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Preface

This collection of articles reflects some of the main subjects discussed at the International Conference on Partial Differential Equations, held at the University of Fez, Fez, Morocco. All the articles in this volume were subject to a strict refereeing process. Most of the papers reflect the authors' contribution to the conference, the purpose of which was to present recent progress and new trends in partial differential equations (PDE). The papers appearing in this volume adhere to this comprehensive goal. Some of the papers are surveys, while others contain significant new results. It is our hope that the volume will be a valuable source for specialists in PDE. Further, by providing extensive references, it should help young researchers to find valuable literature. Topics treated include eigenvalue problems, maximum principle, degenerate equations, elliptic and parabolic systems, and asymptotic behavior of solutions.

The conference was organized by the Faculty of Sciences, Dhar Mahraz, of Fez. Financial support came from the Faculty of Sciences and Technology of Fez, the International Mathematical Union, and the European Mathematical Society. Many colleagues in Fez worked hard in the organization of the conference and in the preparation of this volume, in particular, E. Azroul, A. Benlemlih, A. Elkhalil, and A. Elmahi and the researchers Y. Akdim and S. Elmanouni. It is a pleasure for us to thank all the people and institutions who contributed to the success of the conference and the realization of this volume.

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On the Asymptotic Behavior of Solutions of a Damped Oscillator under a Sublinear Friction Term: From the Exceptional to the Generic Behaviors

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1 Introduction

We study the asymptotic behavior of solutions of the equation

$$mx_{tt} + \mu |x_t|^{\alpha-1} x_t + \omega^2 x = 0 \tag{1}$$

where

$$\alpha \in (0, 1) \tag{2}$$

and $\mu, \omega^2 > 0$. We shall work with the formulation

$$x_{tt} + |x_t|^{\alpha-1} x_t + x = 0 \tag{3}$$

which is attaint by dividing by ω^2 and by introducing the rescaling $\tilde{x}(\tilde{t}) = \beta^{1/(\alpha-1)} x(\lambda \tilde{t})$ where

$$\lambda = \frac{\sqrt{m}}{\omega} \text{ and } \beta = \frac{\mu}{\omega^{2-\alpha} m^{\alpha/2}}. \tag{4}$$

Notice that the x-rescaling uses the assumption (2) (it fails for $\alpha = 1$) and that in formulation (3) we have not written the label $\tilde{}$ for the sake of the notation.

The limit case $\alpha \to 0$ corresponds to the Coulomb friction equation

$$x_{tt} + sign(x_t) + x \ni 0 \tag{5}$$

where $sign$ is the maximal monotone graph of $\mathbb{R}^2$ given by

$$sign(r) = \begin{cases} -1 & \text{if } r < 0, \\ [-1, 1] & \text{if } r = 0, \\ 1 & \text{if } r > 0. \end{cases} \tag{6}$$

The limit equation when $\alpha \to 1$ corresponds with the linear damping equation

$$x_{tt} + x_t + x = 0. \tag{7}$$

We recall that, even if the nonlinear term $|x_t|^{\alpha-1} x_t$ is not a Lipschitz continuous function of x_t (recall (2)), the existence and uniqueness of solutions of the associate Cauchy problem

$$P_\alpha \begin{cases} x_{tt} + |x_t|^{\alpha-1} x_t + x = 0 & t > 0, \\ x(0) = x_0 \\ x_t(0) = v_0 \end{cases} \tag{8}$$

(and of the limit problems P_0 and P_1 corresponding to the equations (5) and (7) respectively) is well known in the literature: see, e.g. Brezis [1]. An easy application of the results of the above reference yields to a rigorous proof of the convergence of solutions when $\alpha \to 0$ and $\alpha \to 1$.

The asymptotic behavior, for $t \to \infty$, of solutions of the limit problems P_0 and P_1 is well known (see, for instance, Jordan and Smith [3]). In the first case the decay is exponential. In the second one it is easy to see that "given x_0 and v_0 there exist a finite time $T = T(x_0, v_0)$ and a number $\zeta \in [-1, 1]$ such that $x(t) \equiv \zeta$ for any $t \geq T(x_0, v_0)$". For problem P_α it is well-known that $(x(t), x_t(t)) \to (0, 0)$ as $t \to \infty$ (see, e.g. Haraux [2]).

The main result of this paper is to show that the generic asymptotic behavior above described for the limit case P_0 is only exceptional for the sublinear case $\alpha \in (0, 1)$ since the generic orbits $(x(t), x_t(t))$ decay to $(0, 0)$ in a infinite time and only two of them decay to $(0, 0)$ in a finite time: in other words, when $\alpha \to 0$ the exceptional behavior becomes generic.

2 Formal results via asymptotic arguments

We can rewrite the equation (3) in as the planar system

$$\begin{cases} x_t = y \\ y_t = -x - |y|^{\alpha-1} y \end{cases} \tag{9}$$

which, by eliminating the time variable, for $y \neq 0$, leads to the differential equation of the orbits in the phase plane

$$y_x = \frac{-x - |y|^{\alpha-1} y}{y} \tag{10}$$

and that allows us to carry out a phase plane description of the dynamics.

We remark that the plane phase is antisymmetric since if $y = \varphi(x)$ is a solution of (10) then the function $y = -\varphi(-x)$ is also solution. So, it is enough to describe a semiplane (for instance $x \geq 0$). On the other hand, it is easy to see that $(1/x, 1/y)$ satisfy a system which has the point $(0, 0)$ as a spiral unstable critical point. For values of $x^2 + y^2 >> 1$ the orbits of the system are given, in first approximation, by $x^2 + y^2 = C$ because $|y|^{\alpha-1} y$ is small compared with x. The effect of this term is to decrease slowly C with time giving the trajectory a spiral character.

We shall prove that there are two modes of approach to the origin and so that the origin $(0, 0)$ is a node for the system (9). The lines of zero slope are given by

$$-x = |y|^{\alpha-1} y. \tag{11}$$

So the convergence to $(0, 0)$ is only possible through the regions

$$\{(x, y) : x > 0,\ y < -x^{1/\alpha}\} \cup \{(x, y) : x < 0,\ y > (-x)^{1/\alpha}\} \tag{12}$$

Let us see that the "ordinary" mode corresponds to orbits that are very close to the ones corresponding to small effects of the inertia. Due to the symmetry it is enough to describe this behavior for the orbits approaching the origin with values of $x > 0$ and $y < 0$. Let $-y = \tilde{y} > 0$. Equation (10) takes the form

$$\tilde{y}\tilde{y}_x = -x + \tilde{y}^\alpha. \tag{13}$$

The line of zero slope is

$$\tilde{y} = x^{1/\alpha} \tag{14}$$

and we search for orbits obeying, for $0 < x << 1$, to the expression

$$\tilde{y} = x^{1/\alpha} + z(x) \tag{15}$$

for some function $z(x)$. If we anticipate the condition $0 < z(x) << x^{1/\alpha}$, equation (10) takes the "linearized form"

$$\frac{1}{\alpha} x^{(\frac{1}{\alpha}-1)} z + x^{\frac{1}{\alpha}} z_x - \alpha x^{(1-\frac{1}{\alpha})} z = 0. \tag{16}$$

Thus the first term can be neglected, compared with the last one, and then the solution can be written as

$$z(x) \sim C \exp\{-[\alpha^2/2(1-\alpha)]x^{-\frac{2(1-\alpha)}{\alpha}} \tag{17}$$

with C an arbitrary constant (which explain the name of "ordinary" orbits). This type of orbits are given, close to the origin, by the approximate equation (11), which for the orbits that reach the origin from below implies that

$$\tilde{y} \sim x^{1/\alpha} \sim -\frac{dx}{dt} \tag{18}$$

and so, integrating the simplified equation

$$\frac{dx}{dt} = -x^{1/\alpha} \tag{19}$$

we get that

$$x(t) \sim [\frac{\alpha}{(1-\alpha)(t+t_1)}]^{\alpha/(1-\alpha)} \tag{20}$$

and so that it takes an infinite time to reach the origin.

Some different orbits approaching the origin can be found by searching among solutions with large values of $|y|$ compared with $|x|^{1/\alpha}$. Thus, close to the origin, the orbits with negative y are "very close" to the solutions of the equation found by replacing (13) by the simplified the equation

$$\tilde{y}\tilde{y}_x = \tilde{y}^\alpha \tag{21}$$

corresponding to a balance of inertia and damping. The solution ending at the origin ($\tilde{y}(0) = 0$) is given by

$$\tilde{y}(x) = -\{(2-\alpha)x\}^{1/(2-\alpha)}. \tag{22}$$

Notice that it involves no arbitrary constant. It is easy to see that this curve is unique in the class of solutions such that $\tilde{y}(x) > 0$ if $x > 0$ (a symmetric curve arises for $y > 0$ and $x < 0$). This justifies the term of "extraordinary" orbit. The time evolution of this orbit is given, for $x << 1$, by integrating the equation

$$-\frac{dx}{dt} = [(2-\alpha)x]^{1/(2-\alpha)} \tag{23}$$

and so

$$x(t) = \frac{1}{(2-\alpha)}[\frac{(2-\alpha)(1-\alpha)}{2\alpha}(t_0 - t)_+]^{(2-\alpha)/(1-\alpha)}, \tag{24}$$

where in general $h(t)_+ = \max\{0, h(t)\}$. This indicate that the motion (of this approximated solution) ends at a finite time. t_0, determined by the initial conditions which, by (23) must satisfy that

$$v_0 \sim \pm[(2-\alpha)|x_0|]^{1/(2-\alpha)} \tag{25}$$

We point out that the two exceptional orbits emanating from the origin spiral around the origin when $x^2 + y^2$ grows toward infinity and so each of them is a separatrix curve in the phase plane.

We end this section by pointing out that the solution of problem (P_α) for $0 < \alpha << 1$ takes an asymptotic form which can be easily described. The differential equations of the orbits "simplify" if $y \neq 0$ is finite and $\alpha \to 0$ to

$$yy_x = -x - 1 \text{ for } y > 0 \tag{26}$$

and

$$\tilde{y}\tilde{y}_x = -x + 1 \text{ for } \tilde{y} = -y > 0. \tag{27}$$

The solutions are circles with center at $x = -1$ if $y > 0$ and center $x = 1$ if $y < 0$ joined. An orbit formed with half circles with centers at $x = -1$ or $x = 1$ when it hits the interval $(0, 1)$ from below it is transformed into an orbit that reaches the origin following very closely that segment, governed by the equation (18) of solution (20). In the limit $\alpha \to 0$ we found that any point $\zeta \in [-1, 1]$ is an asymptotically stable stationary state of (P_0).

3 A rigorous proof of the existence of the extraordinary orbits

We have

Theorem 3.1 *There exists a, b, with $0 < a < (1-\alpha)^{1/(1-\alpha)} < b$, $R > 0$ and $t_0 > 0$ such that for some initial data (x_0, v_0) satisfying*

$$-Rt_0^{(2-\alpha)/(1-\alpha)} \leq x_0 < 0 \tag{28}$$

and

$$at_0^{1/(1-\alpha)} \leq v_0 \leq bt_0^{1/(1-\alpha)} \tag{29}$$

the associate solution $x(t)$ vanishes identically for any $t \geq t_0$. Moreover this solution is unique in a suitable class of solutions.

proof As in the previous section, it is useful to work backwards in time, i.e. we search $X : [-t_0, 0] \to \mathbb{R}$ such that

$$X(-t) = x(t_0 - t), \text{ if } t \in [0, t_0]. \tag{30}$$

with x solution of (3) such that $x(t_0) = 0$. So, $X(-t_0) = x_0$ and $X(0) = 0$. The phase plane becomes now

$$\begin{cases} X_s = Y \\ Y_s = -X - |Y|^{\alpha-1}Y \end{cases} \tag{31}$$

where $s = -t \in [-t_0, 0]$. We define the Banach spaces

$$E = \{X \in C[-t_0, 0] : X(0) = 0, \|X\| < \infty\} \text{ where}$$
$$\|X\| := \sup_{s\in[-t_0,0]} \frac{|X(s)|}{|s|^{(2-\alpha)/(1-\alpha)}}$$

and

$$V = \{Y \in C[-t_0, 0] : Y(0) = 0, |||Y||| < \infty\} \text{ where}$$
$$|||Y||| := \sup_{s\in[-t_0,0]} \frac{|Y(s)|}{|s|^{1/(1-\alpha)}}.$$

We also define the operator $\mathcal{T}$: $E \times V \to E \times V$ given by

$$[\mathcal{T}(\mathcal{X},\mathcal{Y})](f) = (-\int_f^{f'} \mathcal{Y}(\nabla)\lceil\nabla, \int_f^{f'}(|\mathcal{Y}(\nabla)|^{\alpha-\infty}\mathcal{Y}(\nabla) + \mathcal{X}(\nabla))\lceil\nabla). \tag{32}$$

Then, it is clear that if (X,Y) is a fixed point of $\mathcal{T}$ then (X,Y) is the searched solution. We introduce the closed and convex sets

$$\begin{aligned} K_R \; &: \; = \{X \in E : -R\,|s|^{(2-\alpha)/(1-\alpha)} \le X(s) \le 0, \; \forall s \in [-t_0, 0]\} \\ S_{a,b} \; &: \; = \{Y \in V : a\,|s|^{1/(1-\alpha)} \le Y(s) \le b\,|s|^{1/(1-\alpha)}, \; \forall s \in [-t_0, 0]\}. \end{aligned}$$

Let us prove that it is possible to chose a, b, R and t_0 such that $\mathcal{T}$ let a contraction such that $\mathcal{T}(\mathcal{K}_\mathcal{R} \times \mathcal{S}_{4,\lfloor}) \subset \mathcal{K}_\mathcal{R} \times \mathcal{S}_{4,\lfloor}$. In that case the existence of a fixed point would be consequence of the Banach fixed point theorem (which implies also the uniqueness in this class of functions). We shall use the norm

$$\|(X,Y)\| := \max(\|X\|, \||Y\||). \tag{33}$$

Let $X \in K_R$ and $Y \in S_{a,b}$. Then, since

$$0 \ge -\int_s^0 Y(r)dr \ge -\int_s^0 b\,|r|^{1/(1-\alpha)}\,dr = -b\frac{(1-\alpha)}{(2-\alpha)}\,|s|^{(2-\alpha)/(1-\alpha)}, \tag{34}$$

a sufficient condition to have the first component of the condition $\mathcal{T}(\mathcal{K}_\mathcal{R} \times \mathcal{S}_{4,\lfloor}) \subset \mathcal{K}_\mathcal{R} \times \mathcal{S}_{4,\lfloor}$ satisfied is

$$b\frac{(1-\alpha)}{(2-\alpha)} \le R. \tag{35}$$

On the other hand,

$$\begin{aligned} \int_s^0(|Y(r)|^{\alpha-1}Y(r) + X(r))dr \;&\ge\; a^\alpha(1-\alpha)\,|s|^{1/(1-\alpha)} - R\frac{(1-\alpha)}{(3-2\alpha)}\,|s|^{(3-2\alpha)/(1-\alpha)}, \\ \int_s^0(|Y(r)|^{\alpha-1}Y(r) + X(r))dr \;&\le\; b^\alpha(1-\alpha)\,|s|^{1/(1-\alpha)}. \end{aligned}$$

Thus, two sufficient conditions to have the second component of the condition $\mathcal{T}(\mathcal{K}_\mathcal{R} \times \mathcal{S}_{4,\lfloor}) \subset \mathcal{K}_\mathcal{R} \times \mathcal{S}_{4,\lfloor}$ satisfied are

$$a^\alpha(1-\alpha) - R\frac{(1-\alpha)}{(3-2\alpha)}t_0^2 \;\ge\; a \tag{36}$$

$$b^\alpha(1-\alpha) \;\le\; b \tag{37}$$

To see that $\mathcal{T}$ is a contraction it is enough to check that

$$\|D\mathcal{T}(\mathcal{X},\mathcal{Y})\| < 1 \tag{38}$$

$\forall (X,Y) \in K_R \times S_{a,b}$ where $D\mathcal{T}$ is the Gateaux derivative of $\mathcal{T}$. But

$$\langle D\mathcal{T}(\mathcal{X},\mathcal{Y}), (\xi,\eta)\rangle = (-\int_s^0 \eta(r)dr, \int_s^0 \alpha\,|Y(r)|^{\alpha-1}\eta(r) + \xi(r))dr). \tag{39}$$

Moreover,

$$\left|-\int_s^0 \eta(r)dr\right| \le \int_s^0 \||\eta\||\,|r|^{1/(1-\alpha)}dr = \||\eta\||\,\frac{(1-\alpha)}{(2-\alpha)}\,|s|^{(2-\alpha)/(1-\alpha)} \tag{40}$$

and

$$\begin{aligned} \left|\int_s^0(\alpha\,|Y(r)|^{\alpha-1}\eta(r) + \xi(r))dr\right| \;&\le\; \int_s^0[\alpha a^{\alpha-1}|r|^{-1}\,\||\eta\||\,|r|^{1/(1-\alpha)} \\ &\quad + \|\xi\|\,|s|^{(2-\alpha)/(1-\alpha)}]dr \\ &= \alpha(1-\alpha)a^{\alpha-1}\,\||\eta\||\,|s|^{1/(1-\alpha)} \\ &\quad + \|\xi\|\,\frac{(1-\alpha)}{(3-2\alpha)}\,|s|^{(3-2\alpha)/(1-\alpha)} \end{aligned}$$

for any $(X,Y) \in K_R \times S_{a,b}$. Then

$$\|\langle D\mathcal{T}(\mathcal{X},\mathcal{Y}), (\xi,\eta)\rangle\| \le \max[\frac{(1-\alpha)}{(2-\alpha)}\,\||\eta\||, \alpha(1-\alpha)a^{\alpha-1}\,\||\eta\|| + \|\xi\|\,\frac{(1-\alpha)}{(3-2\alpha)}\,|t_0|^2] \tag{41}$$

and so

$$\begin{aligned} \|D\mathcal{T}(\mathcal{X},\mathcal{Y})\| \;&=\; \sup_{\|\xi\|\le 1, \|\eta\|\le 1} \|\langle D\mathcal{T}(\mathcal{X},\mathcal{Y}), (\xi,\eta)\rangle\| \le \max[\frac{(1-\alpha)}{(2-\alpha)}, \\ &\quad \alpha(1-\alpha)a^{\alpha-1} + \frac{(1-\alpha)}{(3-2\alpha)}\,|t_0|^2]. \end{aligned}$$

But $\alpha \in (0,1)$ implies that $\frac{(1-\alpha)}{(2-\alpha)} < 1$ and so the contraction property is assured if

$$\alpha(1-\alpha)a^{\alpha-1} + \frac{(1-\alpha)}{(3-2\alpha)}\,|t_0|^2 < 1. \tag{42}$$

Now, it is easy to check that conditions (35), (36), (42) are satisfied if we take a, b such that

$$[\alpha(1-\alpha)]^{1/(1-\alpha)} < a < (1-\alpha)^{1/(1-\alpha)} \le b, \tag{43}$$

then

$$R \ge b\frac{(1-\alpha)}{(2-\alpha)} \tag{44}$$

and finally

$$0 < t_0 \le \min[\frac{(3-2\alpha)}{R(1-\alpha)}((1-\alpha)a^\alpha - a), \frac{(3-2\alpha)}{(1-\alpha)}(1-\alpha(1-\alpha)a^{\alpha-1})]^{1/2}. \tag{45}$$

It is possible to give a rigorous version of the rest of the results of Section 2. The details will be published elsewhere.

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