A 2D free–boundary problem arising in the magnetic confinement of a plasma in a Stellarator under the presence of a limiter.

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Resumen

One of the main difficulties of the magnetically controlled plasma fusion (in axisymmetric geometric devices as Tokamaks or non axisymmetric geometric ones as Stellarators), is to determinate the conditions on the magnetic field and on the current density in order to maintain the plasma far from the camera walls. A way to prevent mechanically this is to introduce a *limiter*: a solid object which determines the boundary of the plasma since it plays the role of a *thin obstacle* for it. The influence of limiters on plasma confinement has been investigated, from the experimental view point, in the TJ-II Stellarator (CIEMAT, Madrid) and some evidences have been found about how they improve the confinement [2].

In this communication we consider the associated theoretical aspects by studying a 2D free–boundary problem arising in the magnetic confinement of a plasma in a Stellarator device (with current carrying inside of each magnetic surfaces) under the presence of a limiter. Using a similar approach to the already followed for the equilibrium regime [1], our model can be expressed as the following *inverse thin obstacle problem*: Let Ω be an open bounded and regular set of $\mathbb{R}^2$ and let ω (the limiter) be a connected subset of Ω such that $\bar{\omega} \cap \partial\Omega$ is a nonempty connected subset of $\partial\Omega$. Given the parameters $\lambda > 0$, $F_v > 0$ and $\gamma < 0$, the problem is to find (u, F), with $u : \Omega \longrightarrow \mathbb{R}$, $F : \mathbb{R} \longrightarrow \mathbb{R}$, such that $F(s) = F_v$ for any $s \leq 0$ and satisfying:

$$(\mathcal{P}) \begin{cases} -\Delta u + \beta(u\chi_\omega(x)) \ni aF(u) + \frac{1}{2}\left(F(u)^2\right)' + \lambda b u_+ & \text{in } \Omega, \\ u = \gamma & \text{on } \partial\Omega, \\ \displaystyle\int_{\{x:u(x)>s\}} \left[\frac{1}{2}\left(F(u)^2\right)' + \lambda b u_+\right] dx = j(s_+, |u_+(s)|_{L^\infty(\Omega)}) \quad \text{for any } s \in [\inf u, \sup u], & \end{cases}$$

for some $a, b \in L^\infty(\Omega)$. The action of the limiter is modeled by the multivalued maximal monotone graph β given by $\beta(r) = 0$ if $r > 0$, $\beta(0) = [0, +\infty)$ (as usual j is a real function which means the current carrying inside of each magnetic surfaces). The family of integral identities stated in $(\mathcal{P})$ is known as the *Stellarator condition*, and the function u is the averaged poloidal flux. There is a double free-boundary: the boundary of the plasma set $\Omega_p := \{u(x) \geq 0\}$ and the part of it which is in contact with the limiter. We prove the existence of a solution by means of a Galerkin argument for a new family of elliptic problems associated to an equivalent *direct* (but *non-local*) formulation of $(\mathcal{P})$ without any mention to the function $F(u)$.

Palabras clave: Free boundary elliptic problems, Galerkin method, Rearrangement, Plasma fusion, limiter.

Clasificación para el Cedya 2005: Ecuaciones en derivadas parciales

References

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