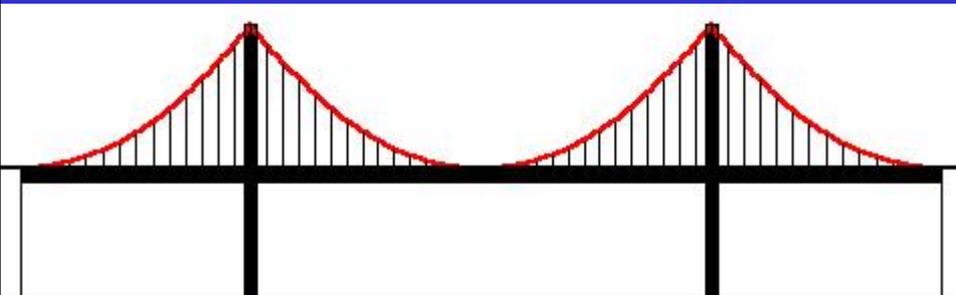


Fig. 8.18. Puente Sancho el Mayor (C.F. Casado, S.L.).

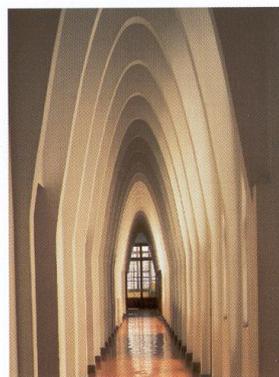
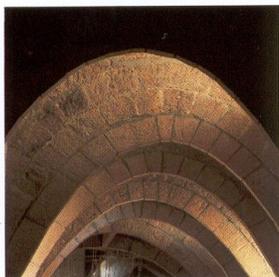


Fig. 8.17. Puente del Alamillo (S. Calatrava).

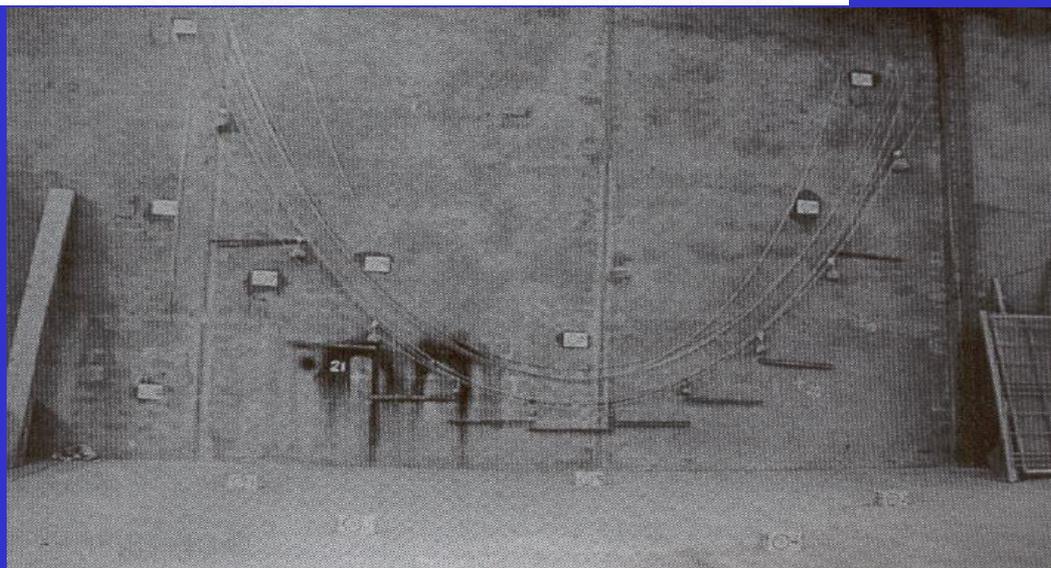
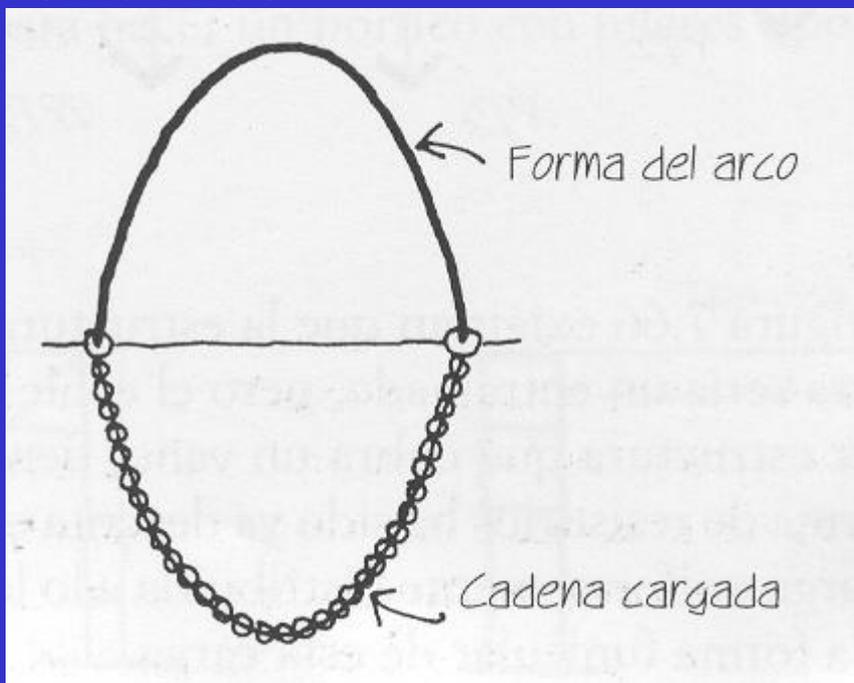
Puente Sancho El Mayor (C.L. Casado S.L.)

Puente del Alamillo (S. Calatrava)

La Catenaria como Arco



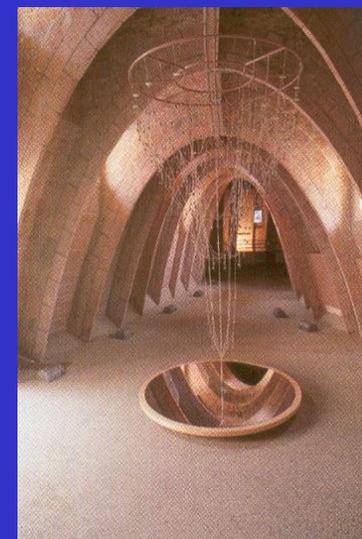
Arcos catenarios de las casas Milà y Batlló y del colegio de las Teresianas



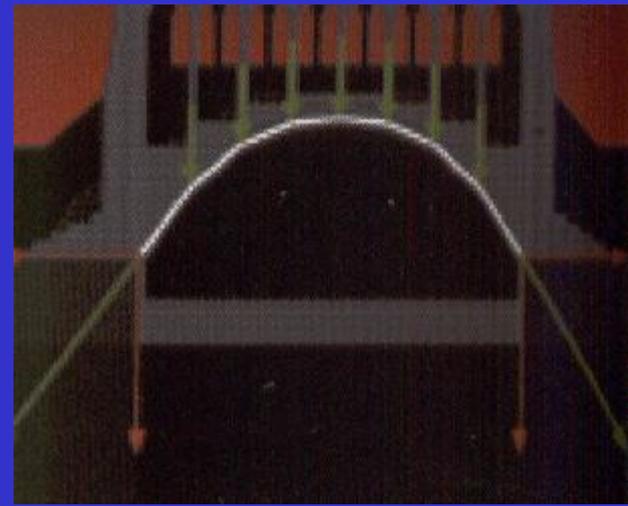
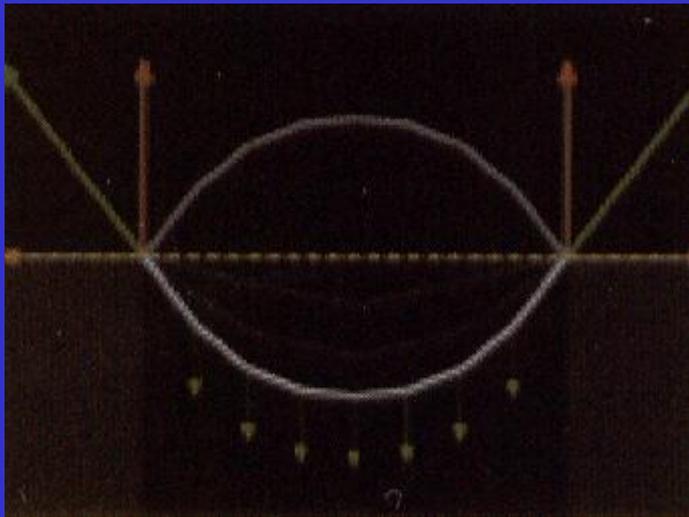
Robert Hooke (1675)

G. Polini (1748)

San Pedro de Roma



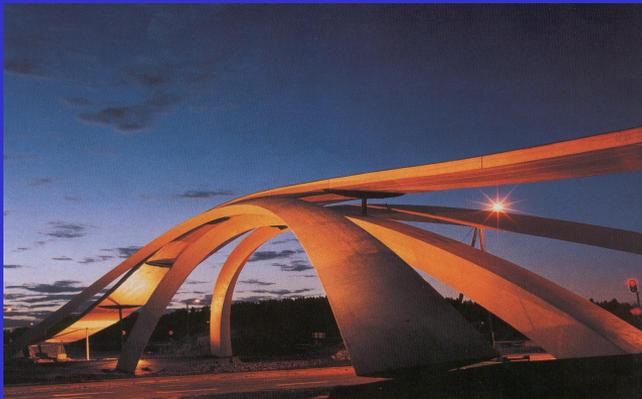
Traslación de arcos catenarios de la Casa Milà. Modelo catenario del Espai Gaudí



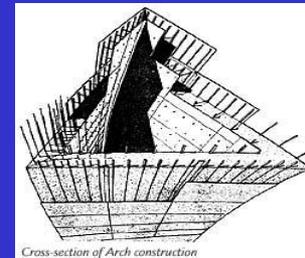
Arco Gateway de Saint Louis (EE.UU.)

Arquitecto Eero Saarinen (1963-67)

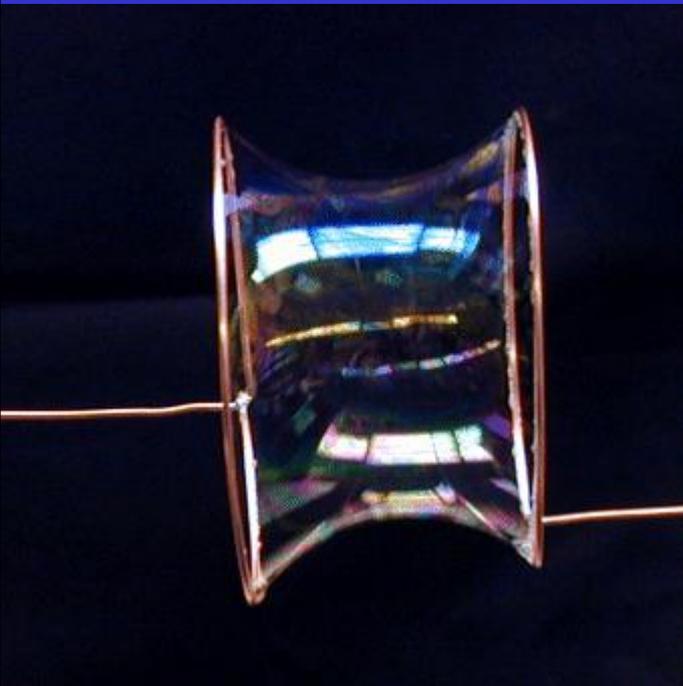
En conmemoración de la expansión al
oeste de los EE.UU. en el sigloXIX



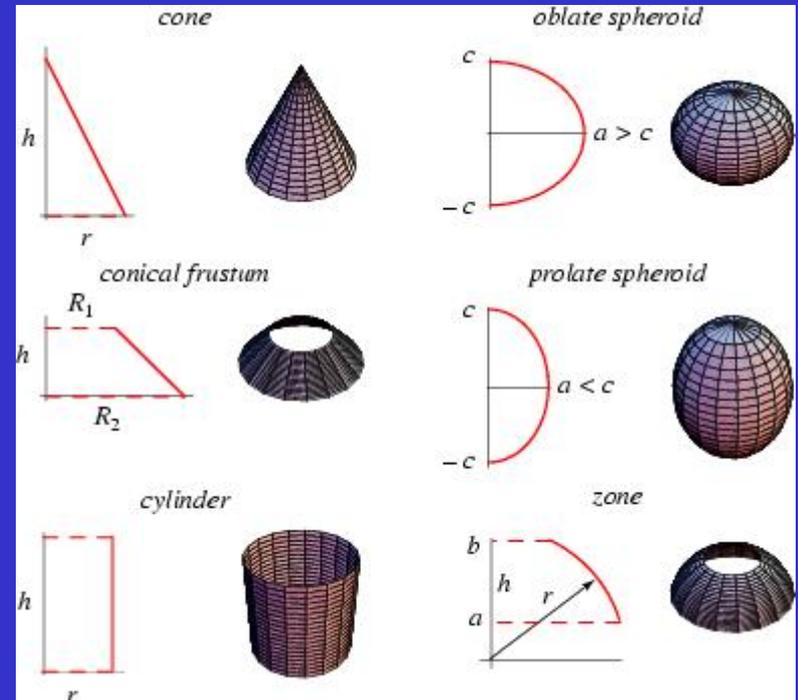
Vebjorn Sand, 2001: Leonardo da
Vinci, Puente del Cuerno de Oro
1502



Catenoide:

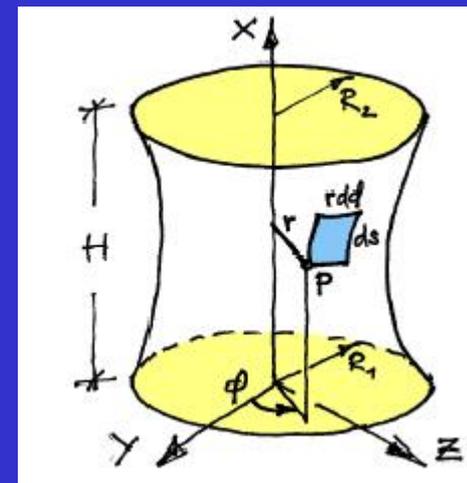


Superficies de revolución



Superficies de área mínima:

Leonhard Euler (1707-1783) 1743..

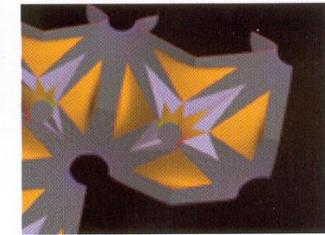
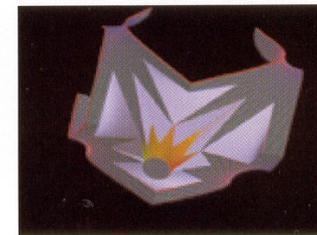
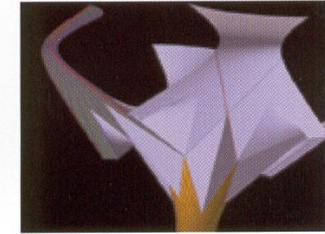
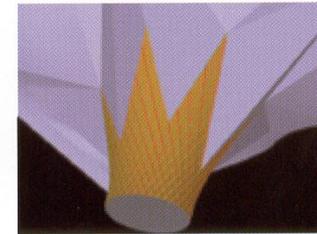
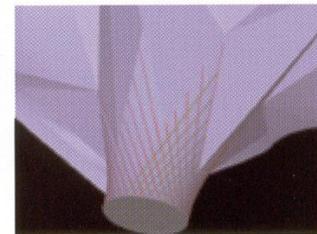
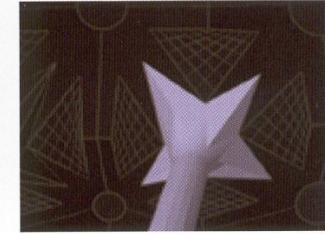
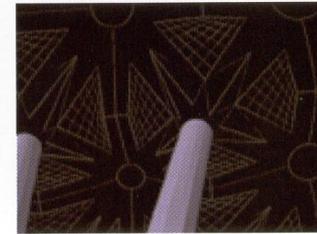
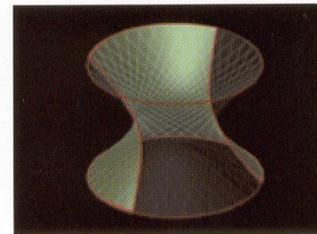
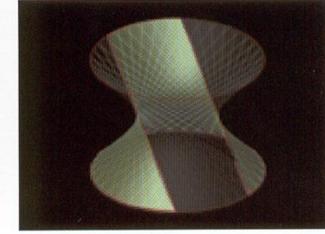
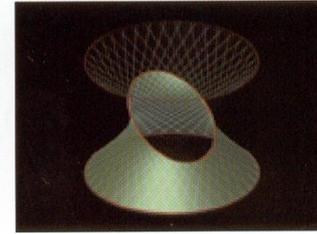
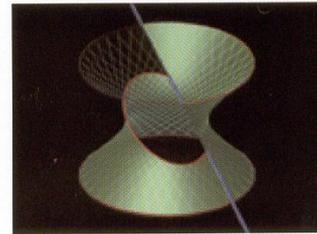
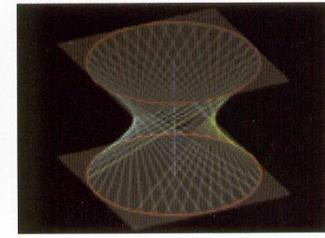
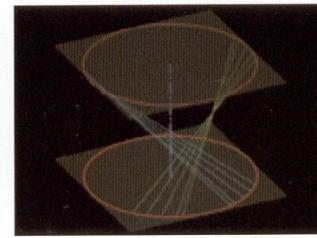
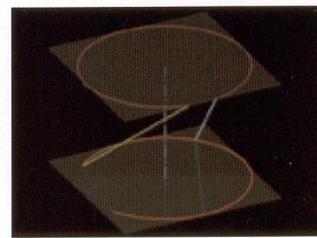


Hiperboloide de una hoja

Superficies regladas

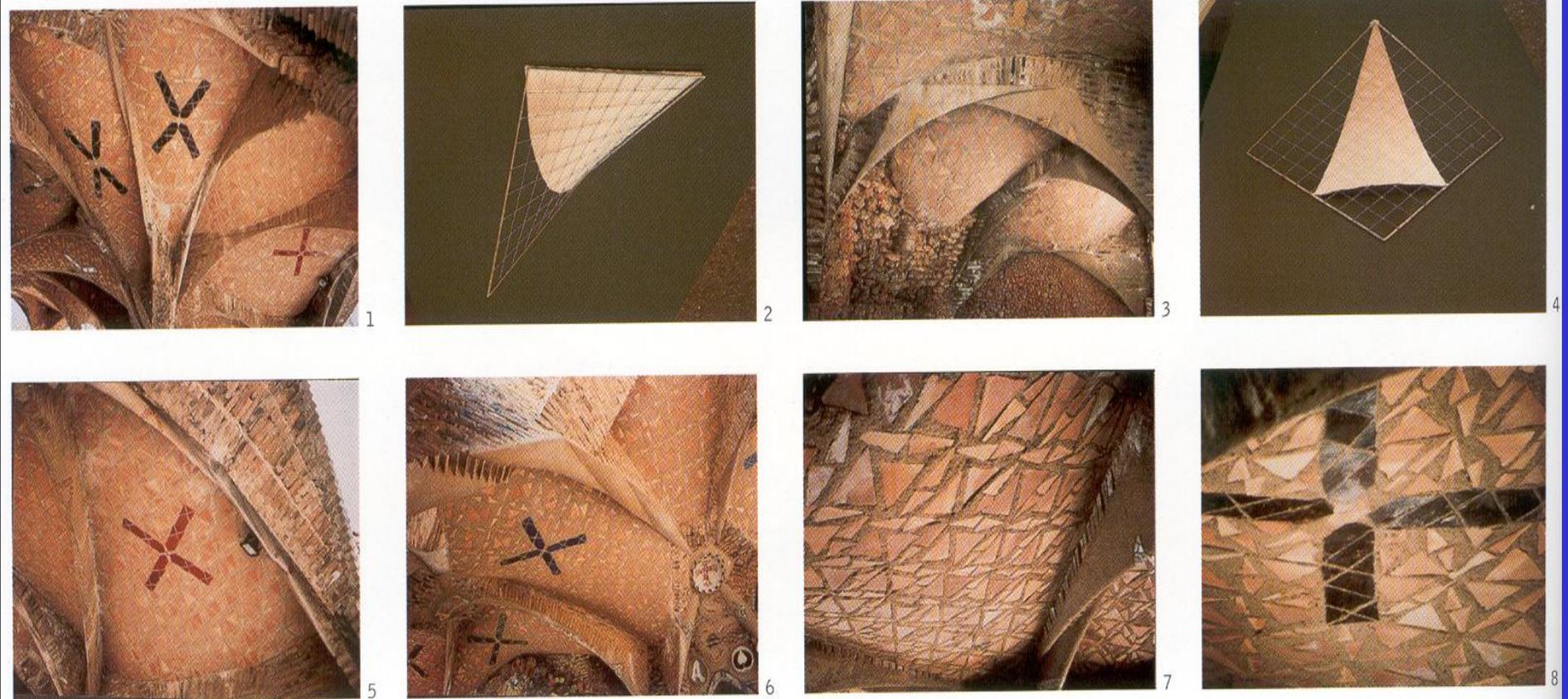
Gaudi:

“El uso de las superficies regladas es lógico por su superioridad plástica y su facilidad constructiva”



Superficies regladas

Parabolide hiperbólico

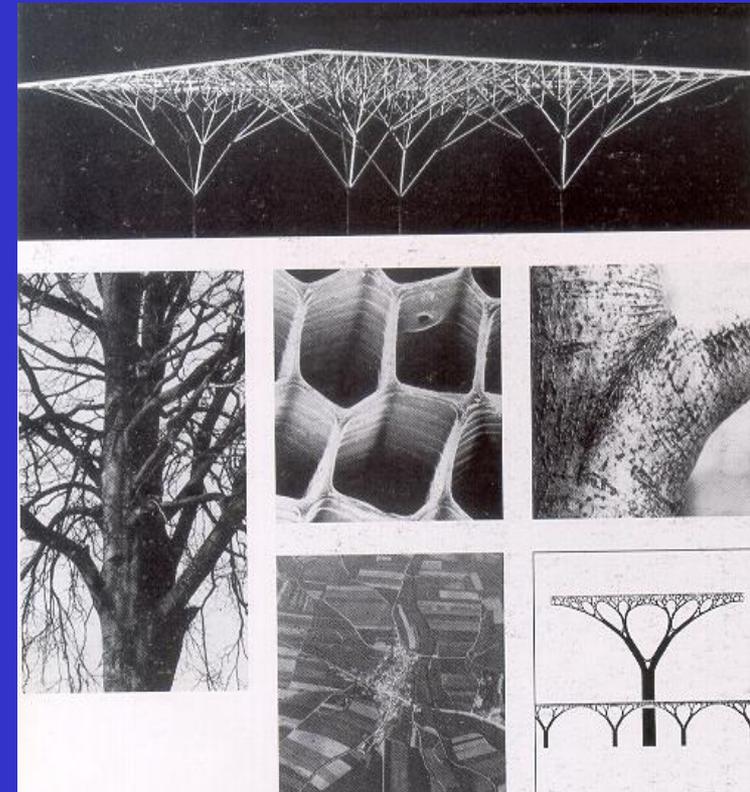
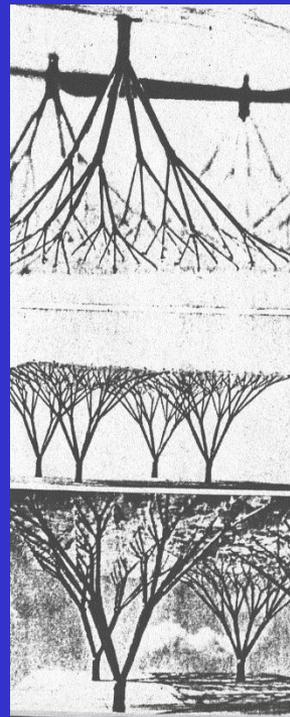


Vicente Candela,...

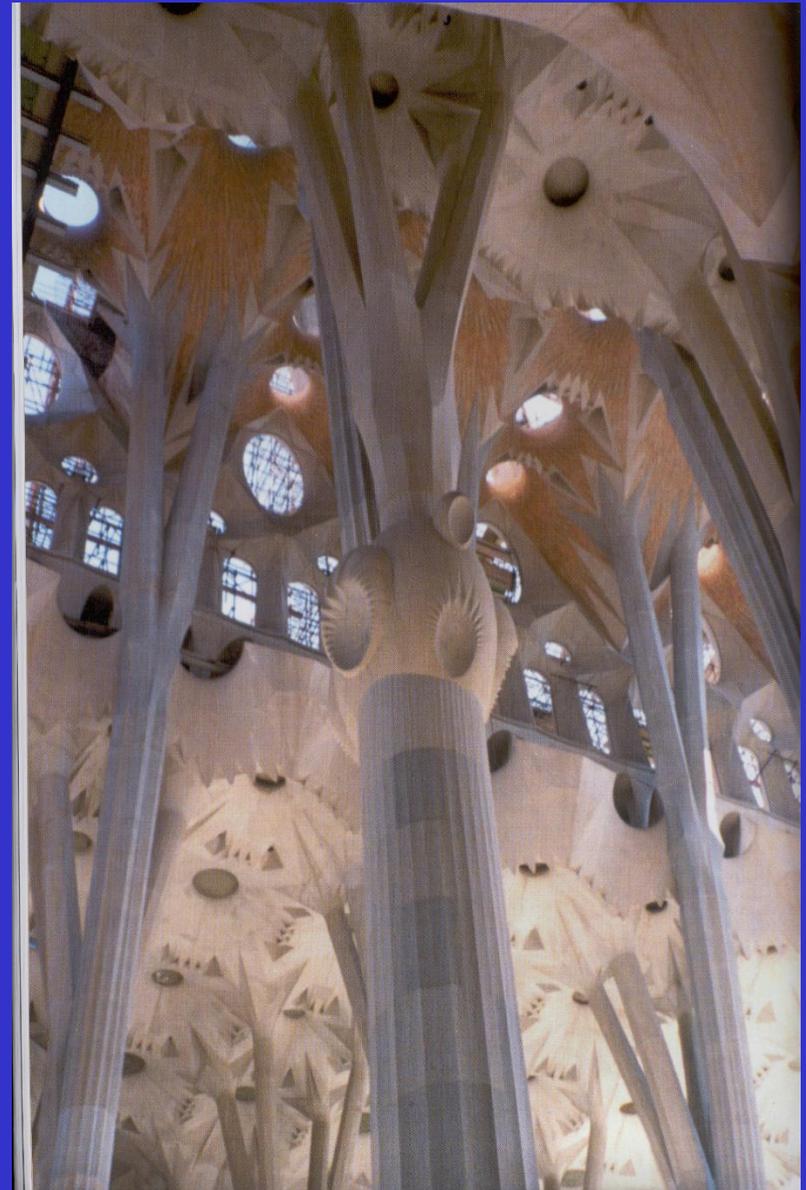
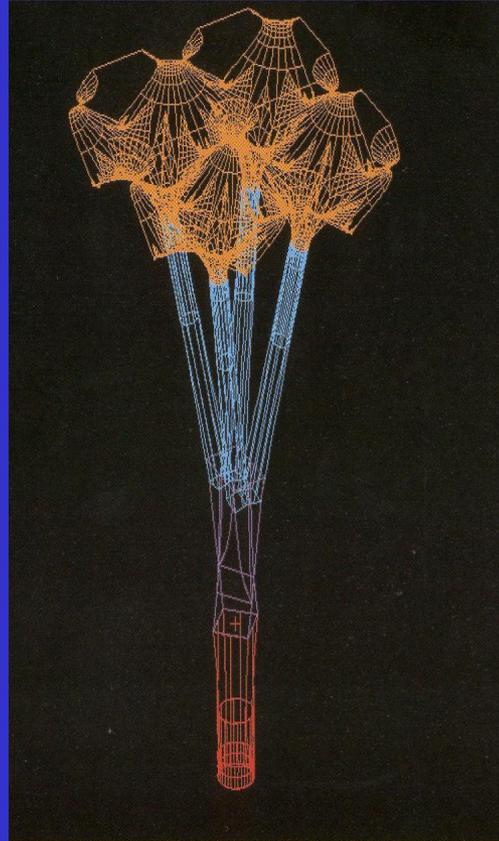
Estructuras arboriformes (fractales) Frei Otto, ...

Trabajo a compresión, menor
pandeo, menor sección

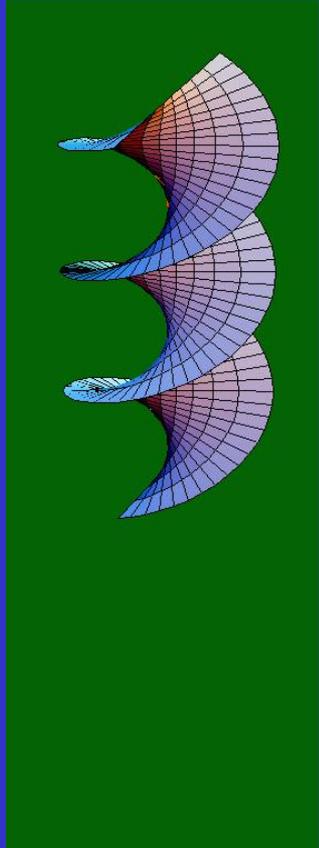
Yale University (1960)



Gaudi: fractalidad en el crecimiento de las ramas de los árboles



Deformación de una catenoide a una helicoide



Gaudí. La búsqueda de la forma

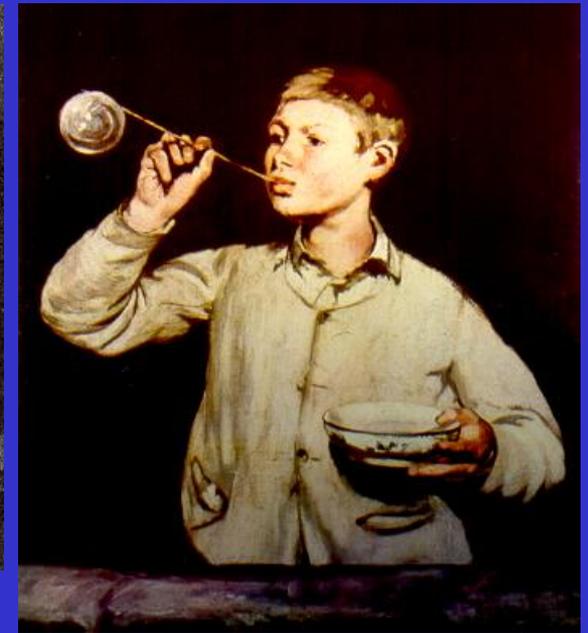


3. Frei Otto

Pompas de jabón. B.E. Murillo (1617-1682)



J.J. de Boissieu, 1799

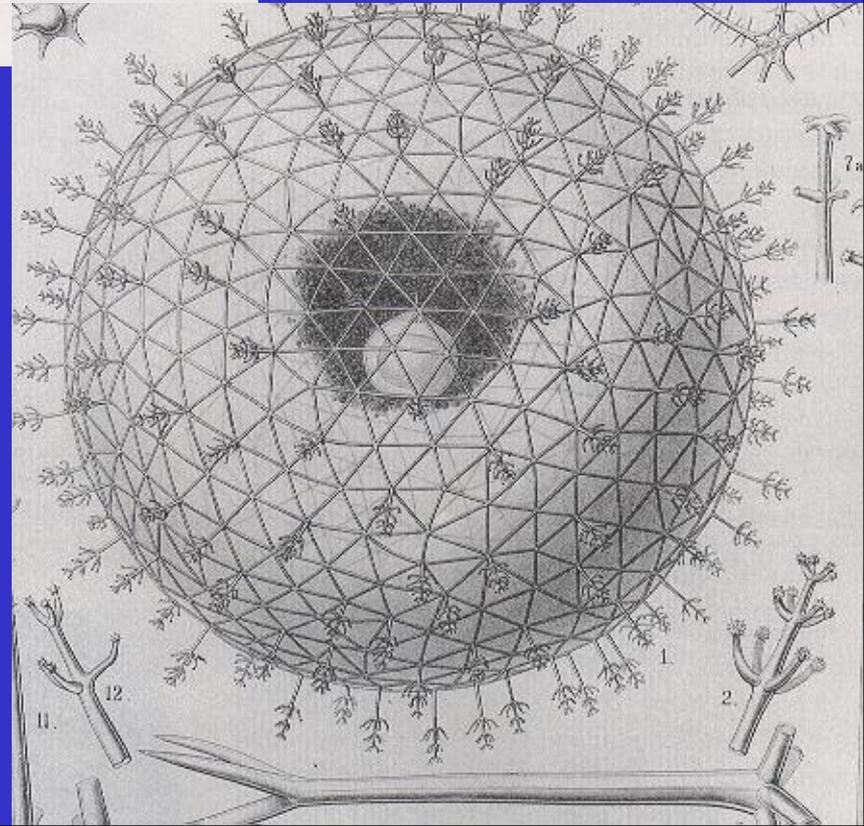
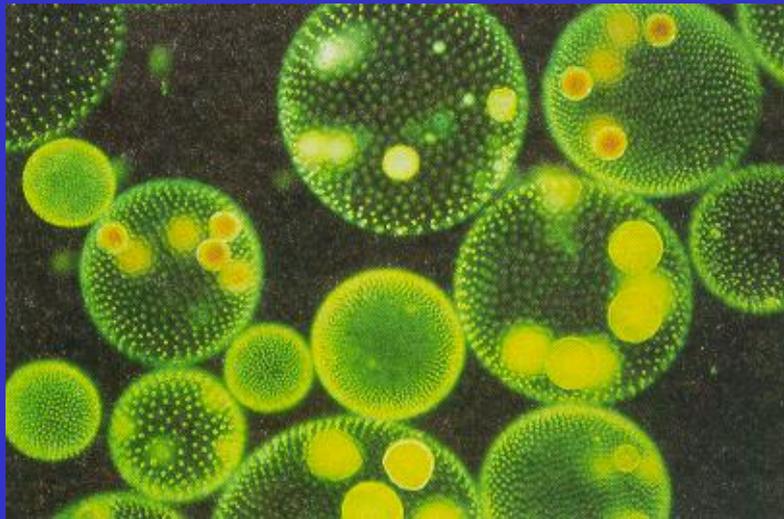
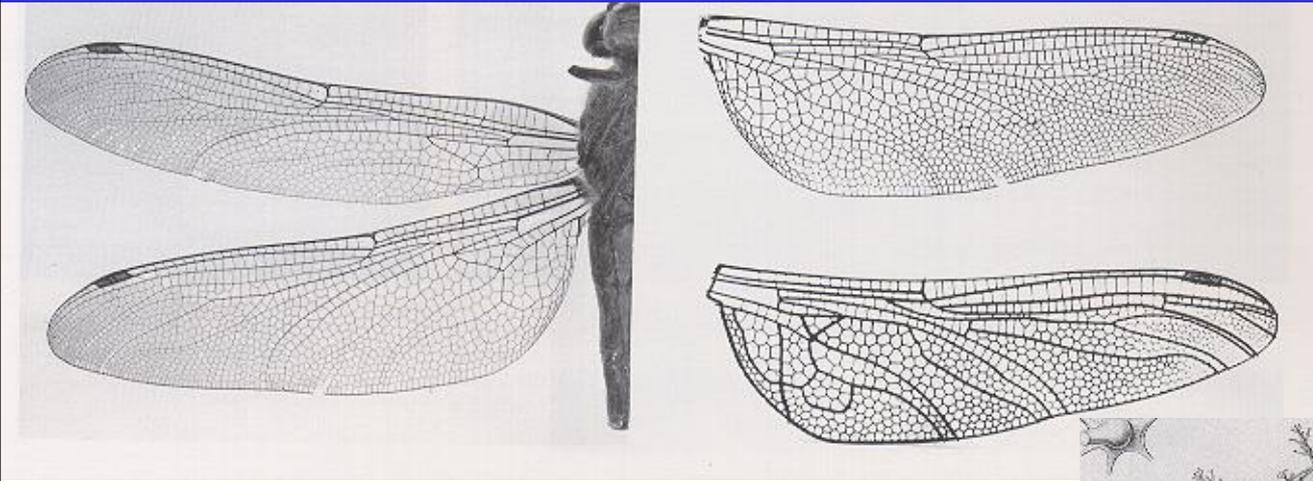


J.B.S. Chardin (1698-1779):
1739

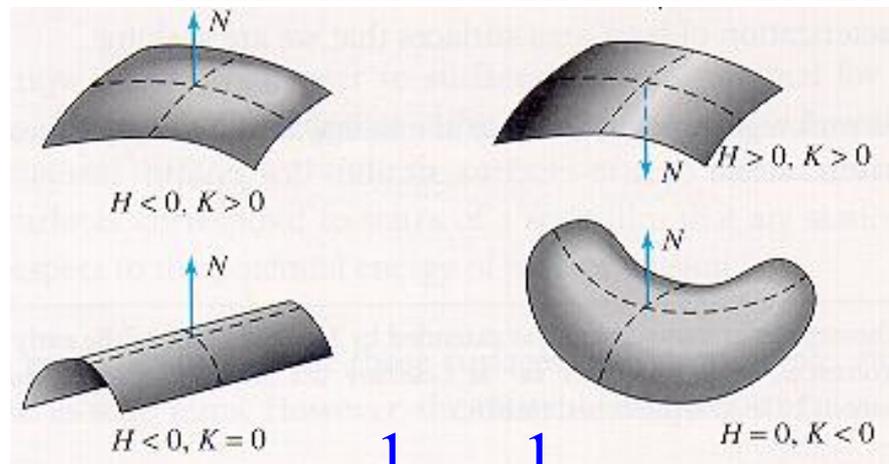
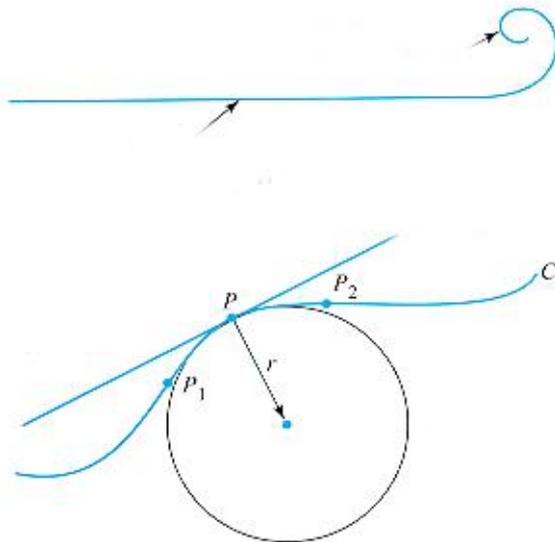
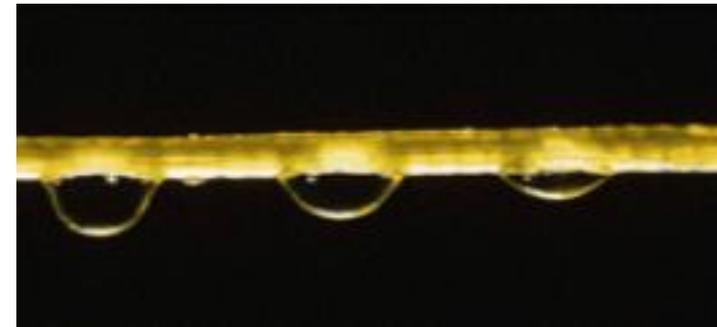
E. Manet (1832-1883): 1867

***Superficies esféricas \Rightarrow área mínima (para un volumen dado).**

En el “mundo animal”

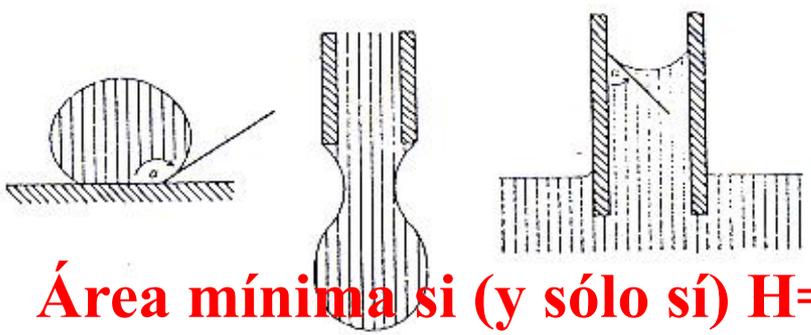


Fluidostática: Curvatura media



$$p = \sigma \left(\frac{1}{R_1} + \frac{1}{R_2} \right) = \sigma 2H$$

P. S. de Laplace (1749-1827),
Th. Young (1773-1829), C.F.
Gauss (1777-1855),



Área mínima si (y sólo si) $H=0$, Superficies jabonosas= minimales

Variación de un funcional, Derivada Gateaux Ecuación de Euler-Lagrange,

$$J(x) = \int_{t_0}^{t_1} L(t, x, x') dt \rightarrow \text{Inf} \quad x(t_0) = x_0, x(t_1) = x_1$$

$$\delta J(x, h) = \int_{t_0}^{t_1} \left(\frac{\partial L}{\partial x} - \frac{d}{dt} \frac{\partial L}{\partial \dot{x}} \right) \Big|_{x(t)} h(t) dt, \quad h(t_0) = h(t_1) = 0.$$

Variación de J

$$\delta J = 0$$

$$x_0(t) + \alpha h(t)$$

Variación de x(t)

Ecuación de Euler-Lagrange

$$-\frac{d}{dt} L_{\dot{x}}(t, x_0(t), \dot{x}_0(t)) + L_x(t, x_0(t), \dot{x}_0(t)) = 0.$$

Cálculo del área de una superficie

$$S = \int_S |\mathbf{T}_u \times \mathbf{T}_v| du dv,$$

$$S = \iint_R \sqrt{\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 + 1} dA,$$

$$a = \int da = \int_{\varphi} \int_{r} ds r d\varphi = \int_0^{2\pi} \int_0^H r \sqrt{1 + \left(\frac{dr}{dx}\right)^2} dx d\varphi$$

Superficies de área mínima y de curvatura media prescrita

MinJ

$$J(x, u, \nabla u) = \int (\sqrt{1 + |\nabla u|^2} + h(x, u)) dx$$

área de la superficie no paramétrica

$$\operatorname{div}\left(\frac{\nabla u}{\sqrt{1 + |\nabla u|^2}}\right) = H(x, u), h' = H$$

**H=curvatura
media,**

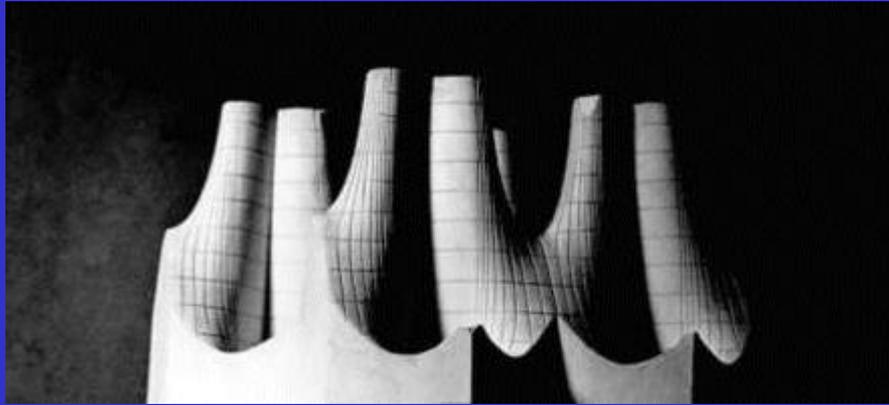
**H=0 área
mínima**



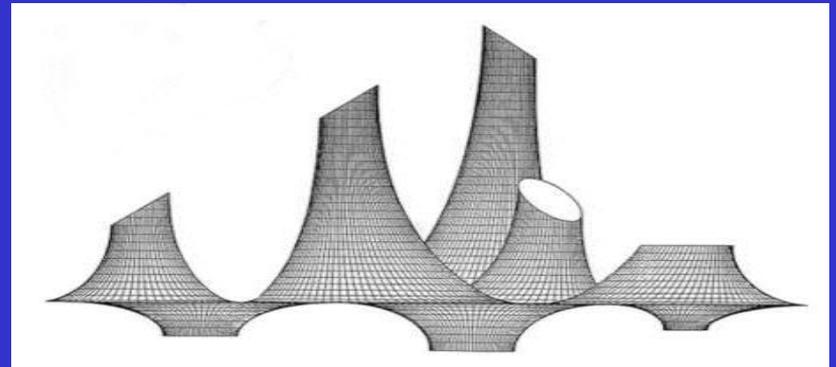
Interior view of the Olympic Stadium.

MINIMAL SURFACES AS ROOFS AND TENT

Otras superficies mínimas: generación.



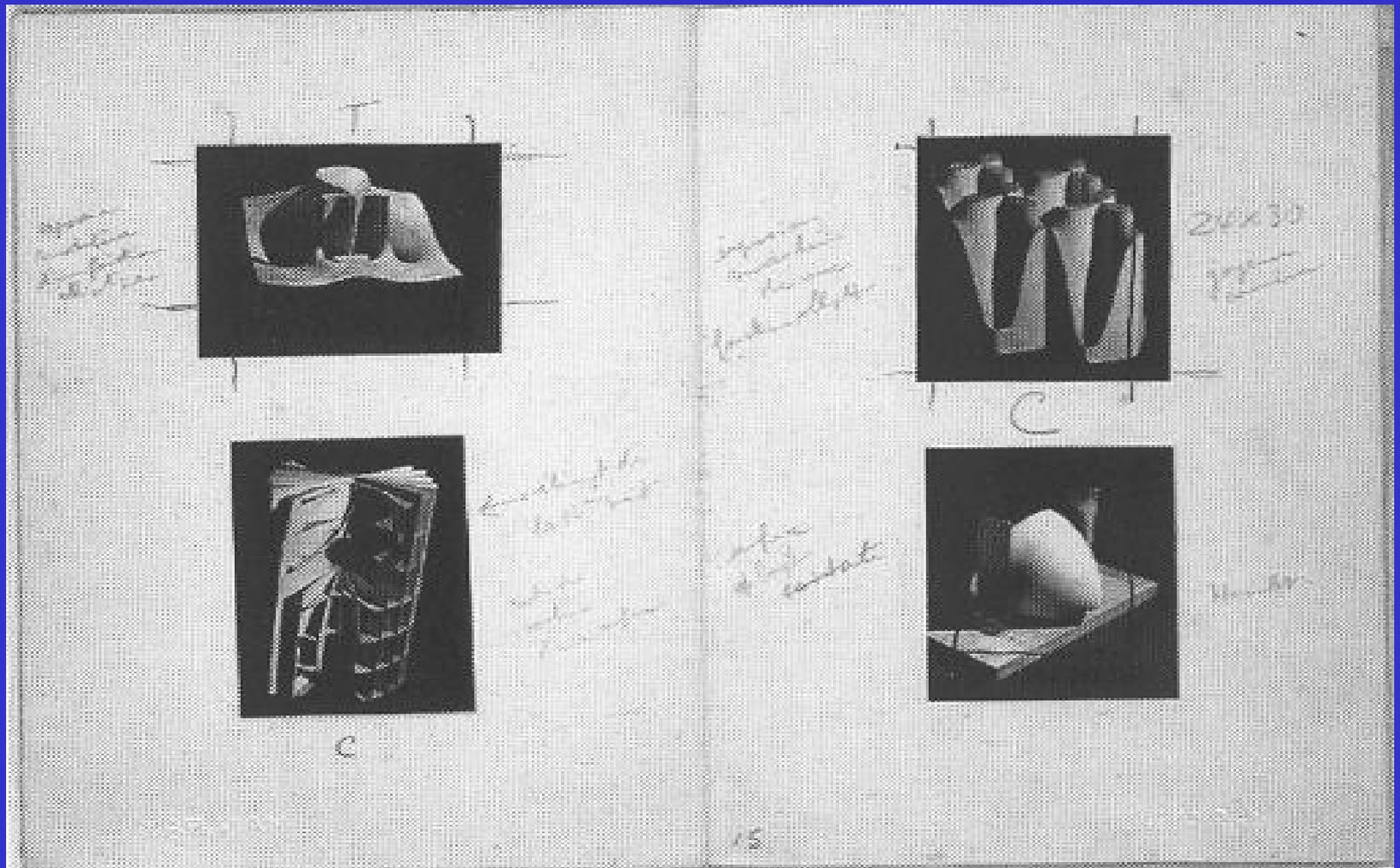
Eduardo Catalano



Modelo para la generación
de superficies en tensión
(School of Textiles and
Materials Technology,
Philadelphia University)

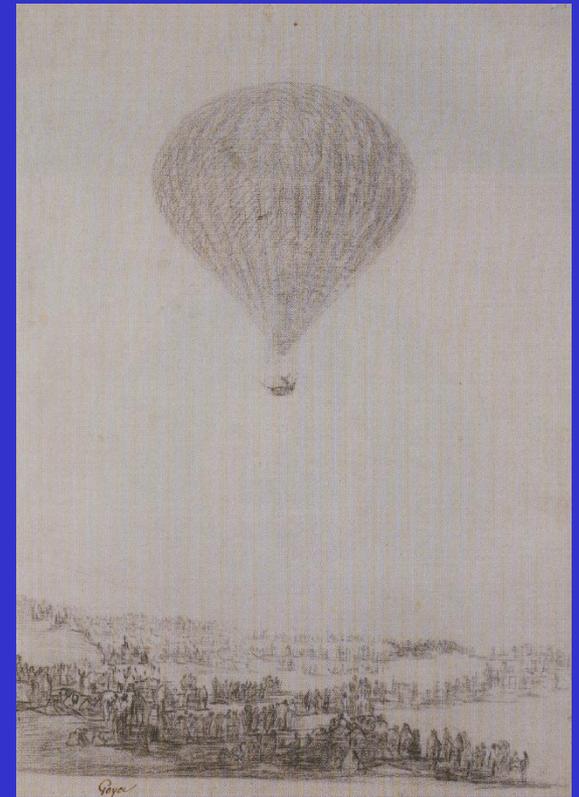
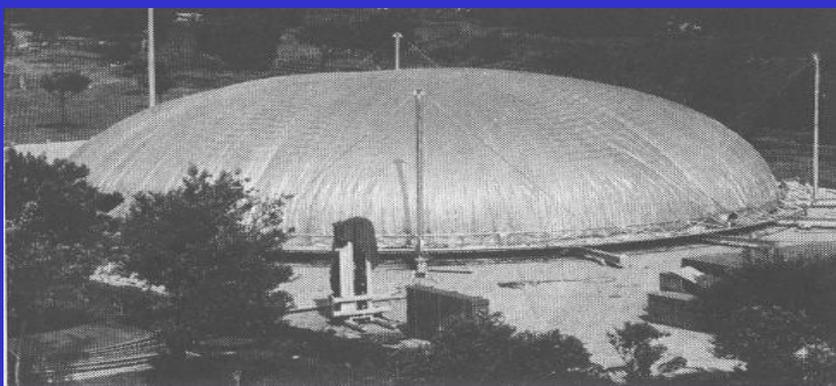


Paraboloide hiperbólico
(Torroja, Candela, ...)



ManRay (Objet mathématique, Shakespeare Equations, 1935-1948)

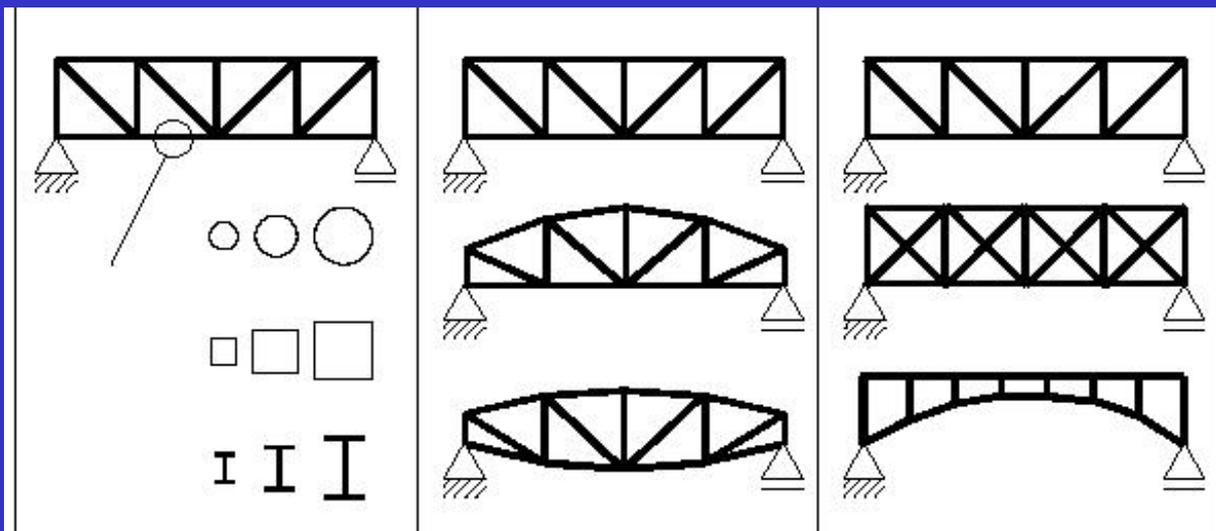
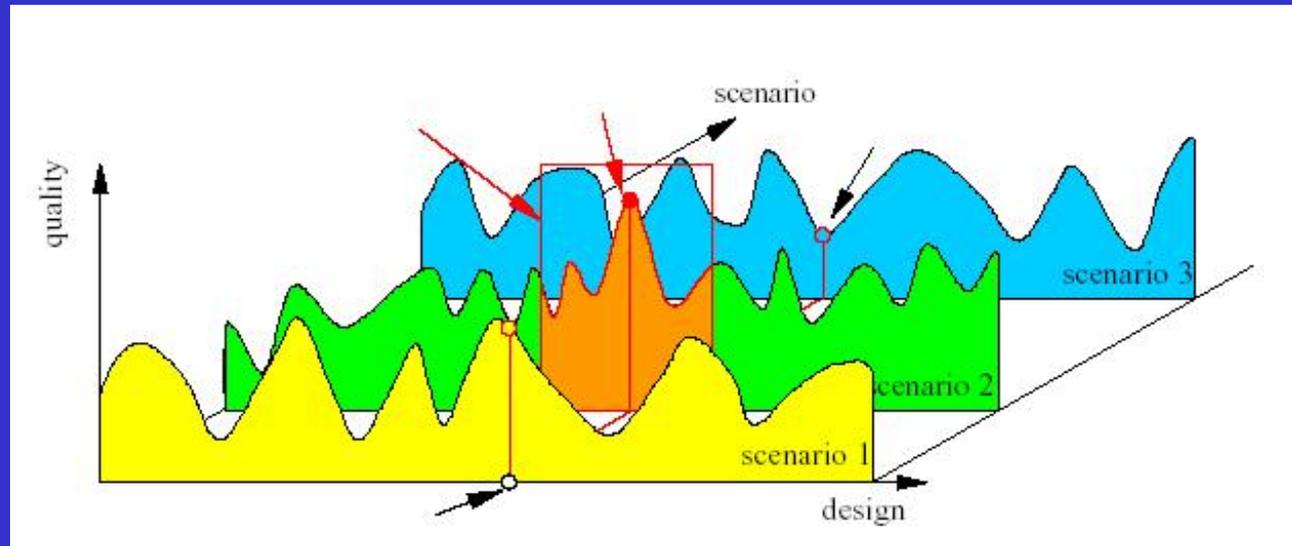
Superficies neumáticas

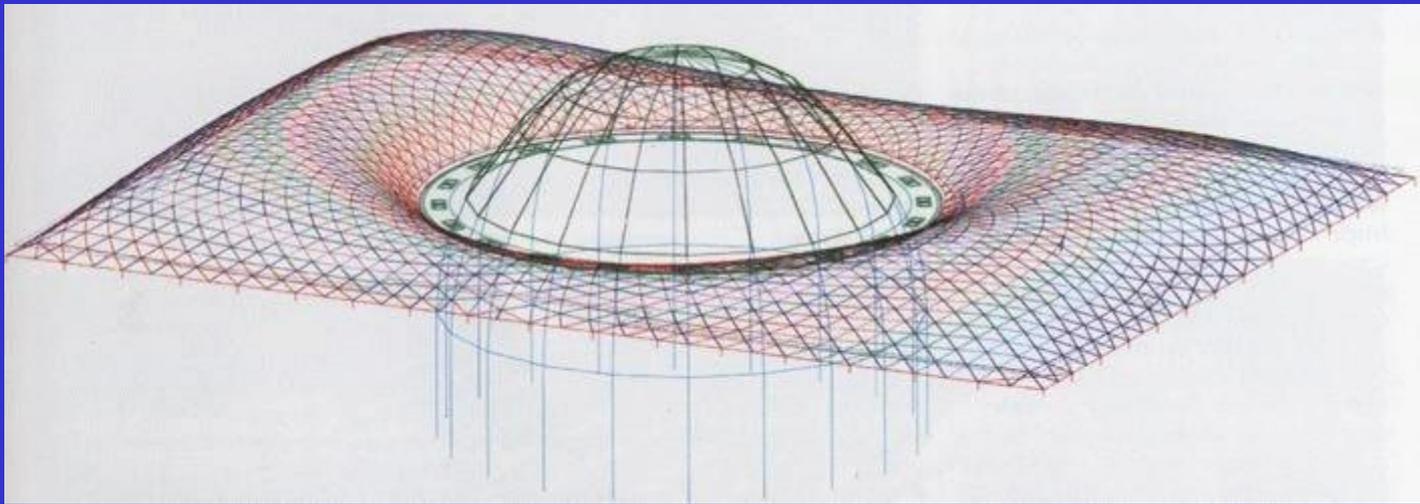
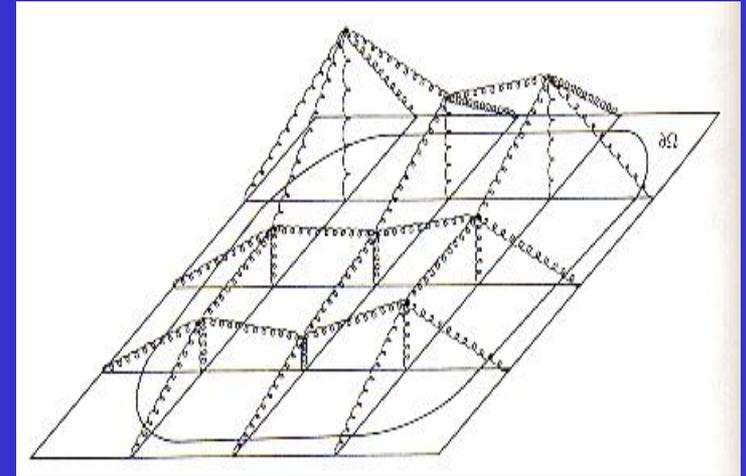
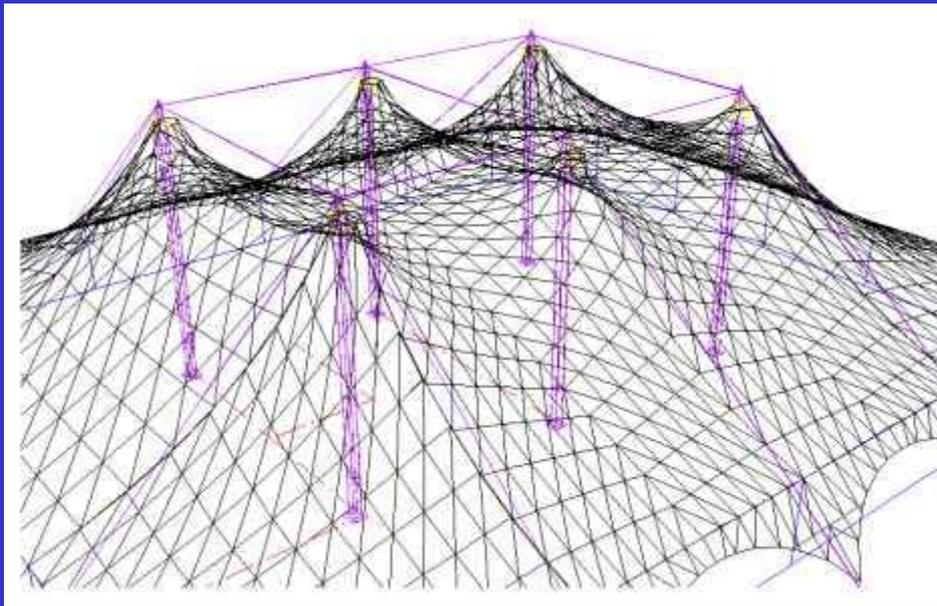


**Goya (El Montgolfier,
1800-1808)**

**Pabellón móvil de la RAI
(Achile y P. G.
Castiglioni, 1967)**

Optimizar subestructuras óptimas





British Museum Roof. Architects Foster and Partners

La cuerda vibrante

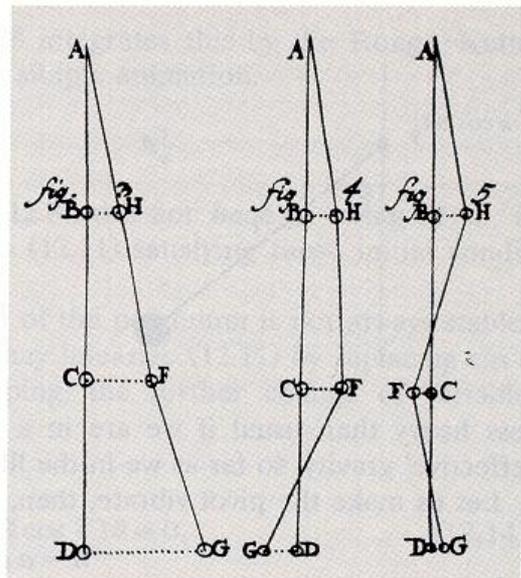
Sonidos musicales y Matemáticas

Pitágoras de Samos: 580-500 a.C. (teoría de la armonía)

Experimentalmente: Vibraciones de una cuerda en modos (mitades, tercios,...): Bien conocido en 1700



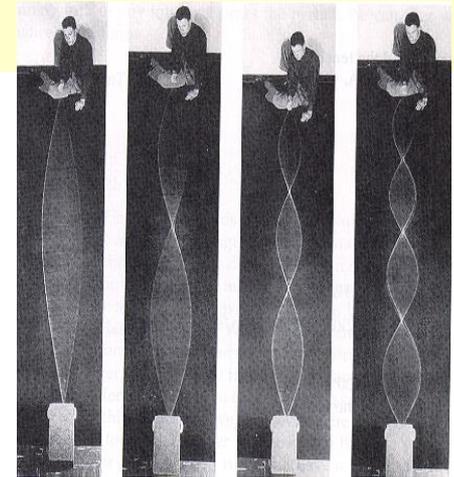
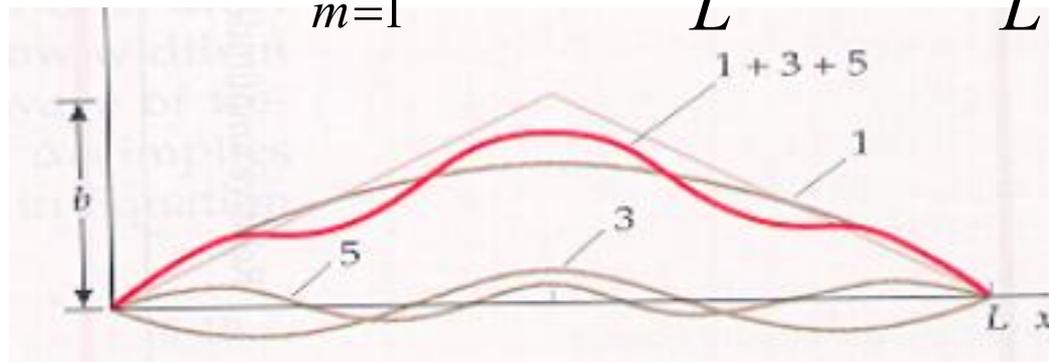
1739: Daniel Bernoulli: *Investigación sobre una nueva teoría de la música, claramente expuesta a partir de incontestables principios de la armonía*



•1746: Jean Le Rond D'Alembert: “Investigaciones sobre la curva que forma una cuerda tensa que se hace vibrar”

1751: Daniel Bernoulli, “coexistencia formal de oscilaciones pequeñas”

$$u(x,t) = \sum_{m=1}^{m=N} a_m \operatorname{sen}\left(\frac{m\pi x}{L}\right) \cos\left(\frac{m\pi ct}{L}\right)$$



1759: Joseph Louis Lagrange,
incorpora velocidad inicial no nula,
pasa al límite $N \rightarrow \infty$



1736-1813

Problema de intercambio de integral y suma infinita,

Problema de derivación de una serie (regularidad de la función límite)

1811: Joseph Fourier “Teoría analítica del calor”.

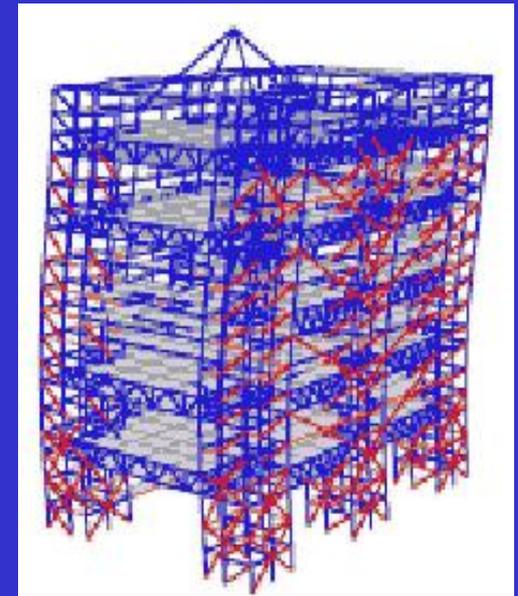
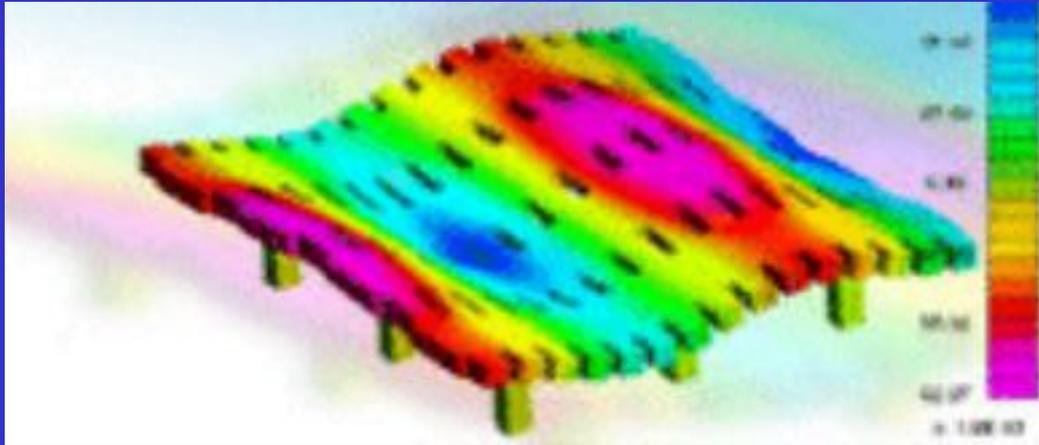


La cuerda vibrante: de Fourier al 2000

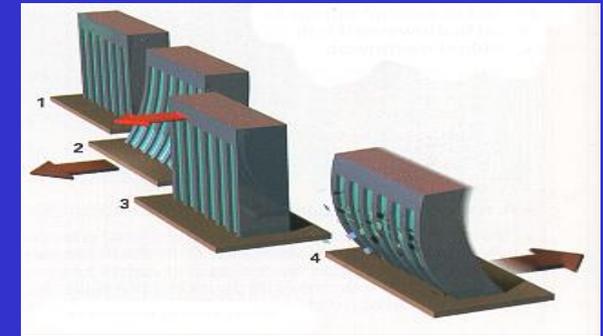
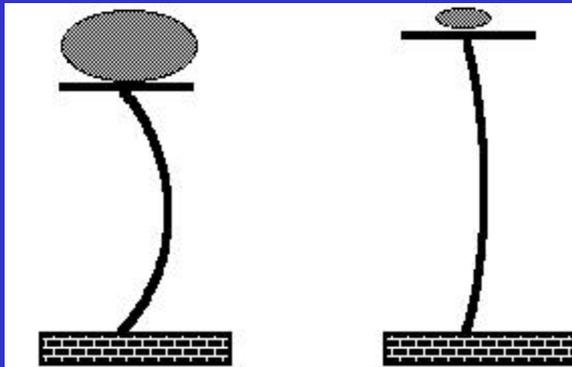
(buen título de una conferencia para matemáticos)

1768-1830

Dinámica:



caos en una viga

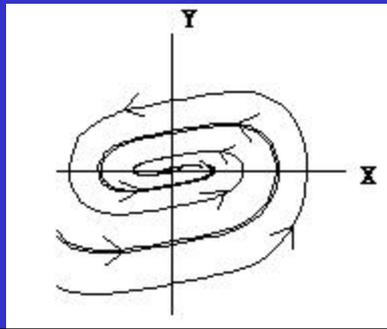


Columna en movimiento (peso despreciable)
con fricción

P. G. L. Dirichlet (1805-1859),

• H. Poincaré (1854-1912),

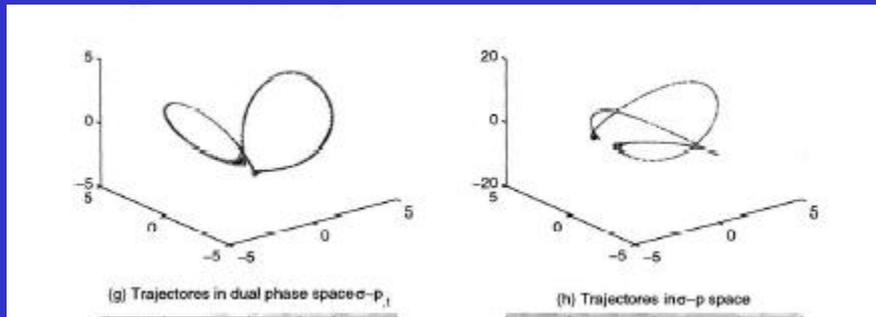
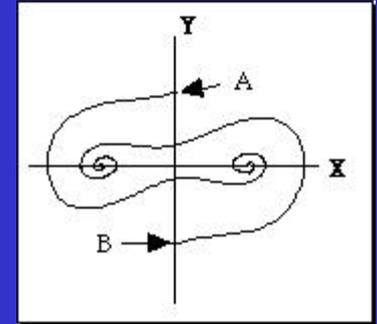
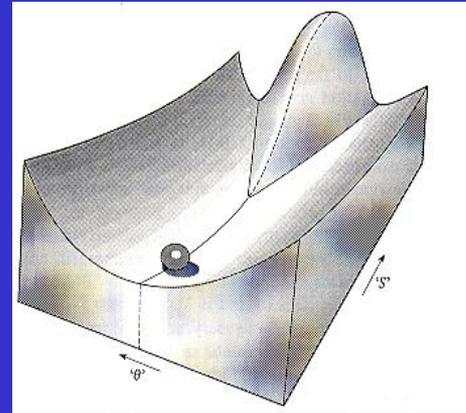
• A.M. Lyapunov (1857-1918),...



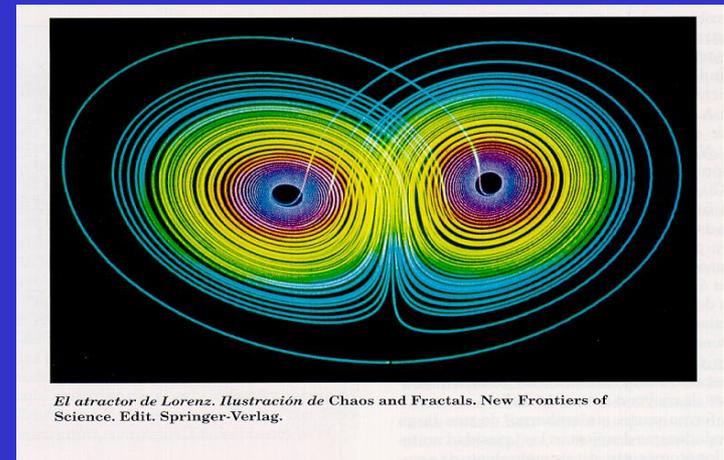
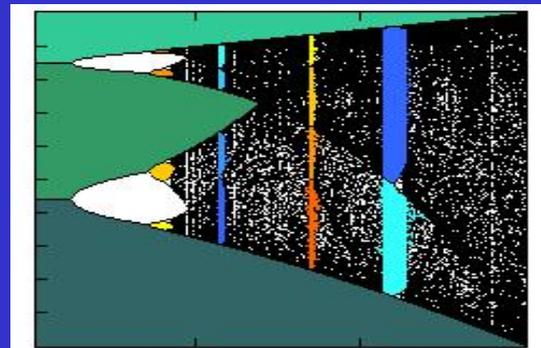
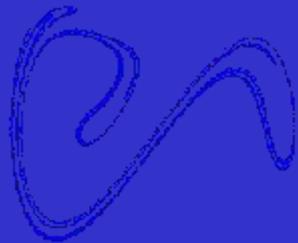
$$\frac{d^2x}{dt^2} + b \frac{dx}{dt} - x + x^3 = F \cos(\omega t)$$

Ecuación de Duffing

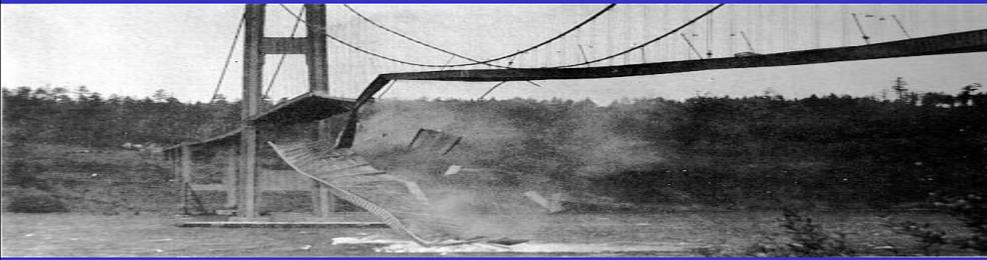
Con peso: dos equilibrios



Cascada de bifurcaciones de soluciones periódicas



El atractor de Lorenz. Ilustración de Chaos and Fractals. New Frontiers of Science. Edit. Springer-Verlag.



**Takoma Narrows
(Washington, 1940)**

DISASTER!
The Greatest
Camera Scoop
of all time!

CARTER FILMS



Interior view of the Olympic Stadium.

MINIMAL SURFACES AS ROOFS AND TENT

