Computational Techniques in Food Engineering

Editors:

E. Balsa-Canto
J. Mora
J.R. Banga
E. Oñate

A publication of:

International Center for Numerical Methods in Engineering (CIMNE)
Barcelona, Spain
2002

COMPUTATIONAL TECHNIQUES IN FOOD ENGINEERING
E. Balsa-Canto, J. Mora, J.R. Banga and E. Oñate (Eds.)
© CIMNE, Barcelona, Spain 2002

ON THE MATHEMATICAL MODELING AND CONTROL OF HIGH HYDROSTATIC PRESSURE FOOD PROCESSING

J.I. Díaz [1,*†] A.M. Ramos [1,†], L. Otero [2,†], A. D. Molina [2,†] and P. D. Sanz [2,†]

[1] *Dept de Matemática Aplicada,
Facultad de Matemáticas,
Universidad Complutense de Madrid,
28040 Madrid, Spain
Email: JI_Diaz@mat.ucm.es

[2] *Instituto del Frio (CSIC),
Ciudad Universitaria, s/n,
28040 Madrid, Spain*

† *Project AGL2000-1440-C02-01 of the MCYT.*

SUMMARY: High hydrostatic pressure (HHP) has become in the last few years in a promising technology in food processing and preservation. HHP treatment of foods always results in a temperature increase due to the work of compression. Later, heat loss through the metal wall of the HPP vessel causes temperature gradients in the processed food that involve inhomogeneities in the pursued pressure effect (enzyme and/or microbial inactivation...). Our main goal (project AGL2000-1440-C0201 involving specialists from Applied Mathematics of the Univ. Complutense de Madrid and the Instituto del Frio of the CSIC) is the modeling, mathematical analysis, control and numerical approximation of the process in devices with different dimensions, different pressurising fluids or different thermoregulating systems. This communication contains a first presentation of our results.

KEYWORDS: High hydrostatic pressure, food processing, mathematical modeling, numerical approximation, optimal control, controllability.

INTRODUCTION

High hydrostatic pressure (HHP) has become in the last few years a promising technology for food processing and preservation. Pressure treatment of foods always results in a temperature increase due to the work of compression. The extent of this temperature increase depends mainly on the food composition, its initial temperature, the pressurising fluid and the applied pressure and can be calculated as described by Otero et al. [1]. Large increments in temperature can be expected, for example, in fatty foods [1,2]. Later, during holding time, heat loss through the metal wall of the high-pressure vessel causes temperature gradients in the processed product. Most of high-pressure applications in food are not only pressure dependent but also temperature dependent. Nevertheless, most food researchers working on pressure treatment of foods do not control the temperature and attribute the results they observe to the applied pressure and

the initial temperature of the treatment. Denys et al. [3, 4] have shown experimentally the thermal gradients that are established in a food model (agar gel) after compression. These authors emphasise that the different pressure-temperature-time profiles perceived during a process at different locations in the high-pressure vessel may result in a pronounced inhomogeneity in enzyme and/or microbial inactivation, nutritional and/or sensorial quality degradation, etc., within the processed product [4]. Then, it is absolutely necessary to know how thermal exchanges in high-pressure treatments are produced and at what rate in order to establish and control the real conditions at which a given process is performed. In this paper, a modelling/simulation of the thermal exchanges taking place in a high-pressure pilot system (including thermoregulating bath) during different processes of pressurisation/depressurisation is presented. A mathematical formulation of the control problem is also given.

MATERIALS, METHODS AND FIRST RESULTS

A GEC ALSTHOM ACB High Pressure equipment (ACB GEC Alsthom, Nantes, France) with a cylindrical chamber of 2.35 l net volume filled with water (which acted simultaneously as sample and compressing fluid) was employed to perform all the experiments. The vessel was thermoregulated at 40°C by means of a laterally surrounding properly isolated coil connected to a heating-cooling bath (RB-12A, Techne, Cambridge, United Kingdom). Pressure was applied at a rate of 2.5 MPa/s up to 200, 300 or 400 MPa in different experiments. Holding times were established as the time necessary to re-equilibrate the vessel at 40°C and were different in each experiment. Pressure was released when the vessel was tempered at 40°C again, at a rate of over 200 MPa/s (by simply manually opening a valve). Temperature was measured in six points of the system: two thermocouples (copper-constantan, type T, steel coated) were placed inside the high-pressure cylinder, two more at the coil around the vessel at its entrance and its exit, one thermocouple inside the thermoregulating bath and the last one at the ambient. The pressure inside the vessel was measured by a pressure gauge located inside the circuit. A data capture system (Fluke Helios I), with eight active channels, connected to a computer was used to register temperature, pressure and time data each 3 s. Temperature variations with pressure were calculated as described by Otero et al. [1] and included in the model. Simulations were performed with Simulink (The MathWorks, Inc.; Natick, MA, USA) using Euler method and a constant size step of 1 s. Modelling all the system includes considering the thermoregulating bath, the refrigerating fluid (flow, mass, physical properties...), the steel mass of the high-pressure vessel, the temperature at the entrance and exit of the coil surrounding the vessel, the ambient temperature, etc..., variables that are not usually taken into account when modelling high-pressure systems. The heat transmission ways considered were convection (ambient air, fluid inside the bath and fluid inside the vessel) and conduction (throughout the different layers that are present). The model includes the calculation of the temperature variation during the compression or the expansion. Taking into account all these parameters allows to have a powerful tool to design and control the conditions of a given high-pressure treatment.

Details on two simulated and experimental time-temperature profiles inside the high-pressure vessel for compression from 0.1 MPa up to 400 MPa and the holding time necessary to recover 40°C in the sample can be found in Otero et al. [1]. For instance, the largest rise in temperature after compression occurred when the highest pressure increase (400 MPa) was applied, as expected. About twenty minutes were necessary to dissipate heat inside the high-pressure vessel and re-equilibrate its temperature at 40°C. Simulated and experimental values of temperature inside the vessel, in the sample, appear to agree satisfactorily with the compression and the holding time. The mean absolute error was 0.6%, being the maximum error found of 1.8%. Mean absolute errors bigger than 1% have never been found in any experiment.

CONSTRAINED CONTROL ON ROBIN BOUNDARY CONDITION

In order to carry out a mathematical study of the control aspects, a simplified two-dimensional model can be introduced. The spatial domain $\Omega=D-\omega$, where D represents the horizontal section of the total device (a symmetric ball) and ω the the coil surrounding the vessel (here represented by a descentred small ball). The problem becomes

$$\begin{aligned}
&\rho c y_t - k\Delta y = 0 && \text{in } \Omega\times(0,T),\\
&k\frac{\partial y}{\partial n} = h(u(t)-y) && \text{on } \partial\omega\times(0,T),\\
&k\frac{\partial y}{\partial n} = h(w(x,t)-y) && \text{on } \partial D\times(0,T),\\
&y(x,0)=y^0(x) && x\in\Omega,
\end{aligned}$$

where u(t) is the control and w(x,t) the external temperature on the boundary of D. A formulation in terms of Optimal Control Theory is the following: Given k, w and y^0 and y_d find the control u(t) minimizing over C, subset of U_{ad}, the set of control constrains due to limitations of the thermoregulating device,

$$U_{ad} = \left\{u \in L^{\infty}(0,T) : u_1 \le u(t) \le u_2 \text{ a.e. } t\in(0,T)\right\}$$

the cost

$$J_k(u) = \frac{1}{2}\|u\|_{L^2(0,T)} + \frac{k}{2}\|y(T,.:u)-y_d\|_{L^2(\Omega)}$$

where C is given by the state constrains (e.g. due to sterilization: Olin Ball &Olson [6]).

$$C=\left\{u\in U_{ad} : e \le \int_0^T F(y(x,t))dt, \text{ a.e. } x\in\Omega\right\} \text{ for some } F\in C^1(\mathbb{R})$$

The Approximate Controllabilty question concerns the case of absence of "economical aspects": given e>0, find u(t) such that

Our mathematical results concerns:

$$\|y(T,.:u)-y_d\|_{L^2(\Omega)} \le \varepsilon$$

i) the existence of an optimal control (it uses suitable compactness theorems),

ii) the *optimality conditions*, given in terms of

$$\begin{aligned}
&\rho c y_t - k\Delta y = 0 && \text{in } \Omega\times(0,T),\\
&k\frac{\partial y}{\partial n} = h(u(t)-y) && \text{on } \partial\omega\times(0,T),\\
&k\frac{\partial y}{\partial n} = h(w(x,t)-y) && \text{on } \partial D\times(0,T),\\
&y(x,0) = y^0(x) && x\in\Omega\\
&-\rho c p_t - k\Delta p = 0 && \text{in } \Omega\times(0,T),\\
&k\frac{\partial p}{\partial n} = -hp && \text{on } \partial\omega\times(0,T),\\
&k\frac{\partial p}{\partial n} = -hp && \text{on } \partial D\times(0,T),\\
&p(x,T) = k(y(x,T)-y_d(x)) && x\in\Omega
\end{aligned}$$

(the proof uses a result due to Bonnans and Casas [7] in a similar way to Martínez Varela [5]),

iii) the *approximate controllability* with constraints: by passing to the limit, in some *a priori* estimates obtained from the optimality conditions, when k increases, it is possible to show the approximate controllability once we assume

$$y(T,.:u_1) \le y_d \le y(T,.:u_2)$$

Numerical experiences for a related problem can be found in Díaz and Ramos [8] where the control rest inactive for some interval previous to t=T.

$$\int_0^T\left[\int_{\partial\omega} g\,d\sigma + r\left(\bar{u}+\int_{\partial\omega} p\,d\sigma\right)\right](u-\bar{u})\,dt \ge 0,\ \forall u\in U_{ad}$$
$$\text{for some } g = g(\bar{u}) \text{ and some } r\ge 0$$

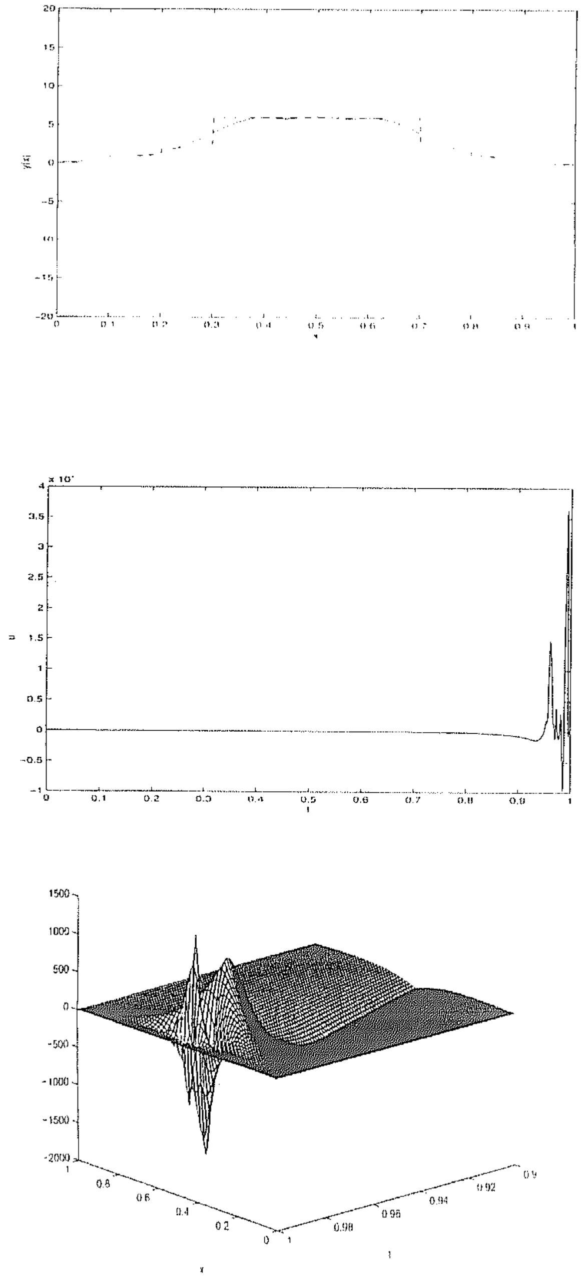

Fig. 1: The desired state, the control and the time dependent solution

REFERENCES

1. Otero, L., Molina-García, A. D. and Sanz, P. D., "Thermal effect in foods during quasi-adiabatic pressure treatments", *Innovative Food Science & Emerging Technologies*, 1, 2000, 119.
2. Shimizu, T., "High pressure experimental apparatus with windows for optical measurements up to 700 MPa", *High Pressure and Biotechnology*, Balny, C., Hayashi, R., Heremans, K. and Mason, P., Eds., Colloque INSERM/John Libbey Eurotext Ltd. Vol. 224, 1992, p. 525.
3. Denys, S., Van Loey, A. M. and Hendrickx, M. E., "A modeling approach for evaluating process uniformity during batch high hydrostatic pressure processing: combination of a numerical heat transfer model and enzyme inactivation kinetics", *Innovative Food Science & Emerging Technologies*, 1, 2000, 5.
4. Denys, S., Ludikhuyze, L. R., Van Loey, A. M. and Hendrickx, M. E., "Modeling conductive heat transfer and process uniformity during batch high-pressure processing of foods", *Biotechnology Progress*, **16**, 2000, 92.
5. Martínez Varela, A, "Una aplicación de la teoría de control óptimo a la esterilización de alimentos enlatados", *Jornadas Hispano-Francesas sobre Control de Sistemas Distribuidos*, Valle, A. Ed. Universidad de Málaga, 1990, pp. 89-102.
6. Olin Ball,C. and Olson, F.C.W. *Sterilization in food technology*, Mc Graw Hill, New York, 1957.
7. Bonnans, J.F. and Casas, E., "Contrôle de systemes non lineaires comportent des constraintes distribuées sur l'etat, Rapport No. 300, INRIA, 1984.
8. Díaz, J.I. and Ramos, A. M., "Numerical experiences regarding the localized control of nonlinear parabolic problems" CD-Rom Proceedings European Congress Computational Methods in Applied Sciences and Engineering (ECCOMAS 2000), Barcelona, 2000, 17 pp.