Mathematics and Mars Exploration

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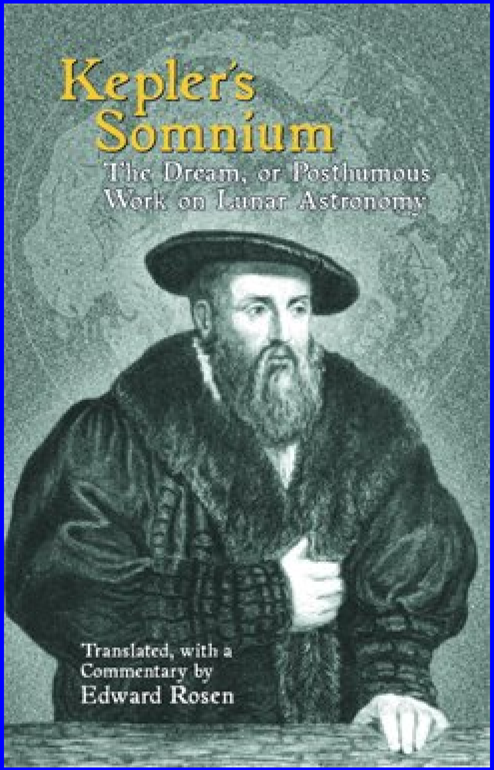
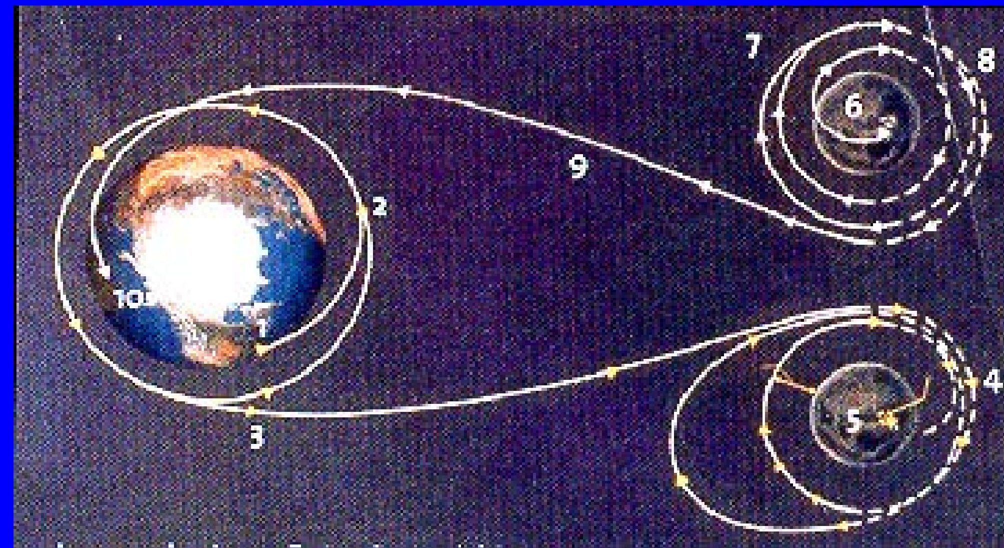
From the “passive study” of Celestial Mechanics to the application of the sharper techniques of Optimal Control Theory (passing through science-fiction),...



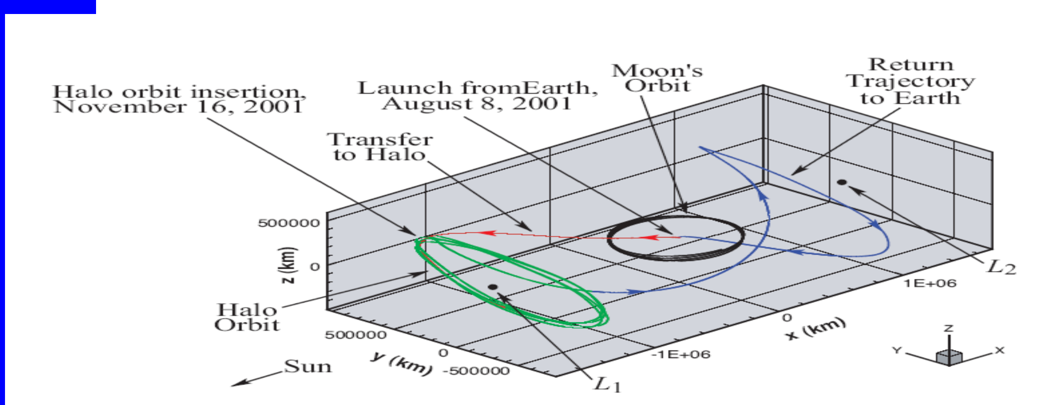
D.F. Lawden 1955, A. Miele 1958,
G. Leitmann 1959, A.I. Lurie 1963,....

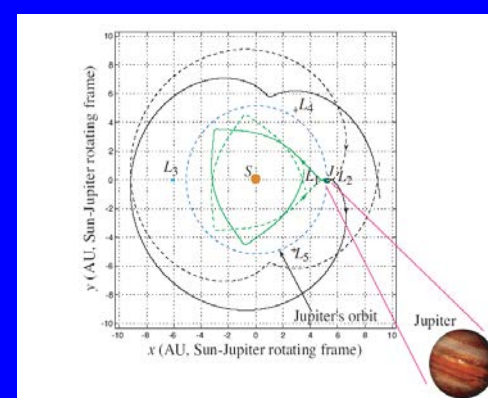
Apollo 11, July, 20, 1969



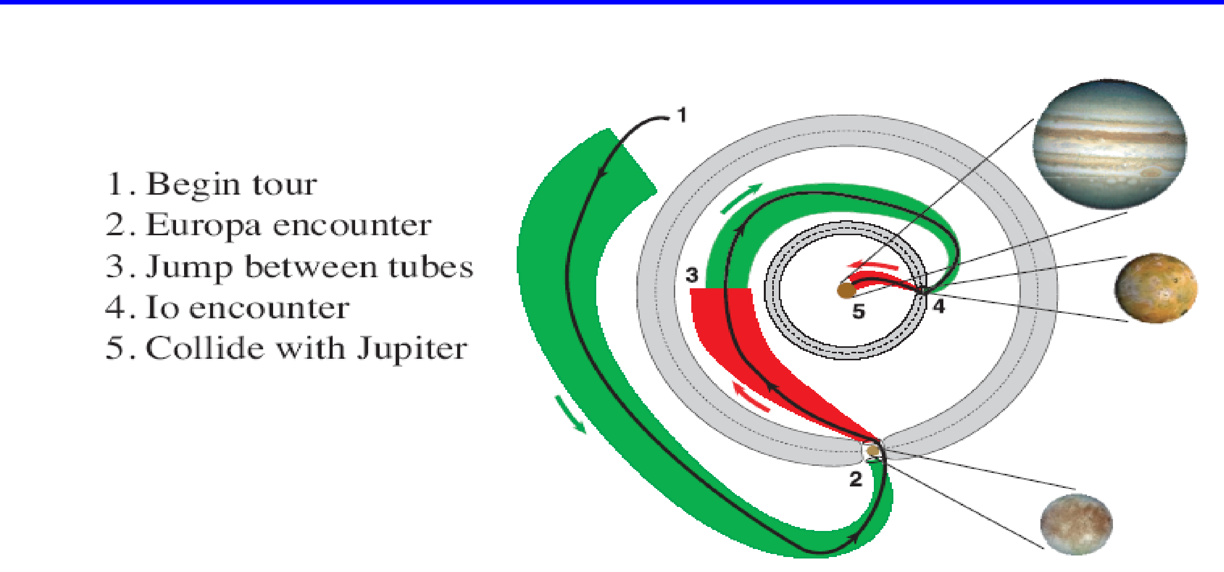
Dynamical systems teams:

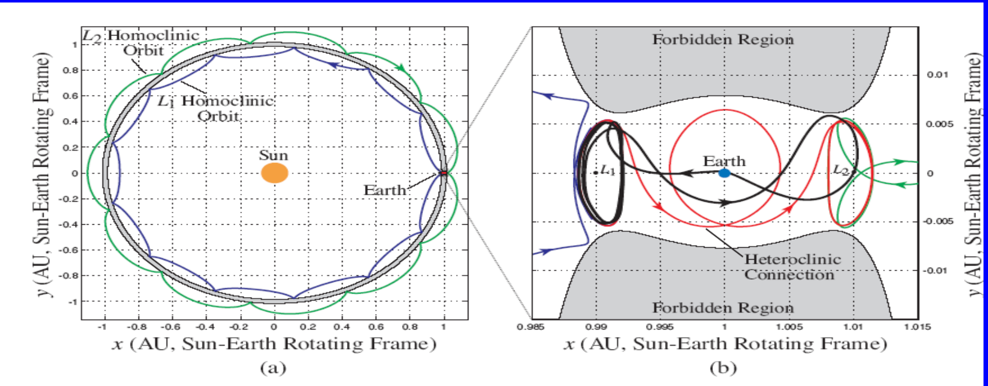
2000–present: Barrabés, Font, Gómez, Nunes, and Simó (part of the Barcelona group) systematically study jumping between resonant orbits; Koon, Lo, Marsden, and Ross at Caltech systematically study jumping between interior and exterior resonances and its application to space mission trajectory design





Júpiter moons





Invariant manifolds for the *Sun–Earth–spacecraft* 3-body problem

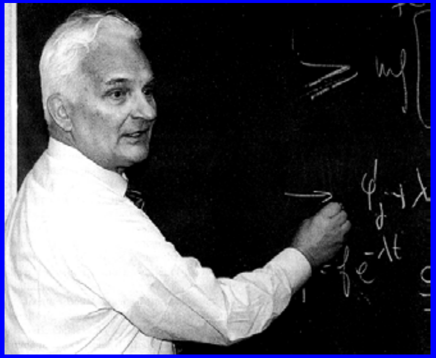
von Neumann: climate and its control

John von Neumann (1955): *Probably intervention in atmospheric and climate matters will come in a few decades, and will unfold on a scale difficult to imagine at present*

To act on the atmospheric climate by acting on the albedo : **the von Neumann problem**

A single “player” and a single control v (terminology of Control Theory and Games Theory).

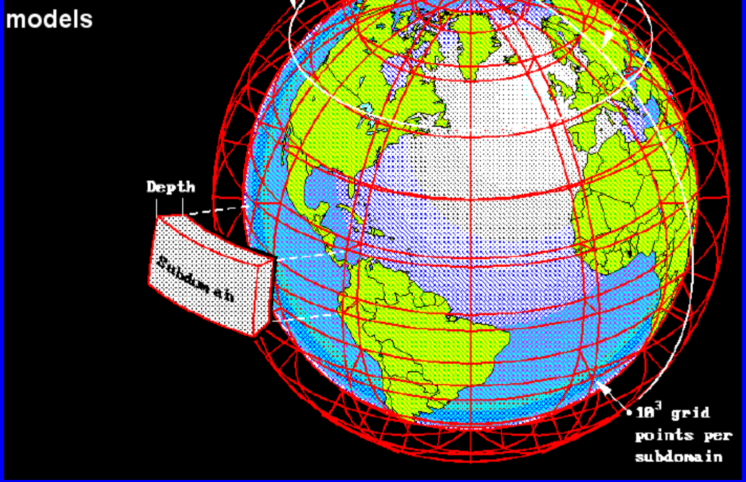
Recent results on a mathematical formulation (raised to me by J.L. Lions: fax of 15 may 1999).



J.L.Lions (1924-2001) President of the CNES

Energy Balance Models, M.I. Budyko and W. D. Sellers, (1969):

diagnostic character and intended to understand the evolution of the global climate on along time scale. Their main characteristic is the high sensitivity to the variation of solar and terrestrial parameters



Used in the study of the Milankovitch theory of the ice-ages (R.North, 1984)

The distribution of temperature y(x,t) is expressed pointwise after a standard average process, where the spatial variable x is in the Earth's surface

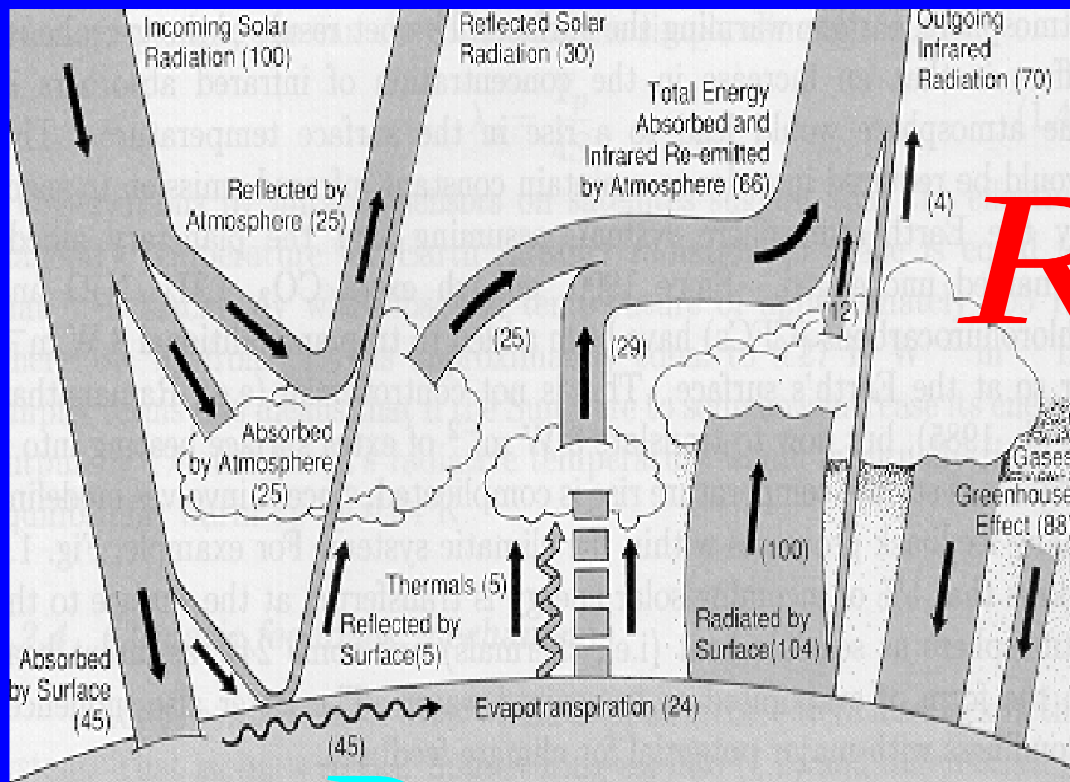
$$y(x,t) = \frac{1}{2\tau |B(x)|} \int_{t-\tau}^{t+\tau} \int_{B(x)} T(y,s)\,dy\,ds$$

S. Arrhenius (1896), ...

W.D. Sellers(1969), M.I. Budyko (1969),....

The Energy Balance on the Earth's surface:

$$c\frac{\partial y}{\partial t} = R_a - R_e + D$$



R_e

Greenhouse effect

R_a

Albedo

D

Constitutive laws

$$R_a = QS(x)\beta(u)$$

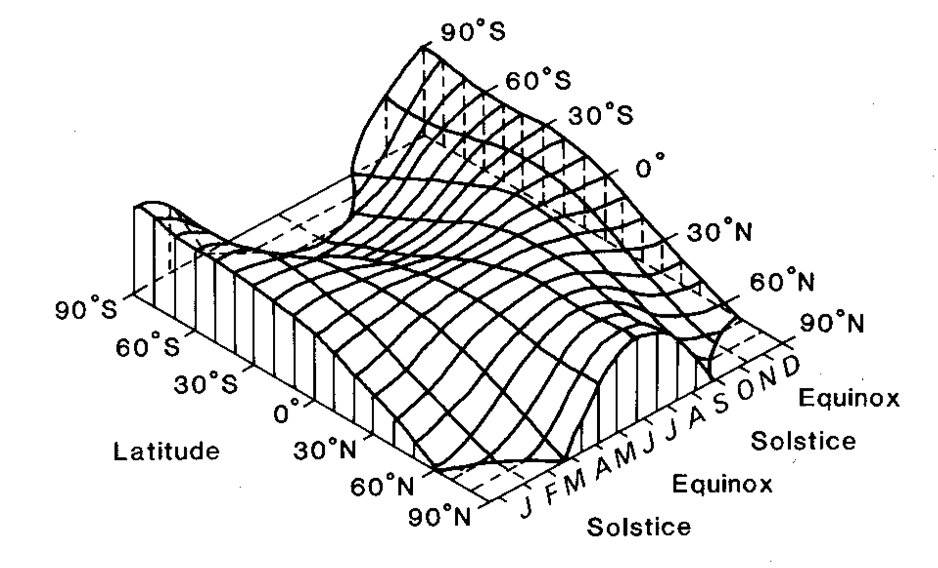
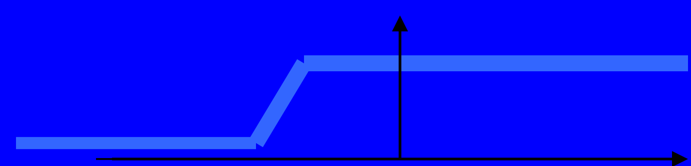


Fig. 2.8. The variation of insolation (at the top of the atmosphere) as a function of

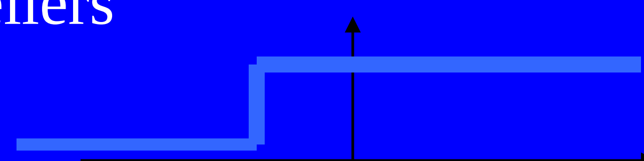
Earth Radiation Budget Satellite

$$\beta(u) = (1 - a(u)) \text{ coalbedo}$$

$$\beta(u) = \begin{matrix} 0.38 \text{ if } u << -10 \\ 0.71 \text{ if } u >> -10 \end{matrix}$$



Sellers



Budyko **u= -10º**

Exploration of Mars

NASA

Imagen del polo Norte marciano obtenida el pasado 12 de septiembre por la cámara de la nave no tripulada *Mars Global Surveyor*, actualmente en órbita del planeta, en la que se observan los depósitos bandeados.

Hacia los polos de Marte

M. I. Hoffert, A. J. Callegari, C. T. Hsieh and W. Ziegler, Liquid water on Mars: An energy balance climate model for CO2/H2O atmospheres, Icarus 47, I 1, 1981, 112-129.

Third International Workshop on Mars Polar Energy Balance and the CO2 Cycle (2009)

On R_e :

Sellers $R_e = \sigma u^4$ Stefan-Boltzman

Budyko $R_e = A + Bu$ Newton

Empirical relation, Depends of the greenhaus gases, anthropogénicos changes,... (internal variables)

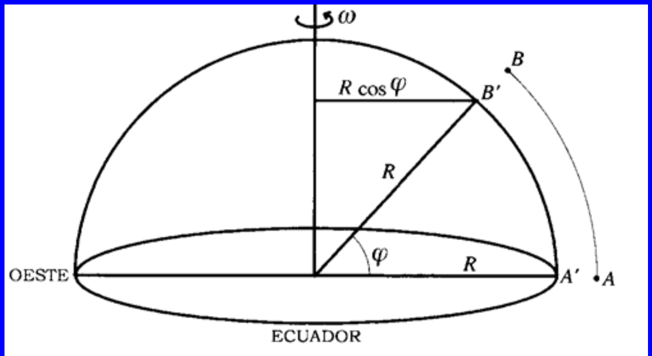
On the diffusion operator, *D*: a jerarchy.

0-dimensional model *D*=0

$$c\frac{du}{dt} = Q\beta(u) - R_e(u)$$

1-dimensional model

$$D = \frac{1}{\cos\varphi}\frac{\partial}{\partial\varphi}\left(k\cos\varphi\frac{\partial u}{\partial\varphi}\right) = \frac{\partial}{\partial x}\left(k(1-x^2)\frac{\partial u}{\partial x}\right)$$



$$x = \cos\varphi$$

In planetary scales O(10^4 Km) the velocity is eliminated by using the eddy diffusive approximation

Bidimensional model $D = div(k(x)\nabla u)$

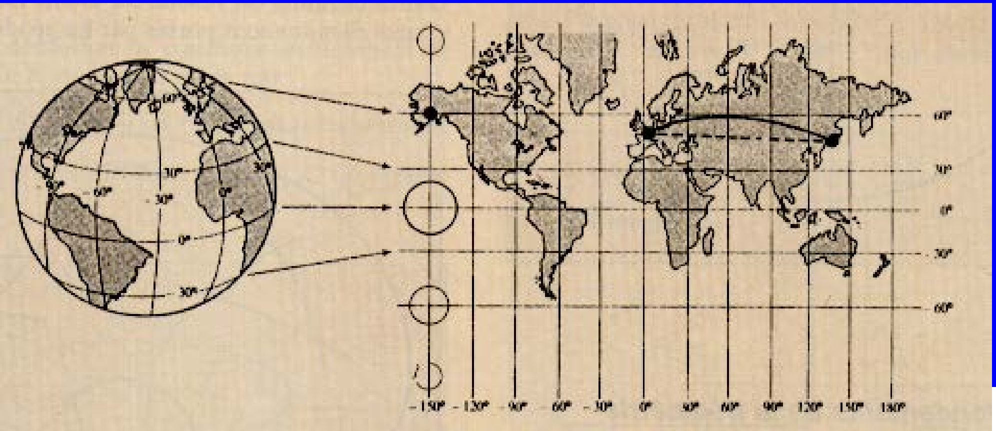
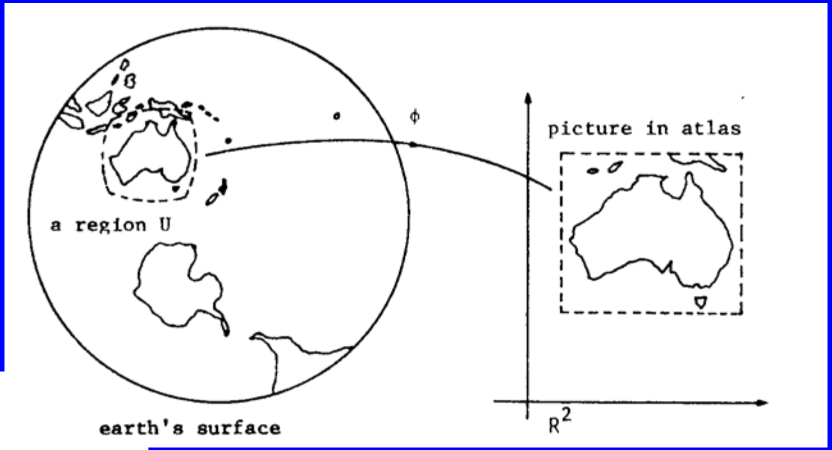
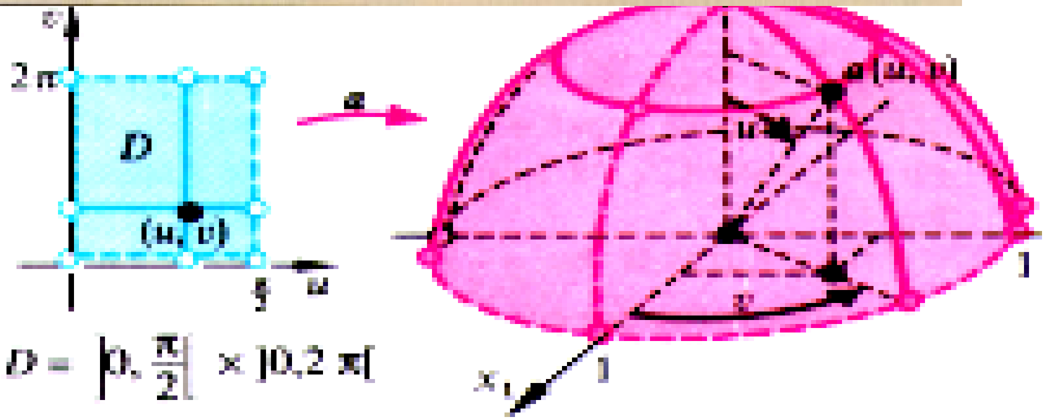
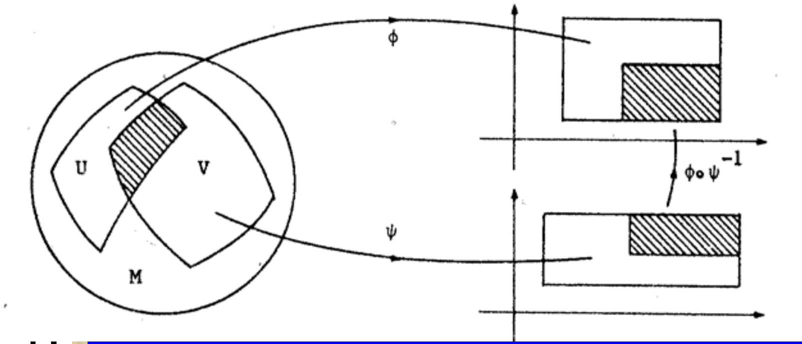


Figure 10 Projection de *Mercator* : $(\varphi,\theta) \longrightarrow (u = \varphi, v = log(tg(\frac{\theta}{2} + \frac{\pi}{4})))$





$$D = \left]0, \frac{\pi}{2}\right[\times]0, 2\pi[$$



$\sigma : D \to \mathbb{R}^3$ déf. par $(u, v) \mapsto \sigma(u, v)$

$$\sigma(u, v) = \begin{pmatrix} \sin u \cos v \\ \sin u \sin v \end{pmatrix}$$

$$c\frac{\partial y}{\partial t} = Q\beta(y) + R_e(x,y) + div(k(x)\nabla y)$$

$$y(x,t_0) = y_0(x)$$

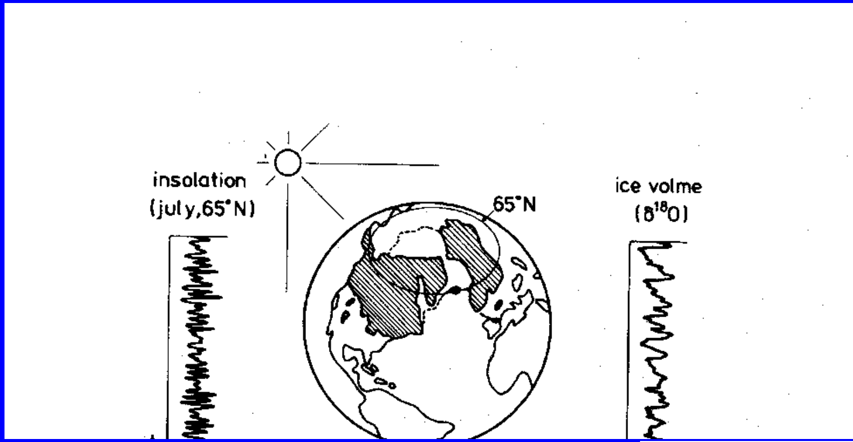
Mathematical treatment (North, Hetzer, Drazin,D, Tello,Badii, Arcoya, HErnandez,...

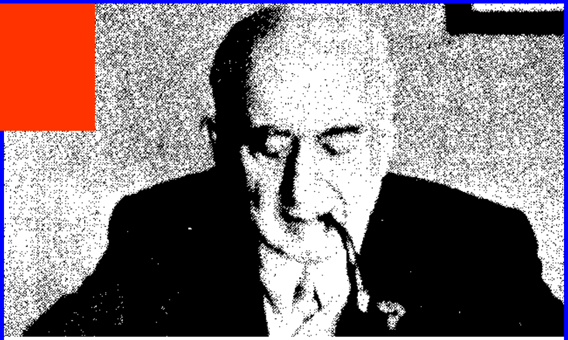
J.I. Díaz and J. L. Lions (eds.), Mathematics, Climate and Environement Masson, Paris, 1993.

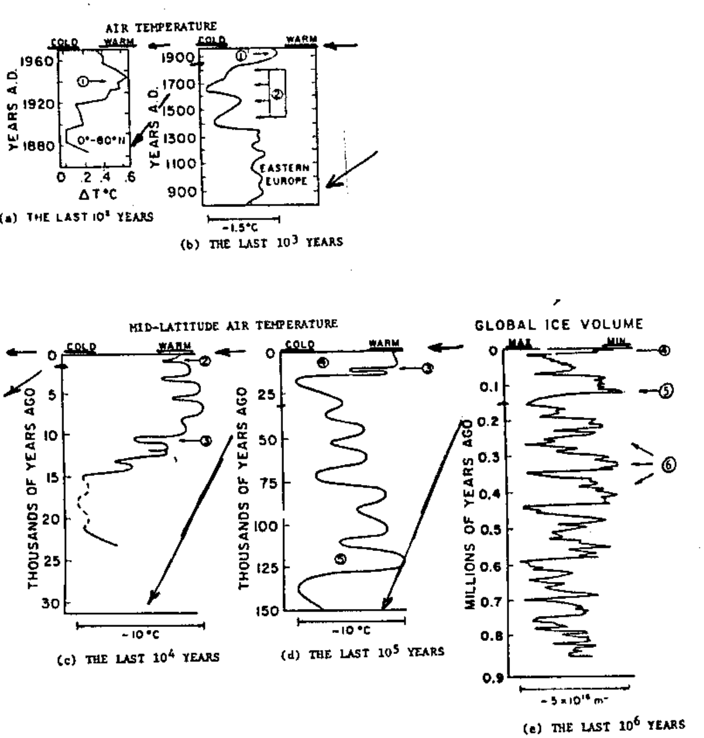
J.I. Díaz (ed), The Mathematics of Models for Climatology and Environement, NATO ASI Series, Springer Verlag, 1997.

J. I. Díaz, On the von Neumann problem and the approximate controllability of Stackelberg-Nash strategies for some environmental problems, Rev. R. Acad. Cien.Serie A Matem, 96, nº 3, 2002, 343-356

Milutin Milankovitch (1879-1958)







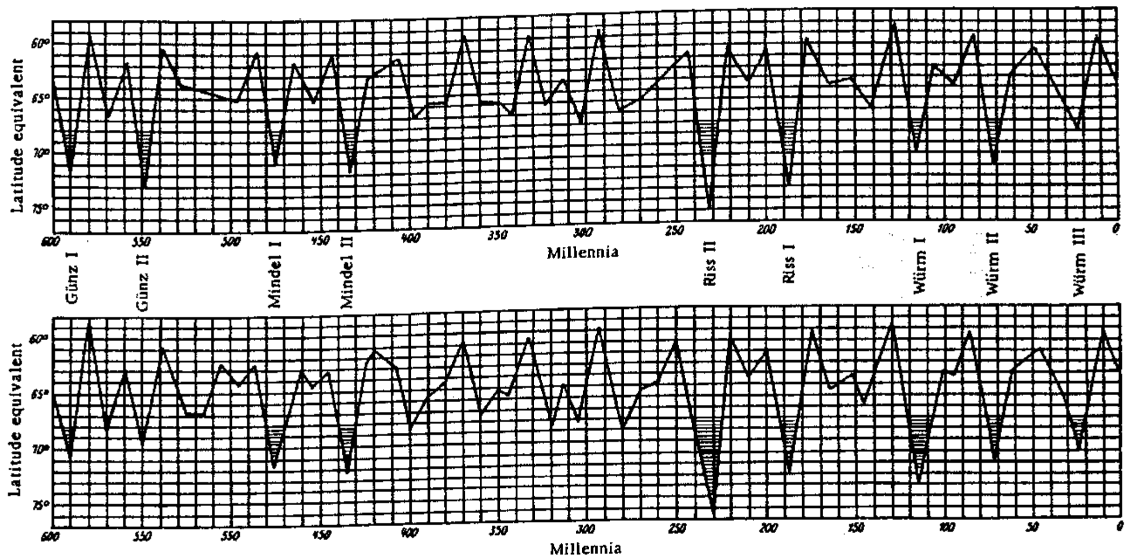


Fig. 7. The Milankovitch radiation curves for latitude 65°N over the past 600,000 years. Milankovitch identified certain low points on the curve with four European ice ages [Milankovitch, 1941]. The top figure is from Milankovitch's 1920 computation according to Stockwell and Pilgrim, and the bottom figure is from his 1941 computation according to Le Verrier and Miskovitch (see Table 1). Vertical axis gives the 65°N Milankovitch equivalent latitudes; time along the horizontal axis is in thousands of years before present.

CLIMATE CHANGE OF MARS: EFFECTS OF OBLIQUITY CHANGES.
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Early Mars Climate Models

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C.Sagan and G. Mullen, Earth and Mars: Evolution of atmospheres and surface temperatures, Science, 177, 1972, 52-56.

A challenging future…