On the optimal control of a free boundary problem related to desalination plants

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Resumen

This communication deals with several optimal control problems related to desalination plants. To fix ideas, we mention that the kind of state equations are given mainly by

$$\begin{cases} \rho(\frac{\partial y}{\partial t} + \mathbf{v}\cdot\nabla y) - \Delta y + \lambda \left|y\right|^{p-1} y = u\chi_{\omega} & \textbf{on } \Omega \times (0,T), \\ y = 0 & \textbf{in } \partial\Omega \times (0,T), \\ y(0,x) = y_0(x) & \textbf{on } \Omega, \end{cases}$$

where $y(t,x)$ denotes the main chemical solute in the brine discharges (for instance cooper sulfate in some tests) and the velocity of the surrounding fluid $\mathbf{v}(t,x)$ is here assumed to be given (for instance, as solution of the Navier-Stokes system uncoupled with the state equation for y). Here $p \in [0,1)$ is the order of the chemical reaction produced in the brine discharges to seawater and we suppose that $y_0 \in L^\infty(\Omega), y_0 \geq 0$ and that $u \in L^\infty(0,t : L^\infty(\omega))$ with $\overline{u} \geq u(t,x) \geq \underline{u} > 0$ for some $\overline{u}$ and $\underline{u}$ expressing the technical limitations of the plant). Since the chemical reaction is less than one it is well known that some free boundary is formed corresponding to the boundary of the support of $y(t,. : u)$ (which we shall denote by $P(y(t,.;u))$ and corresponds to the plume discharges zone). Very often, the brine discharges (depending on the control brine flux $u(t,.)$ on a small open subregion of the spatial domain Ω) must obey to some regulations protecting some given subregion B of Ω (corresponding, for instances to some beach or protected zones in the sea). Moreover, in the raining period of the year the water production is less appreciated in the market and since the discharge is proportional to the production, a suitable governing policy of the desalination plants leads to the minimization of a functional cost of the form

$$J(u) = \int_0^T \int_\Omega \left|\chi_{P(u(t,.))\cap B}\right| dxdt \ + \int_0^T \int_\Omega \frac{\theta(t)}{G(u(t,x))} dxdt,$$

where $B \subset \Omega$ represents the protected sea zone (we suppose that $\underline{u}$ is big enough as to have $\cup_{t\in[0,T]} \left|P(y(t,.;u)) \cap B\right| > 0$), G is a given real continuous increasing function, with $G(\underline{u}) > 0$, representing the economic loss and the given weight function, $\theta(t) \in (0,1]$, attaints its maximum in the dry period of the year.

We mention that this is a nonstandard control problem with some nontrivial aspects: the nonlinear term of the state equation is not Lipschitz continuous and the cost involves the free boundary. Some

previous study (the approximate controllability) was carried out in Díaz and Ramos (1995). One of our main tools is to prove previously the continuous dependence of the free boundary with respect to the control u. This is done by improving several results on nondegeneracy solutions near the free boundary. Some numerical simulations will be also presented. The associate stationary problem (with $\theta \equiv 1$) is also considered.

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