Bifurcation of stationary solutions with respect to the Stefan-Boltzmann emissivity of the outgoing terrestrial radiation in a diffusive Energy Balance Climate Model.

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ETS Arquitectura,
21 de Enero de 2020

Doctorado IMEIO
Actividad formativa

UCM-UPM

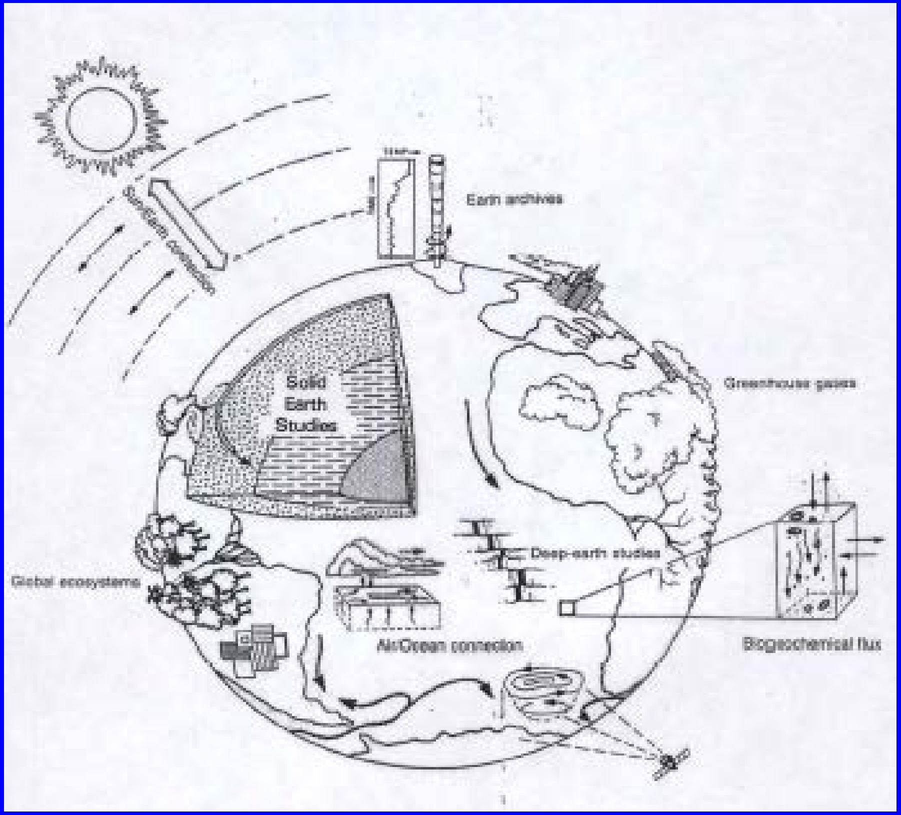
Curso 2019-2020

Modelos No Lineales
en Ingeniería Matemática

Lugar: ETS Arquitectura. UPM.
Aula: XC9
Horario: 20-23 de enero de 2020. (15 horas).
Coordinadora de la actividad: Lourdes Tello. (l.tello@upm.es)

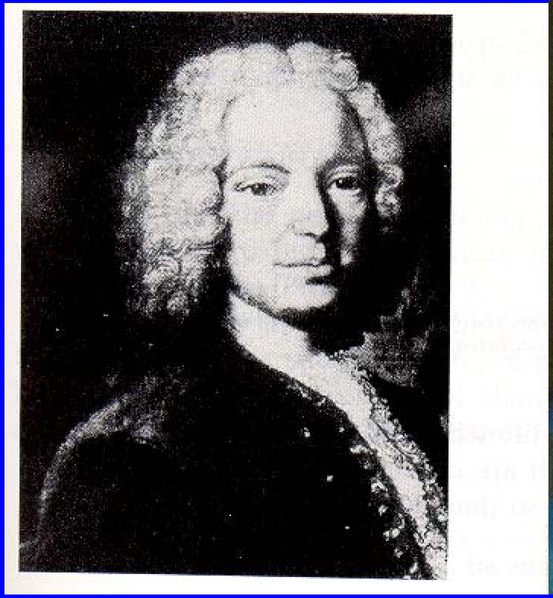
1. Introducción.

El Planeta Tierra: *sistema complejo con numerosos procesos realimentantados*



“Matemáticos” pioneros en el estudio del clima

Convocatoria, en 1738, del Premio de Matemáticas de la Academia Francesa de Ciencias sobre “la causa del flujo y reflujo del mar”. Premiados: ***Daniel Bernoulli, Euler y MacLaurin***



Daniel Bernoulli (1700-1782)

Leonhard Euler (1707-1783)

Joseph Fourier

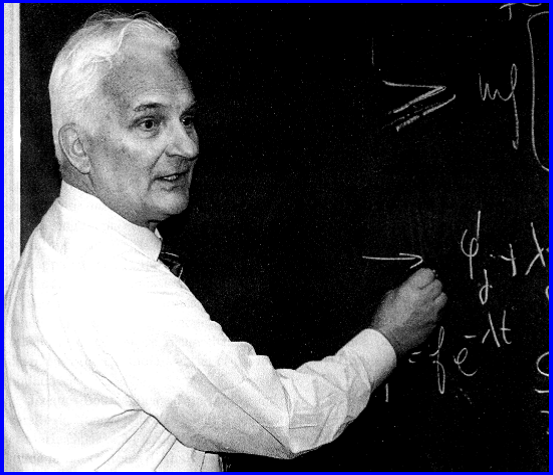
(1768-1830)

(1824)

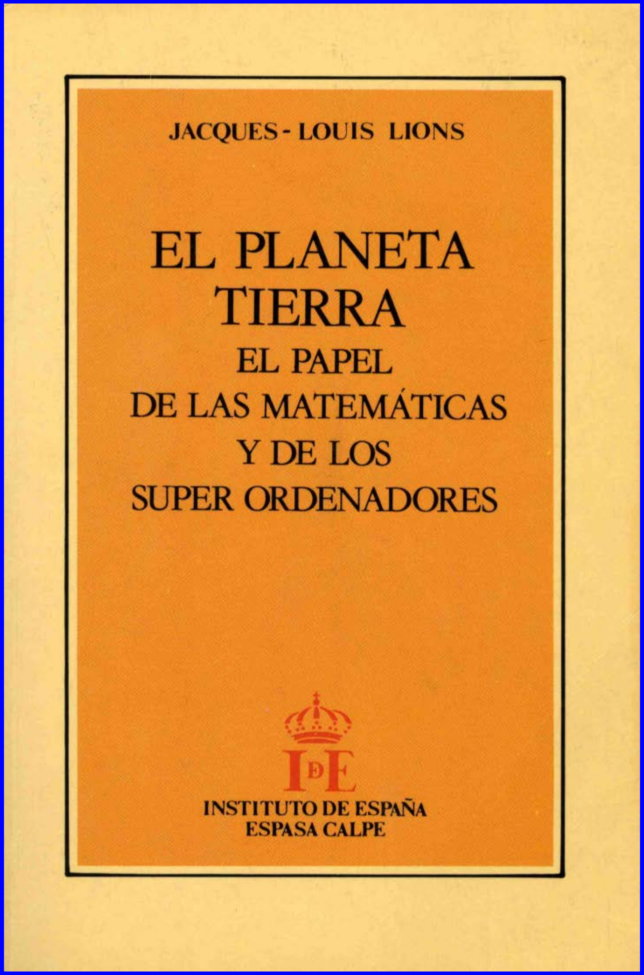
El establecimiento y el progreso de las sociedades humanas, la acción de las fuerzas naturales, pueden cambiar notablemente y dentro de grandes regiones, el estado de la superficie del suelo, la distribución de las aguas y los grandes movimientos del aire. Dichos efectos son capaces de hacer variar el calor medio a lo largo de varios siglos ...

Veremos que el papel activo de numerosos matemáticos en el estudio del clima es hoy día una realidad con un futuro en expansión.

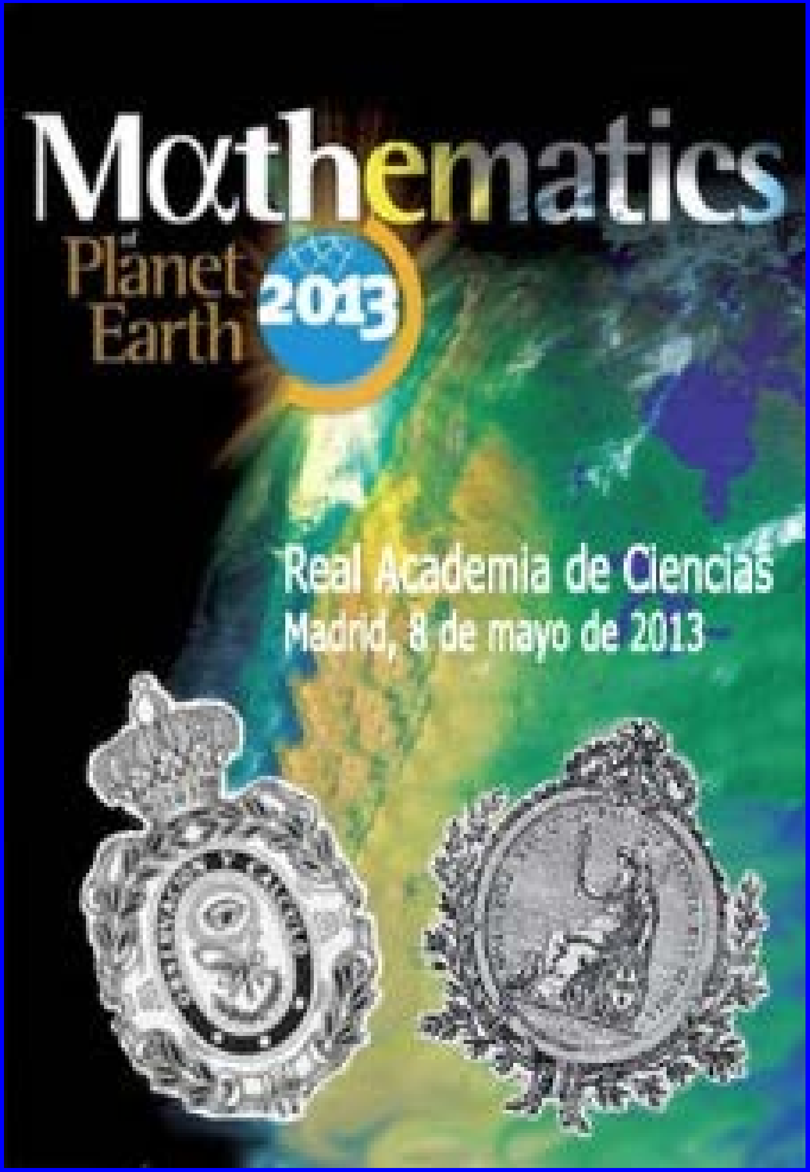
Del libro de 1990 de J.L. Lions
a la creación del
Centre for the Mathematics of Planet Earth, University of Reading, Reading, RG66AX, UK



J.L. Lions
(1928-2001)



Instituto de España, Enero 1990

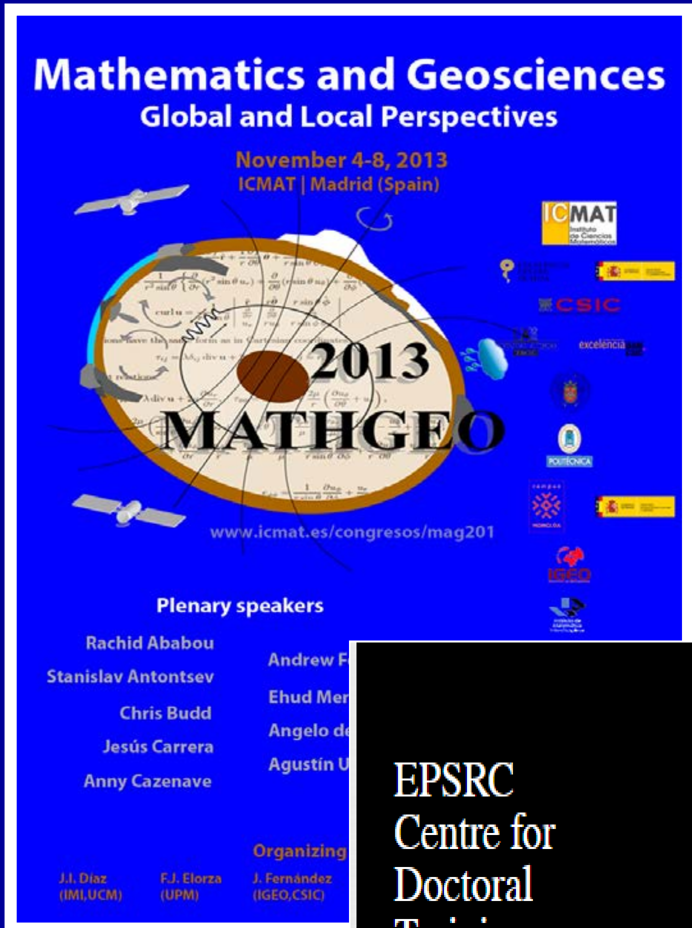


Real Academia de Ciencias, Madrid, 8 de Mayo 2013
Academia de Ciencias, Lisboa, 21 Noviembre 2013

http:///mpe2013.org/

Matemáticas del Planeta Tierra
en España
http://www.icmat.es/mathearth

Matemáticas del Planeta Tierra en Francia
http://mpt2013.fr/



Otros resultados multinacionales de esas “celebraciones”

Numerosos puntos de vista matemáticos para el clima

1. Introducción

1.a. Un articulo con bastantes citas:

A. 194. .J. I. Díaz, A. Hidalgo and L. Tello, Multiple solutions and numerical analysis to the dynamic and stationary models coupling a delayed energy balance model involving latent heat and discontinuous albedo with a deep ocean.Proc. R. Soc. A. 2014 470 20140376; doi:10.1098/rspa.2014.0376 (published 27 August 2014) (PDF). **SESIÓN de L. TELLO**

1.b. Transparencias sobre promedios

2. Existencia de soluciones

2.a. Caso de evolución (trasparencias)

2.b. Evolución con término estocástico:

A.163. J.I. Díaz, J.A. Langa, J. Valero, On the asymptotic behaviour of solutions of a stochastic energy balance climate model. Physica D, 238 (2009), 880-887. (PDF)

A.96. G. Díaz, J. I. Díaz. On a stochastic parabolic PDE arising in Climatology. Rev. R. Acad. Cien.Serie A Matem, 96, nº 1, 2002, 123-128. (PDF)

3. Unicidad y multiplicidad de soluciones

3.a. Caso del problema de evolución parabólico (trasparencias)

3.b. Caso estacionario (trasparencias)

4. Sobre la frontera libre

4.a Mushy region para p>2 (trasnparencias)
4.b. Regularidad de la frontera libre. A.166. J.I. Díaz, S. Shmarev, Lagrangian approach to the study of level sets II: a quasilinear equation in climatology, J. Math. Anal. Appl. 352 (2009) 475–495 (PDF)

5. Problemas de control para modelos climáticos difusivos

J. I. Díaz. On the von Neumann problem and the approximate controllability of Stackelberg-Nash strategies for some environmental problems. Rev. R. Acad. Cien.Serie A Matem, 96, nº 3, 2002, 343-356. (PDF)

J. I. Díaz. Controllability and obstruction for some non linear parabolic problems in Climatology. En el libro Modelado de Sistemas en Oceanografía, Climatología y Ciencias Medio-Ambientales (C.Pares y A.Valle eds.) Universidad de Málaga, 43-58, 1994.(PDF)

J. I. Díaz. Approximate controlability for some simple climate models. En el libro Environment, Economic and Their Mathematical Models (J. I. Díaz y J.L.Lions eds.), Research Notes in Applied Mathematics No 35, Masson, Paris, 29-43, 1994. (PDF)

J. I. Díaz. Mathematical analysis of some diffusive energy balance models in Climatology. En el libro Mathematics, Climate and Environment (J. I. Díaz and J.L.Lions eds.), Research Notes in Applied Mathematics nº 27, Masson, Paris, 28-56, 1993. (PDF)

Un articulo reciente: G. Floridia, Approximate controllability for nonlinear degenerate parabolic problems with bilinear control , Journal of Differential Equations volume 257, issue 9, year 2014, pp. 3382 - 3422

6. Tratamiento numérico de modelos climáticos difusivos

J. I. Díaz, A. Hidalgo and L. Tello, Multiple solutions and numerical analysis to the dynamic and stationary models coupling a delayed energy balance model involving latent heat and discontinuous albedo with a deep ocean. Proc. R. Soc. A. 2014 470 20140376; doi:10.1098/rspa.2014.0376 (published 27 August 2014) (PDF)

A. Hidalgo, L. Tello. A Finite Volume Scheme for simulating the coupling between deep ocean and an atmospheric energy balance model. In the book Modern Mathematical Tools and Techniques in Capturing Complexity. Springer Series in Complexity, Springer, Berlin (2011) 239-255.

J.I. Díaz, S. Shmarev, Langragian approach to level sets: application to a free boundary problem arising in climatology, Archive for Rational Mechanics and Analysis, 194, 2009, no.1, 75-103 (PDF)

R. Bermejo, J. Carpio, J.I. Díaz, L. Tello, Mathematical and Numerical Analysis of a Nonlinear Diffusive Climate Energy Balance Model. Mathematical and Computer Modelling, 2008. (PDF)

R. Bermejo, J. Carpio, J.I. Díaz, P. Galán de Sastre , A finite element algorithm of a nonlinear diffusive climate energy balance model, Pure and Applied Geophysics, 165, nº 6, 2008, 1025-1048. (PDF)

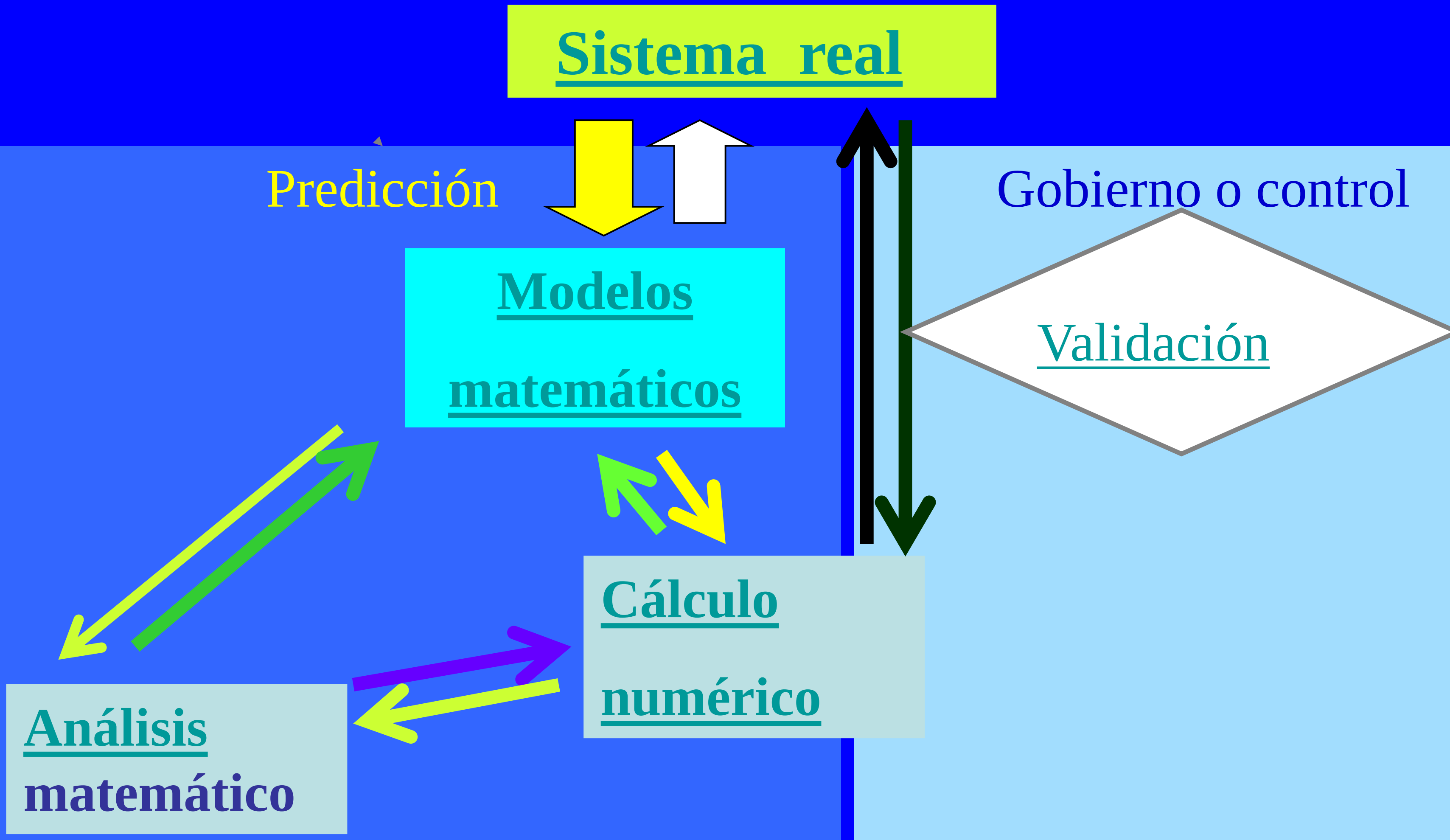
7. Bifurcación y estabilidad de soluciones estacionarias

S. Bensid, J.I. Diaz, Stability results for discontinuous nonlinear elliptic and parabolic problems with a S-shaped bifurcation branch of stationary solutions, Discrete and Continuous Dynamical Systems,
Series B, 22 N 05, (2017) 1757-1778.

S. Bensid and J.I. Diaz, On the exact number of monotone solutions of a simplied Budyko climate model and their different stability, Discrete and Continuous Dynamical Systems, Series B, 24, N 03, (2019), 1033-1047.

S. Bensid and J.I. Diaz, Existence, multiplicity and stability of solutions of quasilinear differential equation arising in climatology.

La “Trilogía Universal” de la Matemática Aplicada



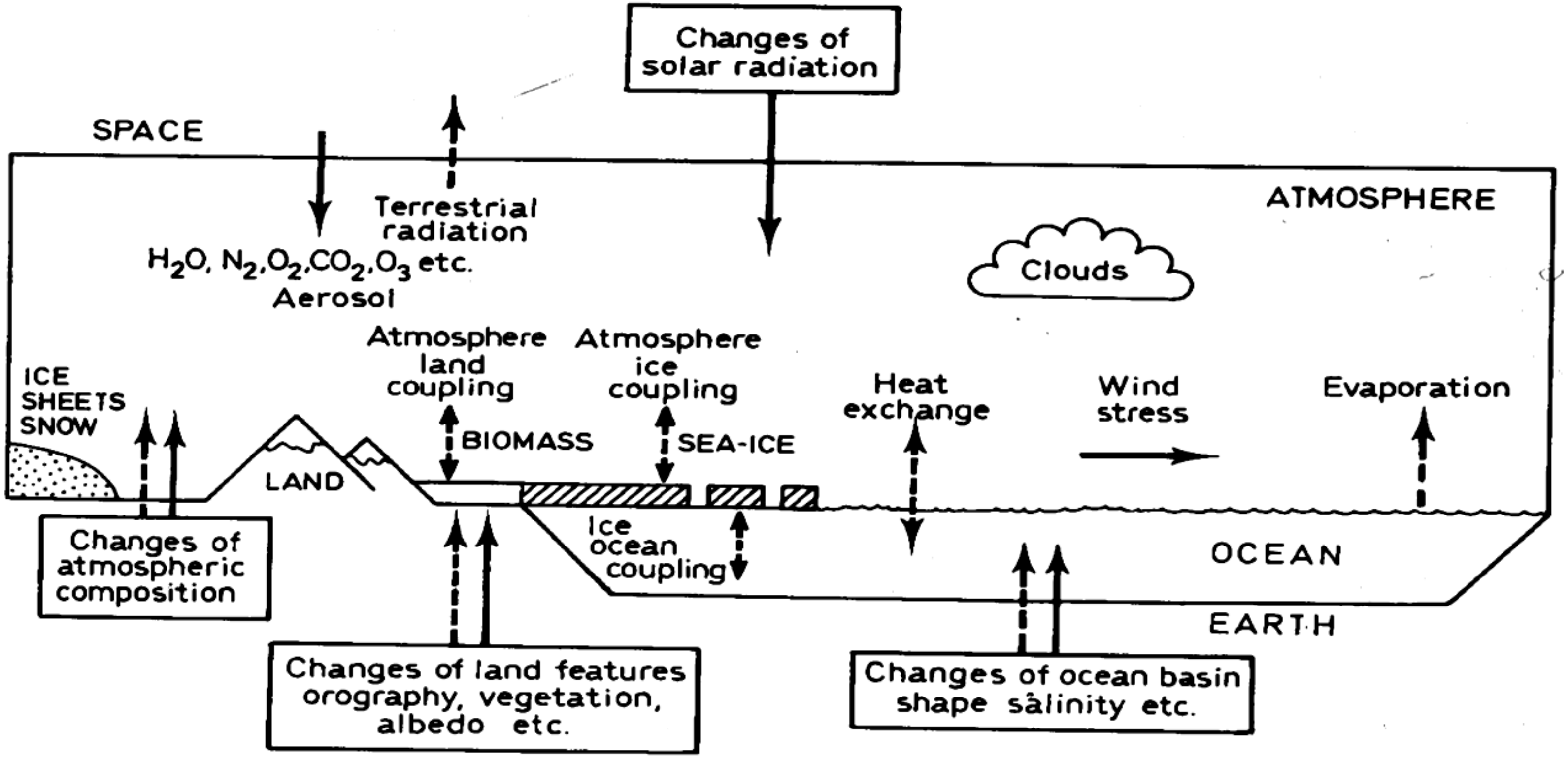
3. La "Trilogía Universal" y el clima global.

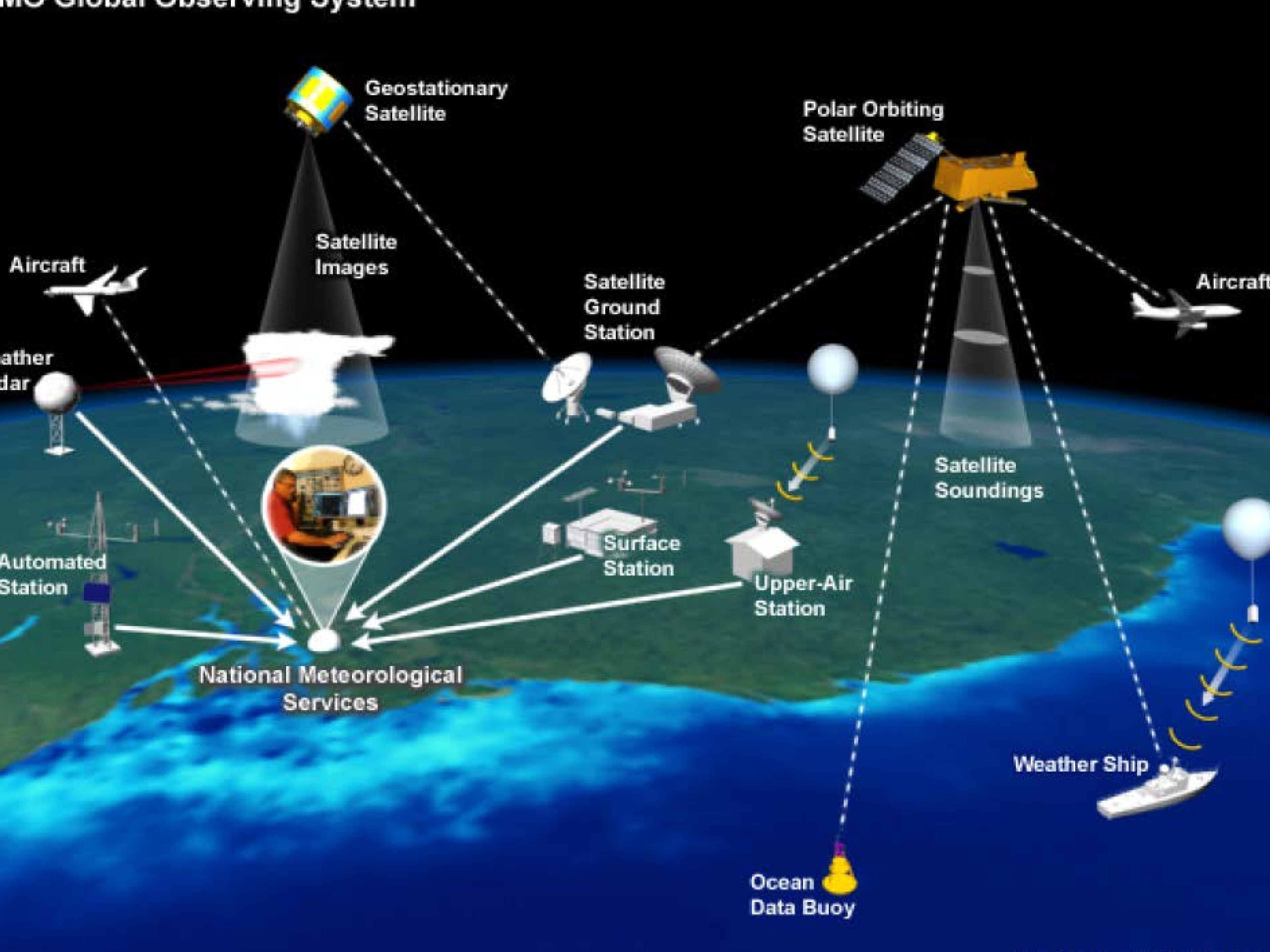
Sistema real

Predicción

Modelización matemática

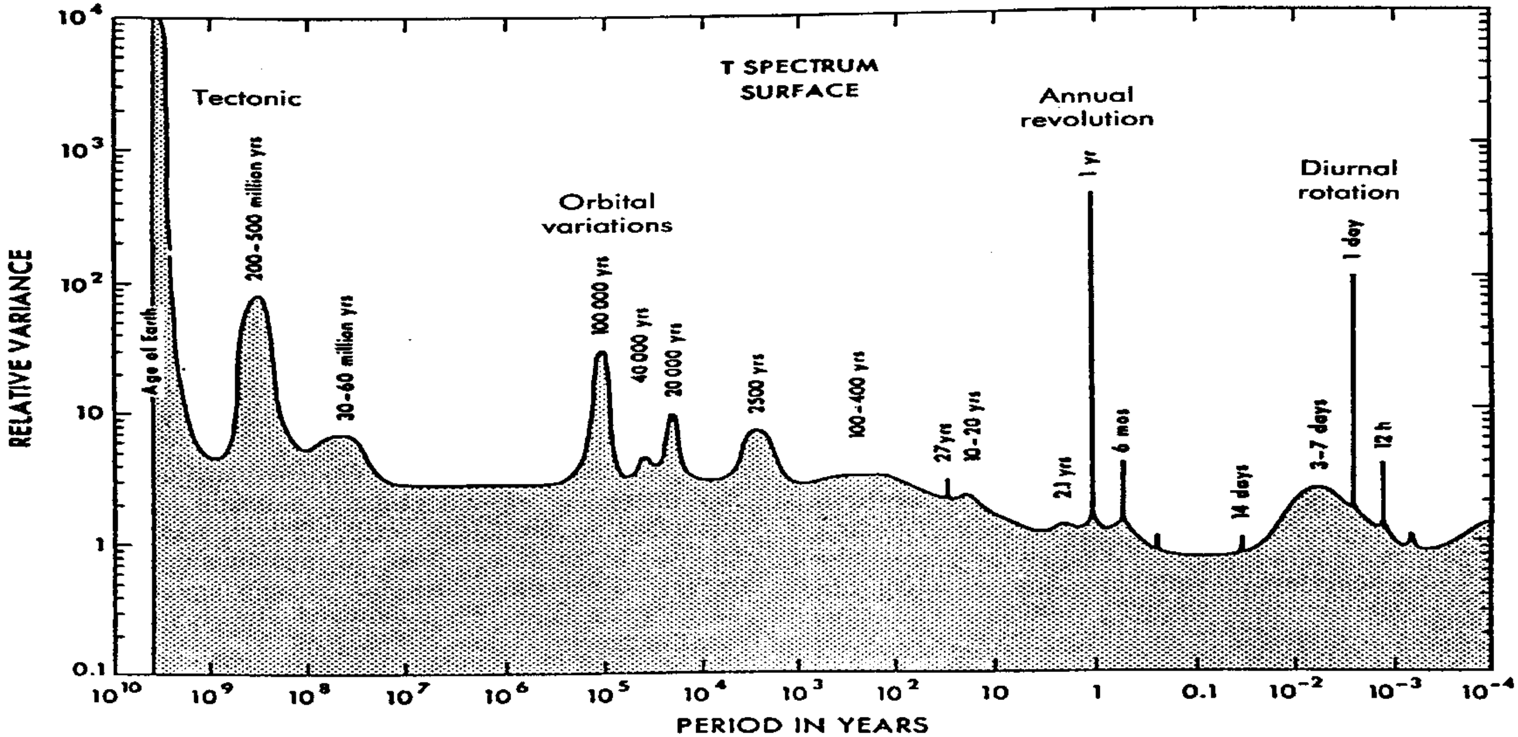
El problema real = CLIMA: componentes externos e internos del sistema climático





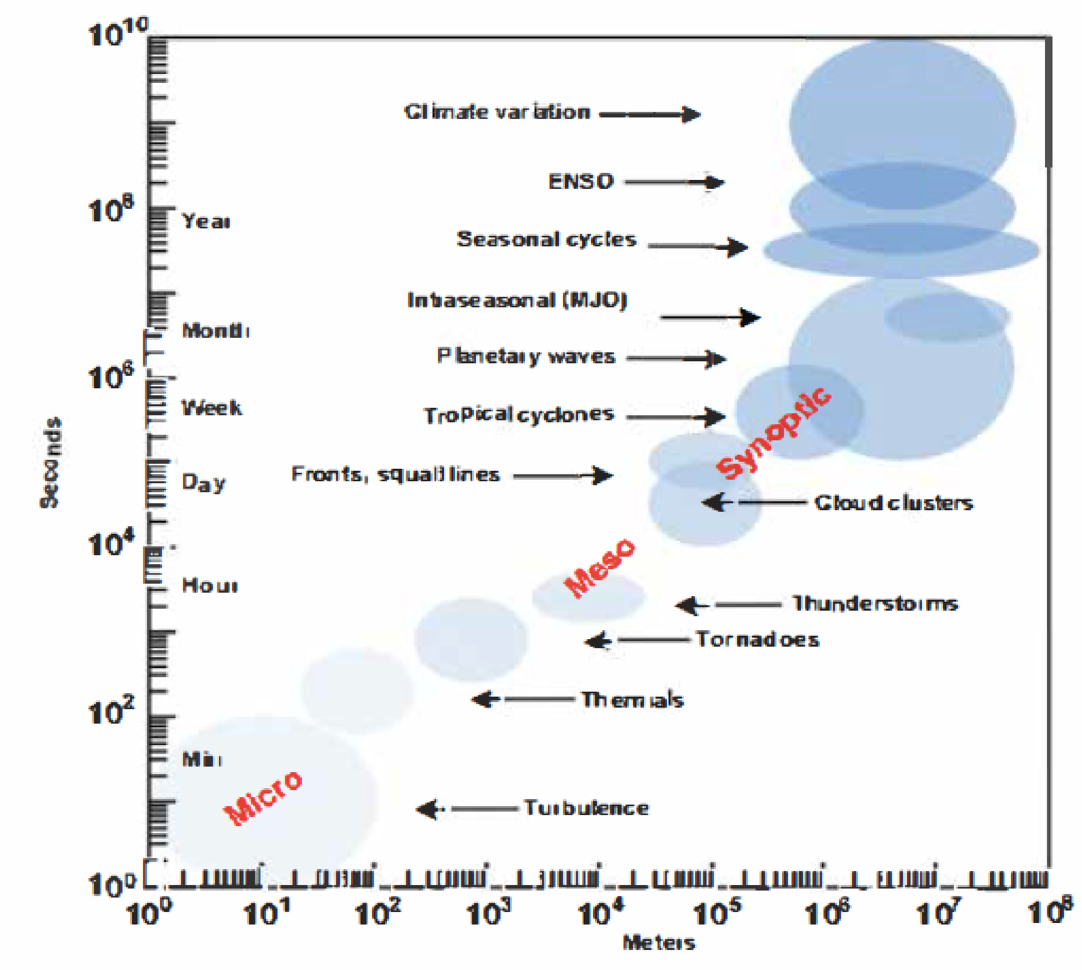
Geostationary Satellite
Polar Orbiting Satellite
Satellite Images
Aircraft
Satellite Ground Station
Satellite Soundings
Automated Station
Surface Station
Upper-Air Station
National Meteorological Services
Weather Ship
Ocean Data Buoy

Escalas temporales y espaciales:
Jerarquía de Modelos
Climatología / Meteorología



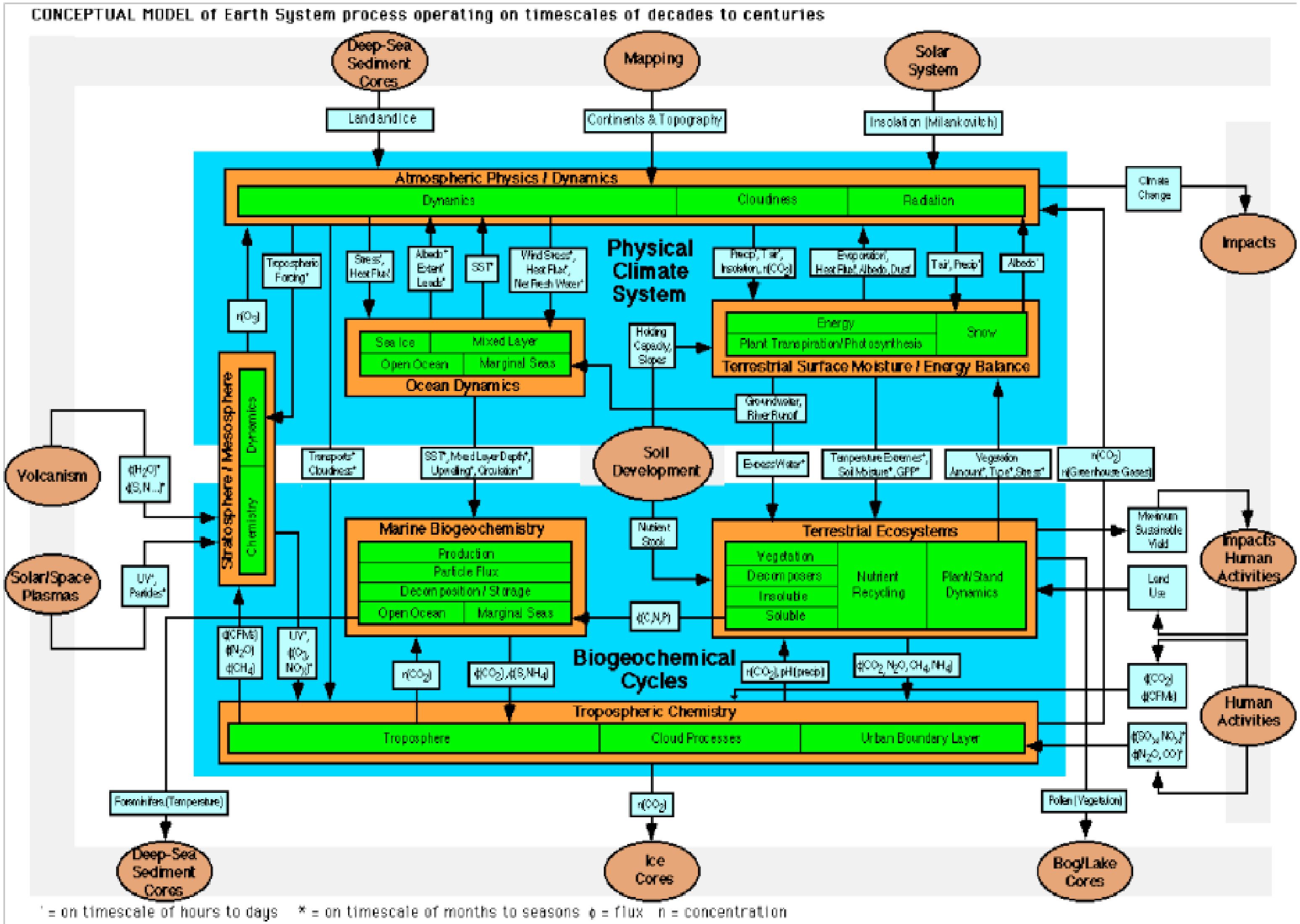
Espectro de variaciones climáticas

Algunas escalas características (atmósfera,…)



(c) COMET diagram of atmospheric variability

Horrendograma de acoplamientos entre los componentes principales del clima



(a) Bretherton horrendogram

Modelos globales / locales.
Modelos globales de Balance de Energía.

Clima: Estado promediado de la atmósfera observado como tiempo meteorológico sobre un periodo finito de tiempo a lo largo de los años (S.H. Schneider,1992)

$$u(x,t) = \frac{1}{2\tau\left|B(x)\right|}\int_{t-\tau}^{t+\tau}\int_{B(x)} T(y,s)\,dy\,ds$$

Predicción del tiempo meteorológico

Modelos climáticos

Pronóstico

Modelos realistas

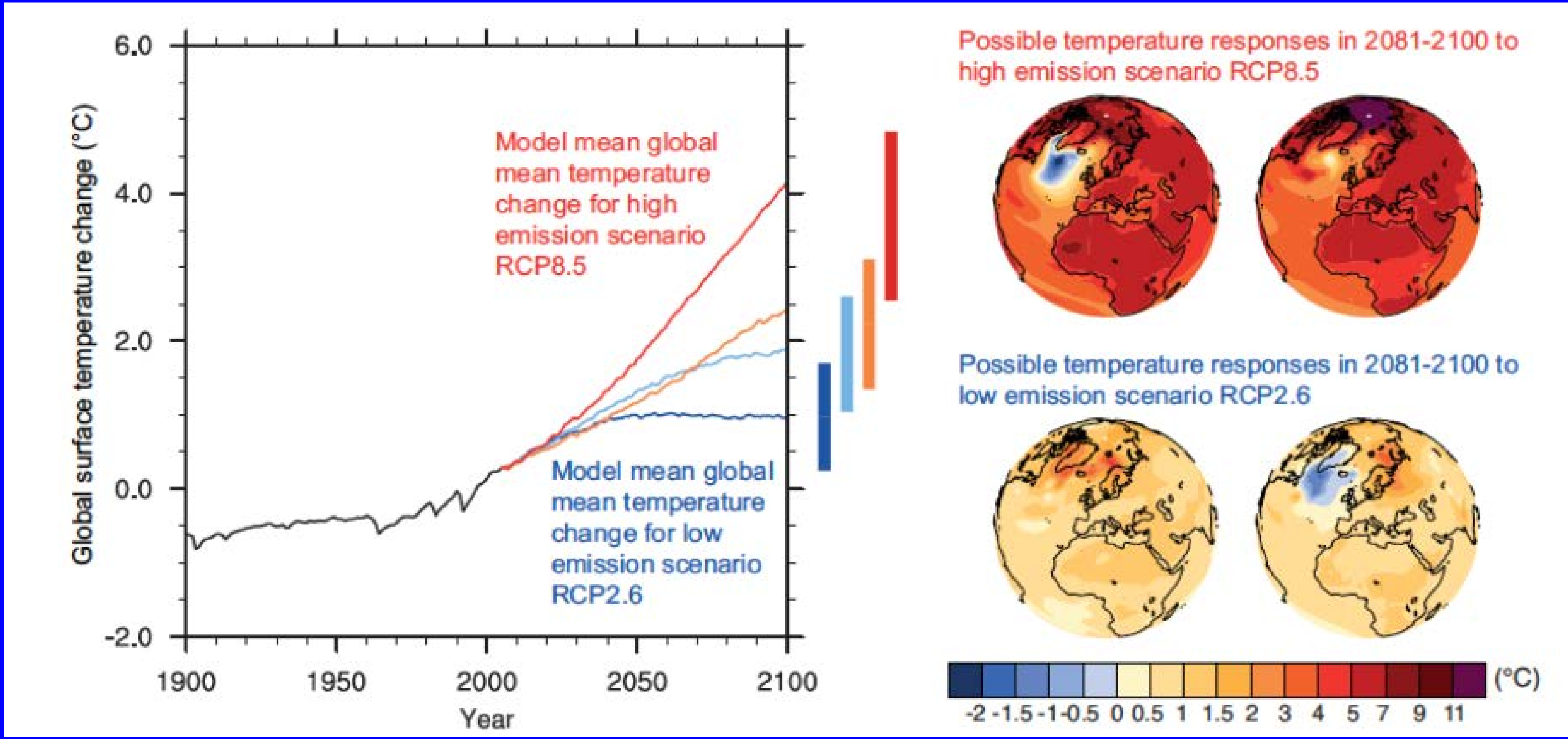
Métodos computacionales

Diagnóstico

Modelos simplificados

Métodos cualitativos

Evolución de los modelos climáticos en los primeros cuatro *Informes de evaluación del IPCC*: desde principios de la década de 1990 hasta mediados de la década de 2000 (IPCC, 2007).

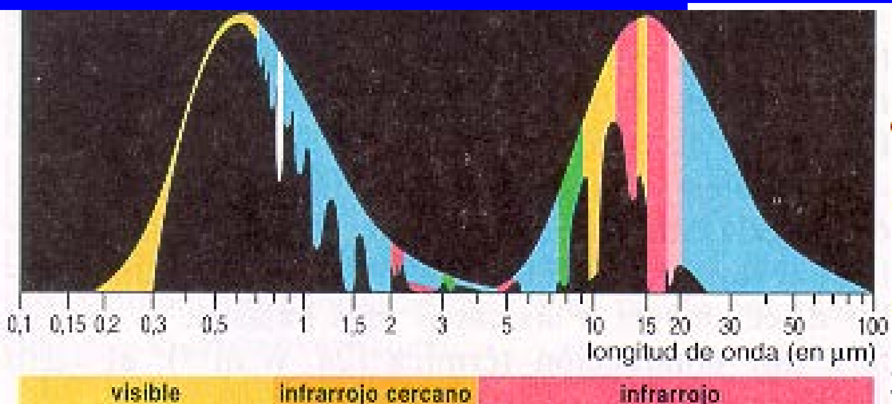


Global surface temperature change (°C)
6.0
4.0
2.0
0.0
-2.0
1900
1950
2000
2050
2100
Year
Model mean global mean temperature change for high emission scenario RCP8.5
Model mean global mean temperature change for low emission scenario RCP2.6
Possible temperature responses in 2081-2100 to high emission scenario RCP8.5
Possible temperature responses in 2081-2100 to low emission scenario RCP2.6
(°C)
-2 -1.5 -1 -0.5 0 0.5 1 1.5 2 3 4 5 7 9 11

El sistema climático de la Tierra recibe prácticamente toda su energía del Sol. Esta energía viene en forma de **radiación electromagnética, que se origina desde el Sol**.

Parte de la energía se elimina por absorción en la fotosfera solar, pero una buena parte del espectro de energía solar llega a la la atmósfera de la Tierra similar a la de **un cuerpo negro ideal a una temperatura de 5.780 K**.

La diferente absorción por los gases de la atmósfera le da al espectro solar (sobre la superficie de la Tierra) unos perfiles más irregulares



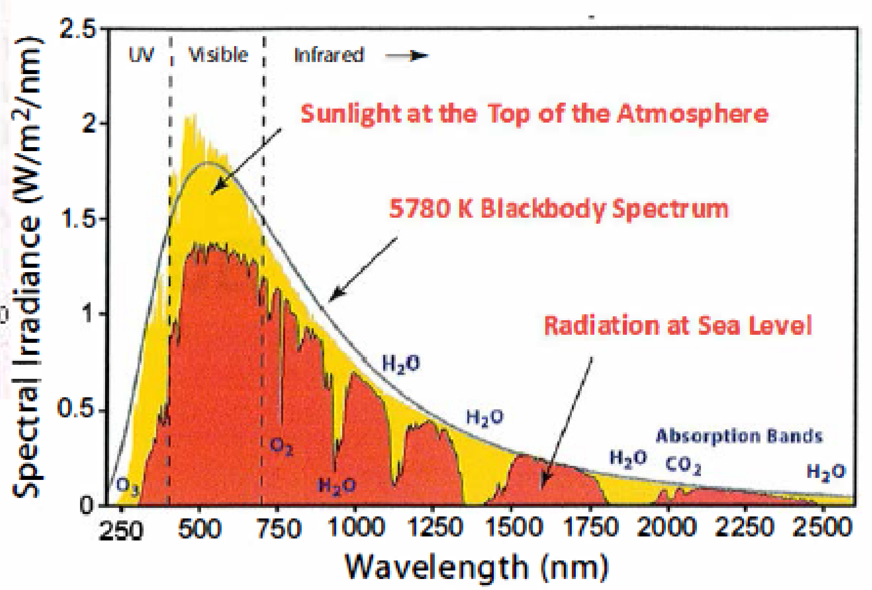
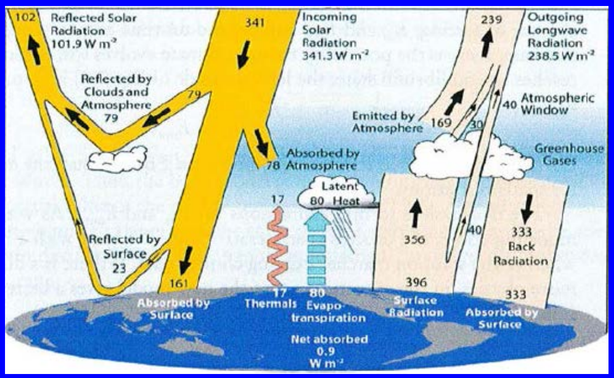


Figure 2.1. *The observed solar spectrum at the top of the atmosphere (yellow) and at sea level on a clear day (red). The solid line (black) is the spectrum the Sun would have if it were a black body at the temperature of 5,780 K. Image courtesy of Global Warming Art.*

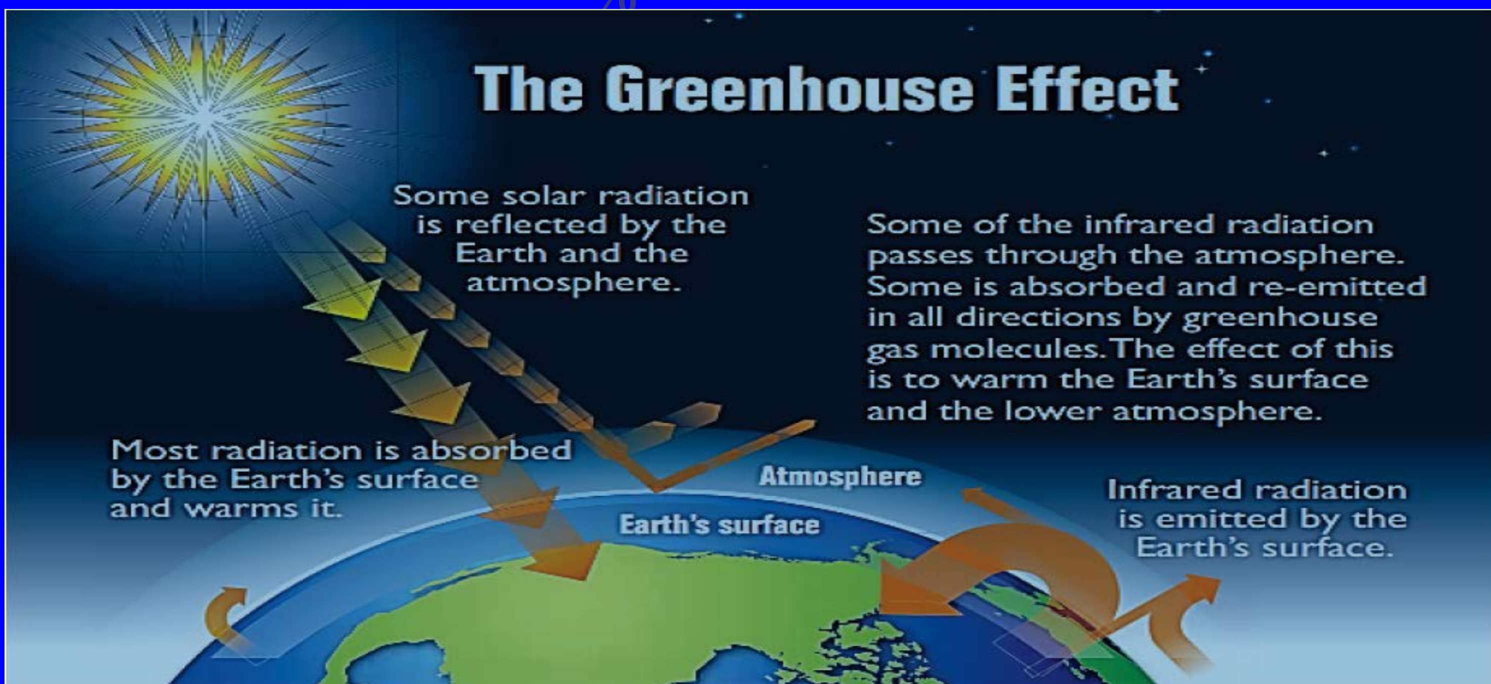


~342Wm^{-2}

30%

49 %

T= -18°c

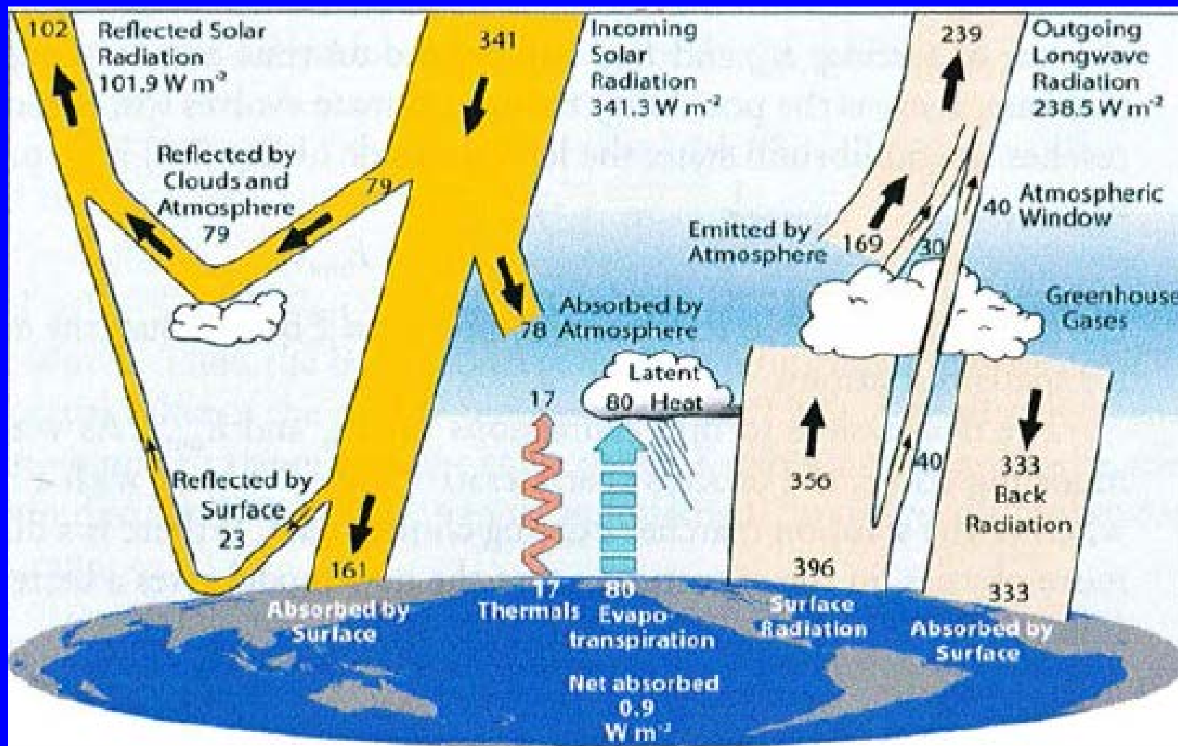


Balance de energía radiativa

S. Arrhenius (1896), ...

W.D. Sellers(1969),

M.I. Budyko (1969),....



R_a

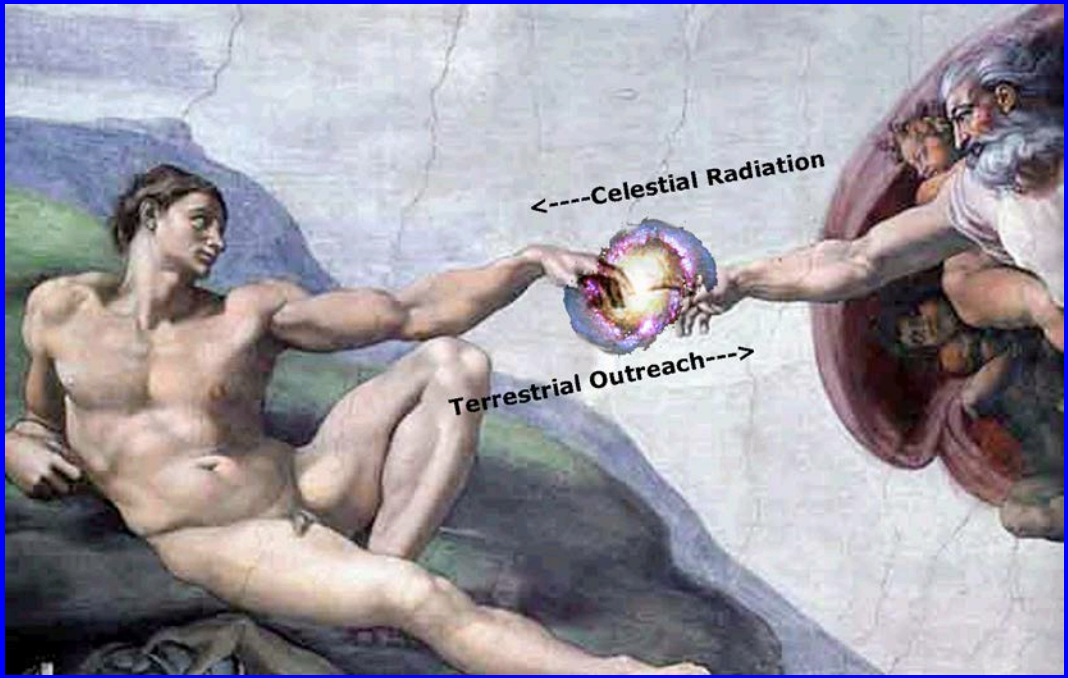
Albedo

R_e

Efecto invernadero

D

$$c\frac{\partial u}{\partial t} = R_a - R_e + D$$

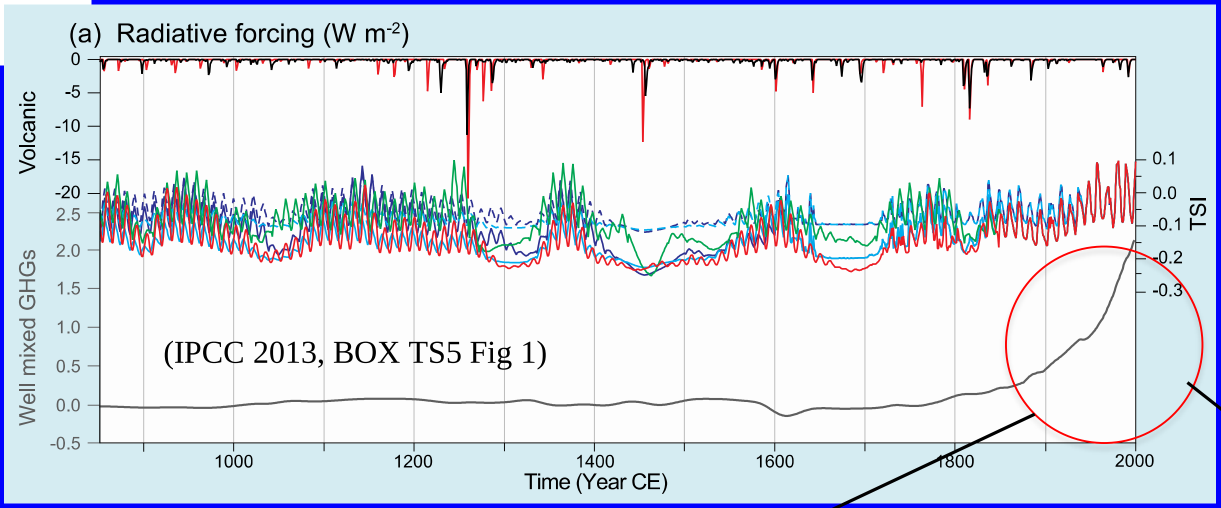


Los cambios en el sistema climático se deben alteraciones del equilibrio radiativo.

Causas potenciales:

1. Variaciones en la radiación solar entrante
2. Variaciones en la radiación solar reflejada.
3. Cambios en la radiación terrestre fuera de la atmósfera (por ejemplo, CO_2, CH_4, N_2O, H_2O_v)

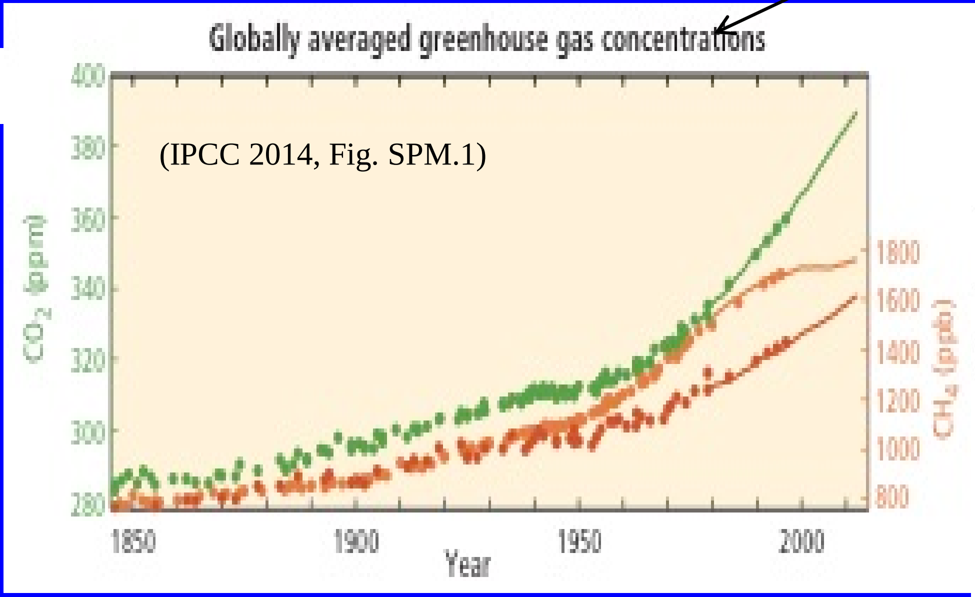
Factores de forzamiento radiativo
Naturales y de origen humano

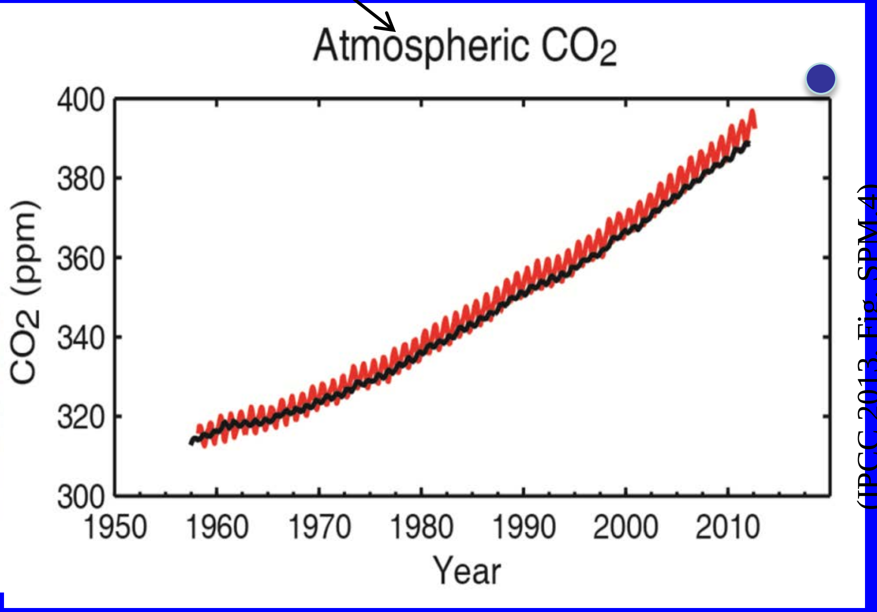


Las concentraciones de CO2 superan los valores de los últimos 800 ka

Mean rates of growth unprecedented in the last 22 ka high conf.)

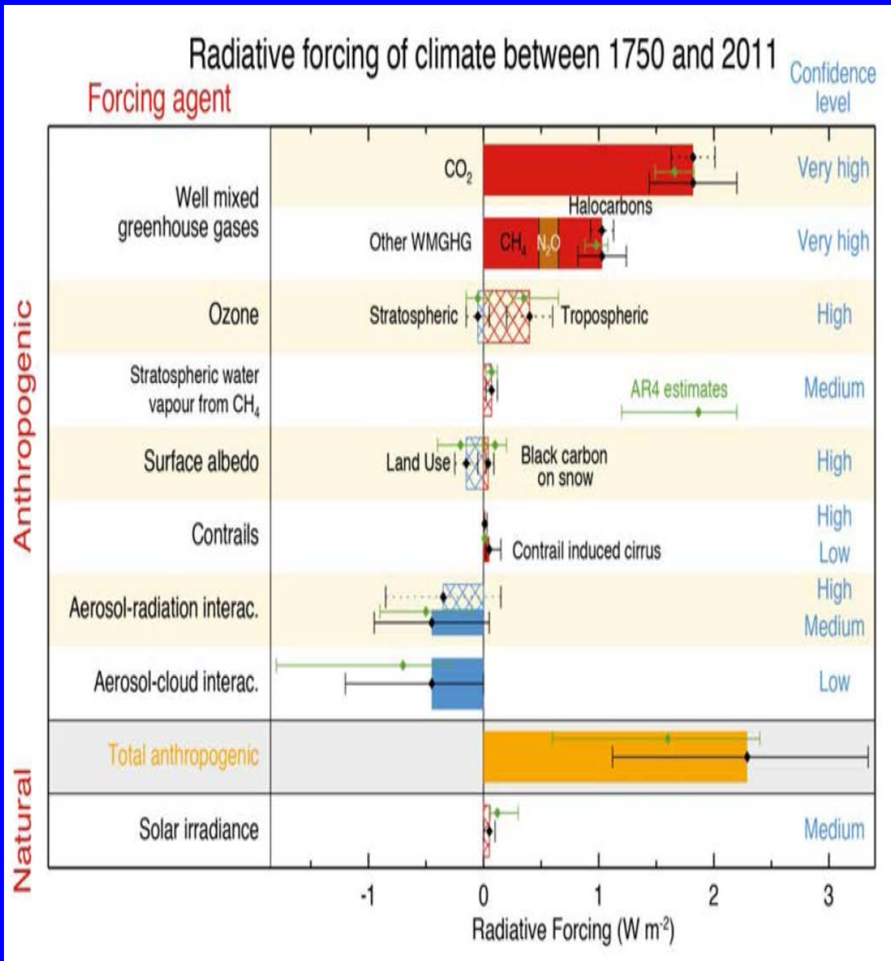
$\Delta[CO_2]$ ~ 40 % desde niveles preindustriales





-Ciais et al., IPCC, 2013: Ch6-

What causes system changes?: drivers of climate change



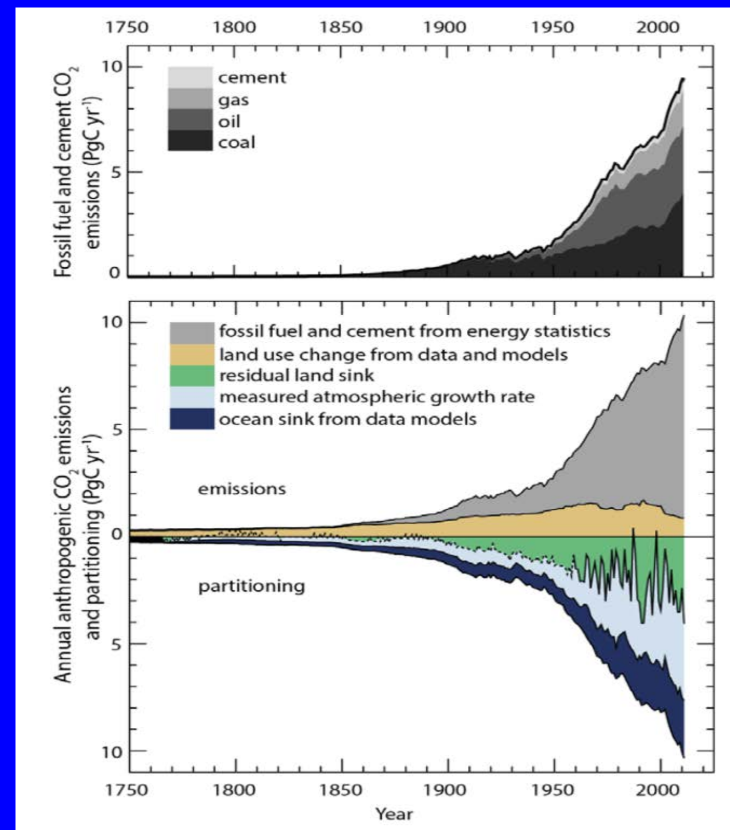
-Boucher et al., IPCC, 2013: Ch7-

- Myhre et al., IPCC, 2013: Ch8-

(IPCC 2013, Fig. TS.6)

Total radiative forcing is positive , and has led to an uptake of energy by the climate system

The largest contribution by Δ [CO2] since 1750



(IPCC 2013, Fig. TS.4)

-Ciais et al., IPCC, 2013: Ch6-

Regreso a modelos simplificados: ***Leyes de estado***

$$R_a = QS(x)\beta(u)$$

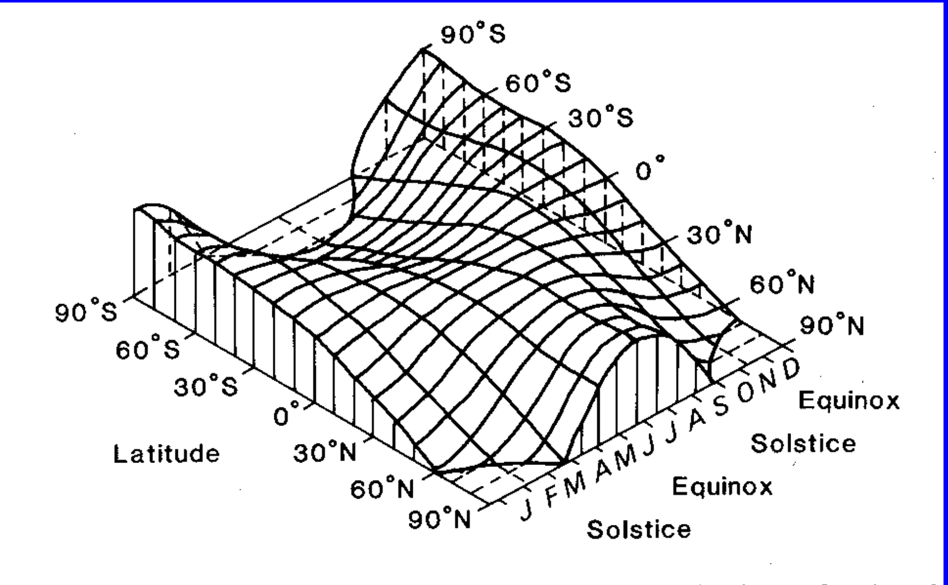


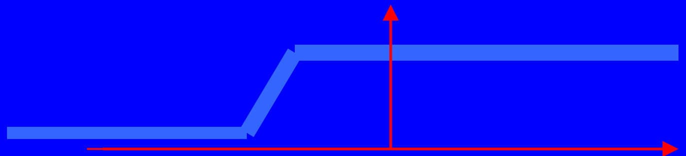
Fig. 2.8. The variation of insolation (at the top of the atmosphere) as a function of

Earth Radiation Budget Satellite

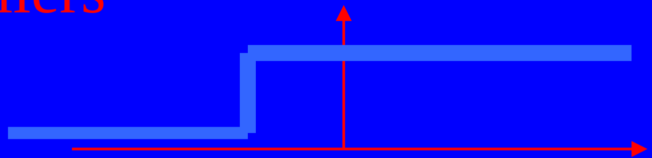
Satélite (ESA) ***Ingenio*** (CDTI),…

$$\beta(u) = (1 - a(u)) \text{ coalbedo}$$

$$\beta(u) = \begin{cases} 0.38 \text{ si } u << -10 \\ 0.71 \text{ si } u >> -10 \end{cases}$$



Sellers



Budyko **u= -10º**

Distribucion espacial (promediada anualmente) de la función insolación S(x)

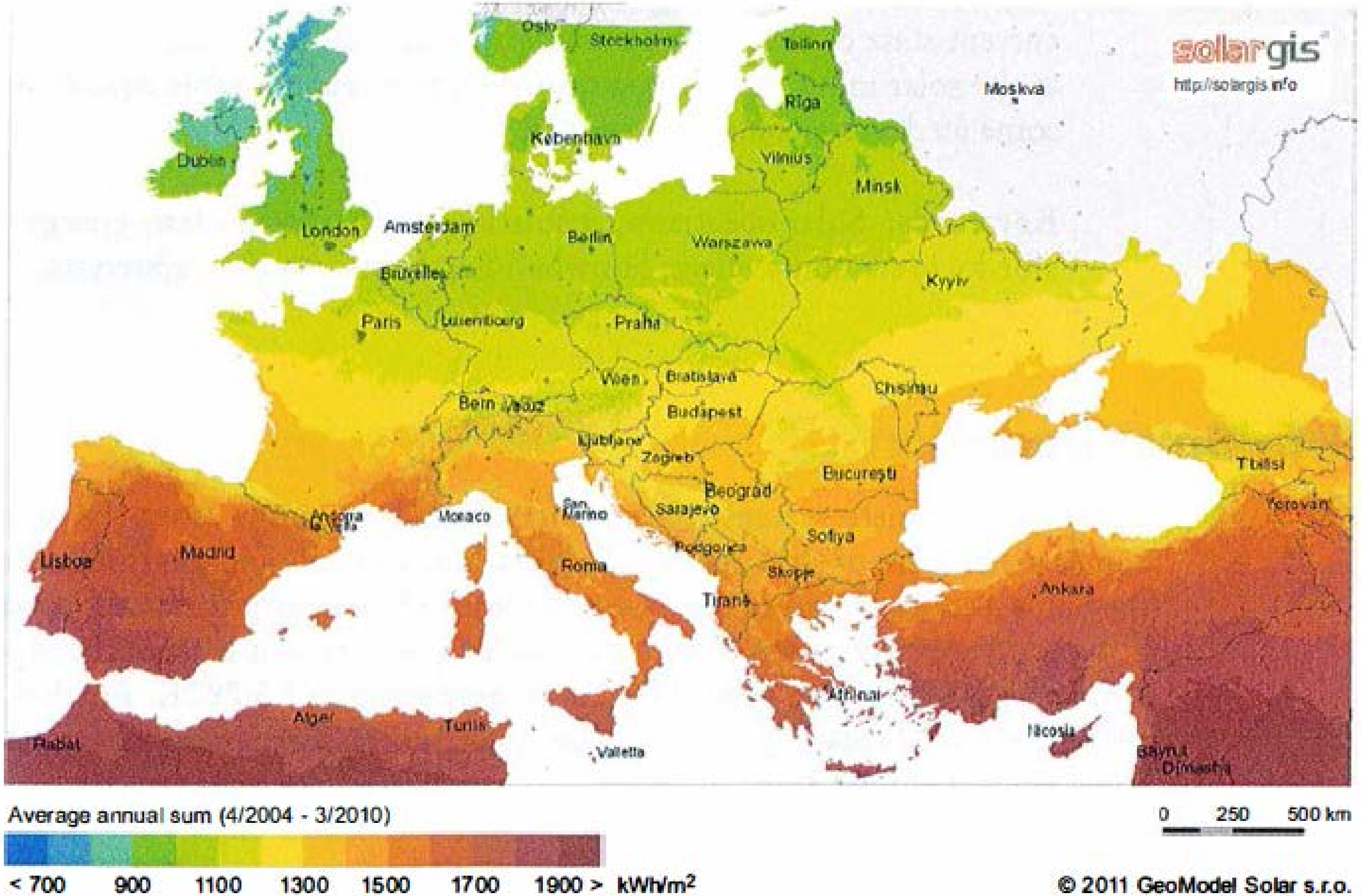


Figure 2.2. *Average annual surface insolation in Europe. Reprinted with permission from SolarGIS.*

$R_e = \sigma u^4$ Ley de Stefan-Boltzman **Sellers**

$R_e = A + Bu$ Ley de enfriamento de Newton **Budyko**

Relación empírica, Depende de gases de invernadero, cambios antropogénicos,... (variables internas)

Sobre el operador de difusión *D*

Jerarquía

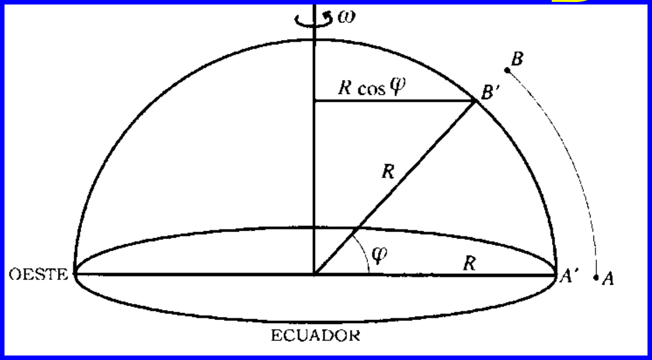
Modelo 0-dimensional *D*=0

$$c\frac{du}{dt} = Q\beta(u) - R_e(u)$$

Modelo 1-dimensional

$$D = \frac{1}{\cos\varphi}\frac{\partial}{\partial\varphi}(k\cos\varphi\frac{\partial u}{\partial\varphi}) = \frac{\partial}{\partial x}(k(1-x^2)\frac{\partial u}{\partial x})$$

$$x = \cos\varphi$$



Difusión bidimensional

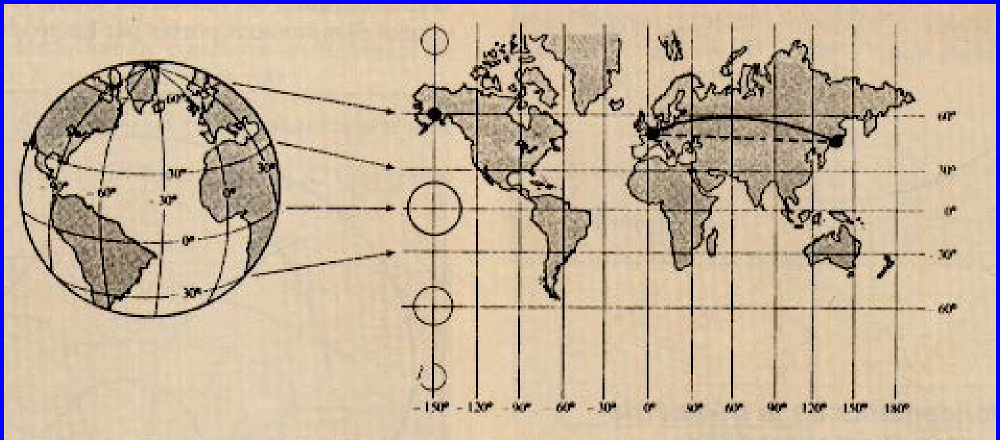
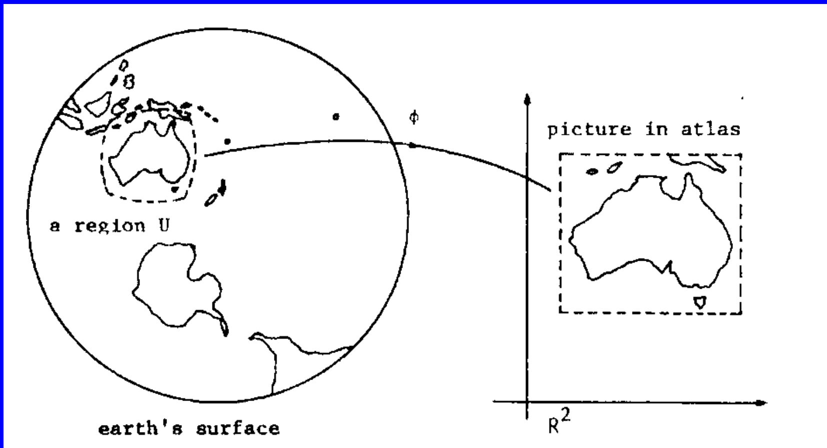
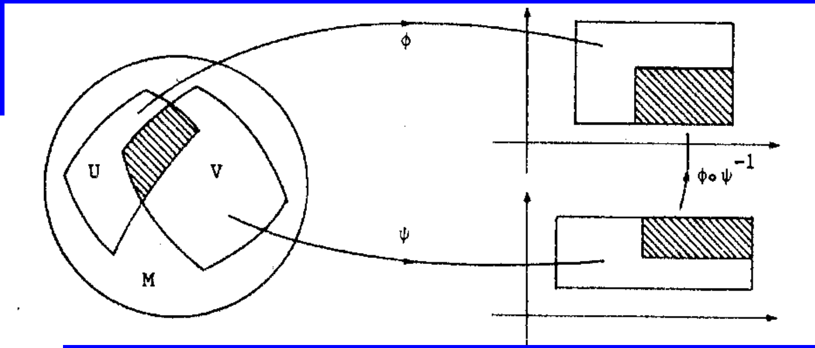
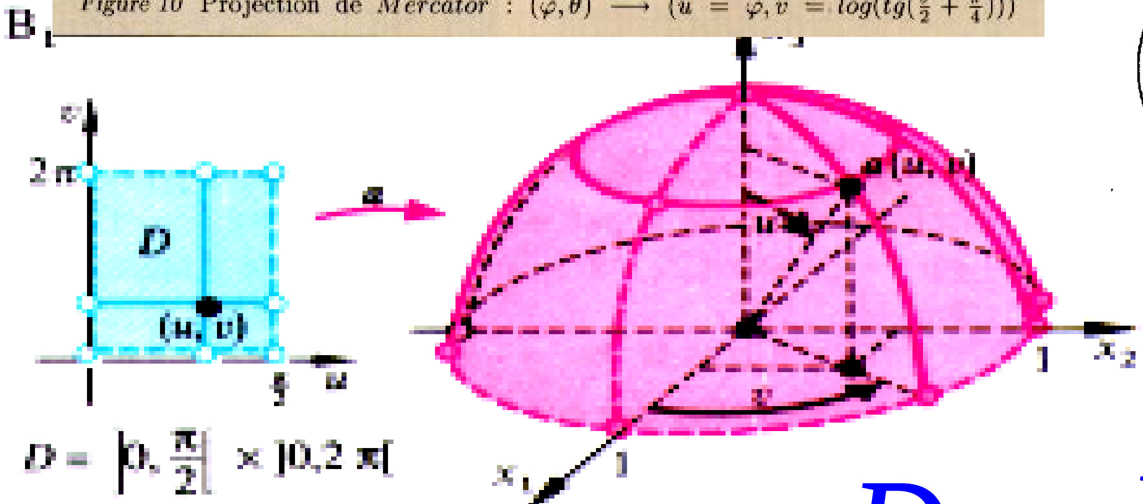


Figure 10 Projection de *Mercator* : $(\varphi, \theta) \longrightarrow (u = \varphi, v = log(tg(\frac{\theta}{2} + \frac{\pi}{4})))$







$D = \left]0, \frac{\pi}{2}\right[\times]0, 2\pi[$

$a : D \to \mathbb{R}^3$ déf. par $(u, v) \mapsto a$

$$a(u, v) = \begin{pmatrix} \sin u \cos v \\ \sin u \sin v \\ \cos u \end{pmatrix}$$

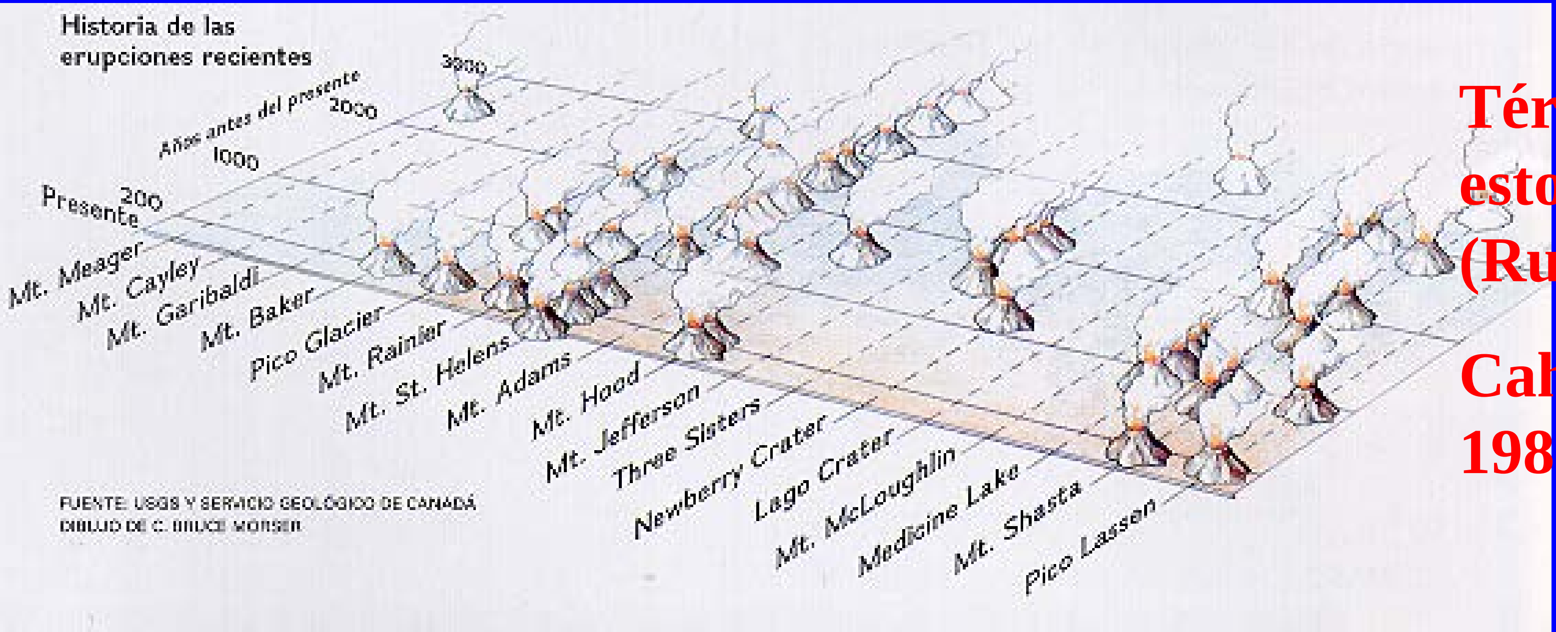
$$D = div(k(x)\nabla u)$$

$$c\frac{\partial u}{\partial t} = Q\beta(u) + R_e(x,u) + div(k(x)\nabla u)$$

$$u(x,t_0) = u_0(x)$$

Modelos estocásticos :Volcanes

$R_e(x,u)$



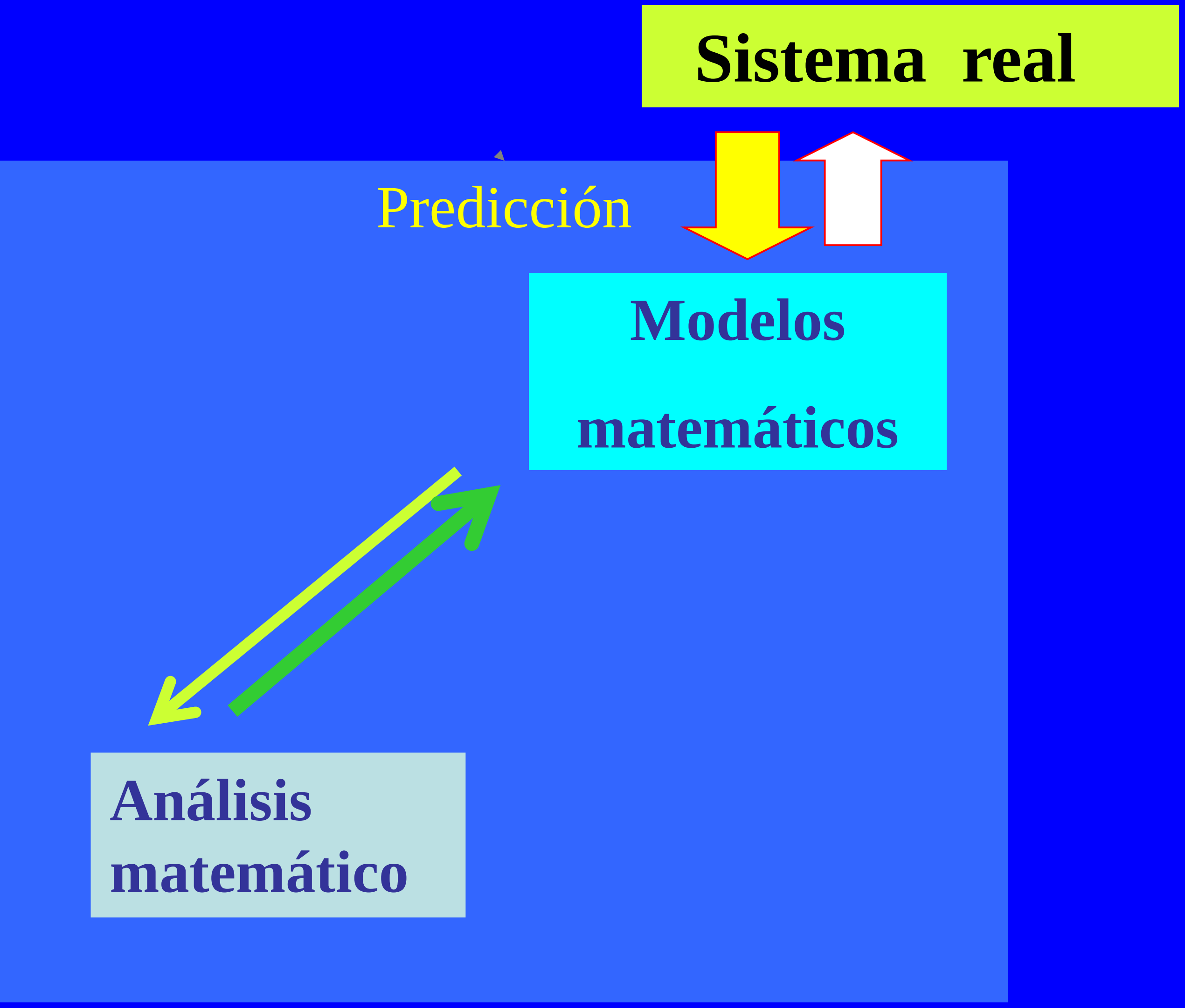
Término estocástico (Ruido blanco)

Cahalan-North, 1982

Modelos más complejos:

- Términos de retardo (promedios,…)
- Acoplamiento con las ecuaciones de de la energía interna del océano profundo
- Acoplamiento con las ecuaciones de la dinámica de grandes masas de hielo
- Acoplamiento con las ecuaciones de la Mecánica Celeste
- Acoplamiento con las ecuaciones del manto como medio visco-elástico
- Acoplamiento con modelos para la biosfera

La “Trilogía Universal” de la Matemática Aplicada



1 Introduction

In this paper, we are concerned by the study of non-negative solutions a quasilinear problem which arises as a simplified climate model. We consider

$$(P)\begin{cases} -(|u'|^{p-2}u')' + \epsilon g(u) = \lambda f(u) & \text{in} \quad (0,1), \\ u'(0) = 0, u(1) = u_0, \end{cases}$$

where u represents the temperature in Kelvin degrees, $u_0 \geq 0$ is given, $p > 2$ (case which is justified to take into account some turbulence effects according Stone [31]) and ϵ is a variable parameter (mainly $\epsilon \in (0,1)$) representing the different emissivity due to the variations of the greenhouse gases according the Stefan-Boltzmann law

$$g(u) = u^4, \text{ for } u > 0.$$

Here $x = \cos\Phi$ with Φ the latitude. The nonlinear term $f(u)$ is co-albedo when it is represented in an abrupt way near the color change of the ice sheets ($\mu = -10^0 C$) the discontinuous function given by

$$f(u) = f_0 + (1 - f_0)H(u - \mu), \tag{1.1}$$

for some $\mu > 0$, under the key assumption that the given parameter

$$f_0 \in (0,1), \tag{1.2}$$

with $H(s)$ the Heaviside discontinuous function

$$H(s) = 0 \quad \text{for} \quad s < 0, \qquad H(s) = 1 \quad \text{for} \quad s \geq 0.$$

In a previous paper by the authors we study the so called Budyko-Sellers EBM model corresponding to the semilinear case $p = 2$ with a linearization of the Stefan-Boltzmann law (the Newton cooling law) and then $\epsilon = \lambda\omega^2$ and $g(u) = u$. The main tool was the maximum principle and the construction of some explict solutions of some linear auxiliary boundary value problems. In that case the variability of the parameter λ was attached to the eventual variability of the Solar constant (usually denoted by Q in the climate modelling literature after the work by Budyko [5]). The main goal of this paper is to show that some similar arguments can be applied in the quasilinear case, even if the associate auxiliary problems can not be explicitly solved. Moreover, here we will study the case in which the solar constant is maintained as a fixed constant and the variability (multiplicity of solutions and the different stability) is exclusively due to different values of the emissivity ϵ.

This work deal with two main questions. The first concern the existence and multiplicity of non-negative (monotone decreasing) solutions of problem (P). Note that by a solution $u_{\epsilon,\mu}$ of problem (P), we mean a function $u \in C^2((0,1) \setminus \{x_{\epsilon,\mu}\}) \cap C^1([0,1))$, for some $x_{\epsilon,\mu} \in [0,1)$ where $u(x_{\epsilon,\mu}) = \mu$ (called as *the free boundary associated to* u) and with $u \geq 0$, $u \neq 0$, such that $-(|u'|^{p-2}u')' + \epsilon g(u) = \lambda f(u(x))$, for any $x \in (0,1) - \{x_{\epsilon,\mu}\}$, and $u'(0) = 0$, $u(1) = u_0$, for $u_0 > 0$.
So, we will obtain a bifurcation curve $\|u_\epsilon\|_\infty = \Psi(\epsilon)$, where $\|u\|_\infty = \max\limits_{x\in[0,1]} |u(x)|$ and we show that is a reflected $S-$shaped curve giving the exact multiplicity of solutions in terms of the parameter ϵ.

We remark that any solution of (P) is also a weak solution of the *multivalued problem*

$$P^*(\epsilon,\beta)\begin{cases} -(|u(x)'|^{p-2}u'(x))' + \epsilon g(u(x)) \in \lambda\beta(u(x)) & a.e.\ x \in (0,1), \\ u'(0) = 0,\ u(1) = u_0, \end{cases}$$

where β is the maximal monotone graph of $\mathbb{R}^2$ given by

$$\begin{cases} \beta(r) = f(r) & \text{if } r \neq \mu, \\ \beta(\mu) = [f_0, 1]. \end{cases}$$

Observe that solutions of problem (P) are stationary solutions associated to the parabolic problem

$$PP^*(\epsilon,\beta,\widetilde{u}_0)\begin{cases} u_t-(|u_x|^{p-2}u_x)_x+\epsilon g(u)\in\lambda\beta(u) & x\in(0,1), t>0,\\ u_x(0,t)=0, \quad u(1,t)=u_0 & t>0,\\ u(x,0)=\widetilde{u}_0(x), & x\in(0,1).\end{cases}$$

which mathematical treatment was is an special case of the study made in Díaz-Tello ([16]). So, the second question treated in this paper will be the different stability of non-negative solutions of problem (P) as stationaty solutions of $PP^*(\epsilon,\beta,\widetilde{u}_0)$.

[16] J. I. Diaz and L. Tello. On a nonlinear parabolic problem on a Riemannian manifold without boundary arising in Climatology. *Collectanea Mathematica,* Volum L, Fascicle 1 (1999), 19-51.

As mentioned befor, our interest to consider problem $PP^*(\epsilon,\beta,u_0)$ is motivated by its application in climatology. In term of Energy Balance Models, problem $PP^*(\epsilon,\beta,u_0)$ is a simplified version of Budyko's model. Let us recall that in 1969, M.I. Budyko [5] and W.D. Sellers [29] introduced an Energy Balance Model for the averaged atmospheric temperature. Essentially, they arrived to a balance of the type

$$\text{heat variations} = R_a - R_e + D,$$

where D stands the heat diffusion, R_a the absorbed solar and R_e the outgoing emitted terrestrial radiation energy. More in general, the model must be formulated on a Riemanian manifold without boundary representing the Earth atmosphere. To simplify the formulation, we consider the 1-D model corresponding to the case in which the surface temperature is assumed to depend only on the latitude component. Hence the distribution of temperature $u(x,t)$ in position x at time t verifies the quasilinear balance equation

$$u_t-(|u_x|^{p-2}u_x)_x = R_a - R_e,$$

where

$$R_a = QS(x,t)\beta(x,u) \quad \text{and} \qquad R_e = \epsilon g(u).$$

Here S represent the insolation function, Q the solar constant and β is the discontinuous function with respect to u representing the absorptivity or the co-albedo which is lower over-ice covered regions than over ice-free regions. In the Sellers model, β is assumed to be more regular function (at least Lipschitz continuous: [29]). Following many papers (see for instance [10],[16], [13], [27], [30]) the co-albedo function depends strongly on the temperature in a neighborhood of a critical temperature $-10°C$ which is responsible of the modification of the white color. We can take $\beta(u)$ as a step function

$$\beta(u) = \begin{cases} a_w & u > -10°C, \\ a_i & u < -10°C, \end{cases}$$

where $a_i, a_w \in (0,1)$. So, we can write

$$\beta(u) = a_i + (a_w - a_i)H(u+10),$$

where H is the Heaviside function.

We can avoid the complication of insolation, the emitted terrestrial term and simplify the spatial domain to a semisphere $(0,1)$ where the condition $u_x(0,t) = 0$ is the assumption of symmetry in the equator and the condition $u(1,t) = 0$ represents the renormalized temperature at the North pole (i.e. we are assuming that $u = T + T_N$ where $T_N < -10°C$ represents the North pole temperature and thus $\mu = -10 - T_N > 0$). Hence, if we denote the solar constant by λ, then we have our equation in problem $PP^*(\epsilon, \beta, u_0)$.

Notice that, since the Stefan-Boltzmann law is formulated in terms of Kelvin degrees then the North pole temperature u_0 will be, obviously $u_0 > 0$. Nevertheless, it can be easely shown that the simple change of variables $U := u - u_0$ leads to a problem which can completely studied with the same techniques of this paper. So, for simplicity (independetly of the physical meaning) we will asume in the rst of the paper

$$u_0 = 0 \tag{1.3}$$

Notice also that the discontinuity of the co-albedo function generates a *free boundary* separating the *ice-free region* $\{x \in (0,1), u(x,t) > \mu\}$ from *the ice-covered region* $\{x \in (0,1), u(x,t) < \mu\}$. Moreover, in order to study $PP^*(\epsilon, \beta, \widetilde{u}_0)$ under an uniqueness of solutions framework, a special class of solutions of $PP^*(\epsilon, \beta, \widetilde{u}_0)$ must be used: namely the so called *non degenerate solutions*. I.e., solutions $u(t, x : \widetilde{u}_0)$ such that

$$meas\{x \in (0,1), \mid u(t, x : \widetilde{u}_0) - \mu \mid \leq \theta\} \leq C\theta,$$

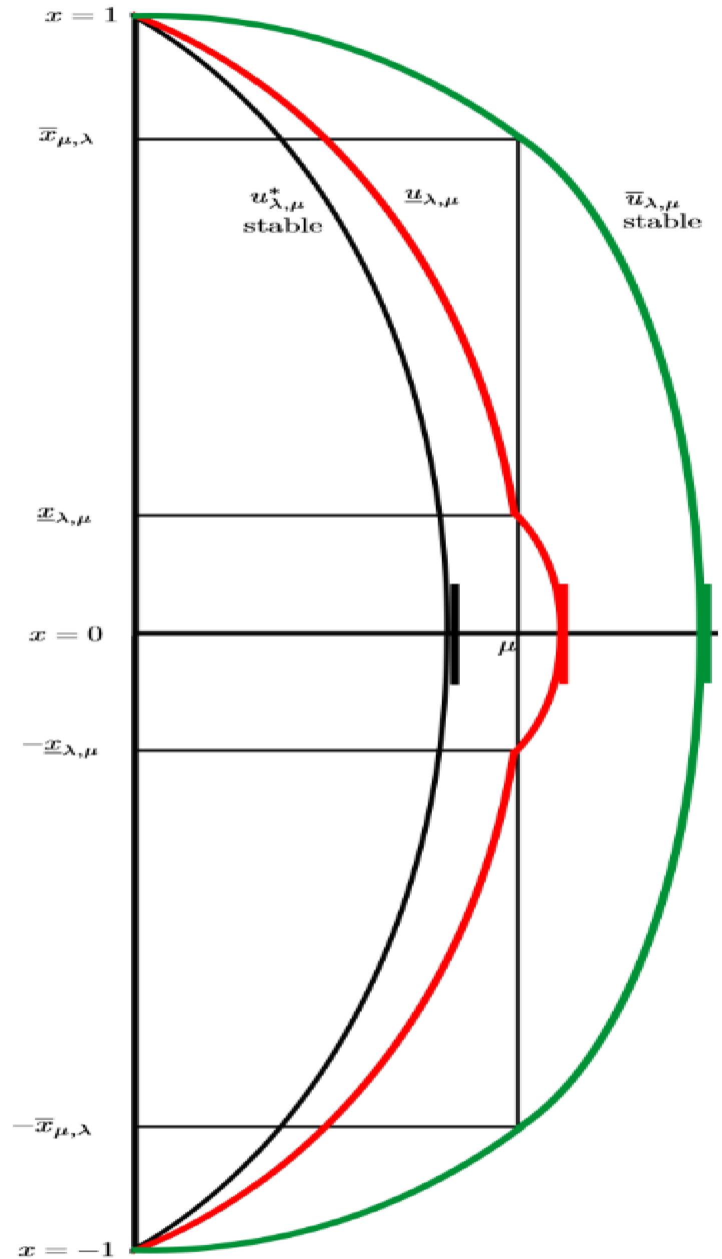
for any $\theta \in (0, \theta_0)$ and for any $t > 0$, for some $C > 0$ and $\theta_0 > 0$. Here $meas(.)$ denotes the Lebesgue measure. We refer the reader to [16], or [8] for an exposition of this type of results.
In a similar way to solutions of linear equations with an absorption term $-u'' + \omega^2 u = M$, the solutions of the reaction-diffusion problem (P) may oscillate as functions of x. As a matter of fact, there are several results in the literature indicating that the semilinear Budyko diffusive energy balance climate model admits an infinity of stationary solutions. That was shown in the [28] and [23] by considering a non-autonomous term $\lambda f(x, u)$ and in [17] for the mere autonomous case. The main goal of this paper concerns the study of non-oscillating solutions of the autonomous framework (which in the title is referred as "monotone solutions").

In this work, our main goal is to get a bifurcation diagram in terms of the emissivity parameter ϵ and have some informations to treat a very important question of stability of our solutions. We mention that due to the multivalued term and the quasilinear diffusion operator the very sharp bifurcation and stability results obtained through the famous Crandall-Rabinowitz theorem [6] can not be applied in our framework (since such a theorem requires in a fundamental way the differentiability of the nonlinear terms and , in addition, our diffusion is a degenerate operator which does not admits a good linearization operator).

[6] M.G.Crandall, P.H. Rabinowitz, Bifurcation from simple eigenvalues, J. Funct. Anal, 8, (1971), 321-340.

In [3], the authors derived a bifurcation diagram in term of solar constant λ and studied the case $p = 2$ using a strategy based on explicit form of solutions. We show the exact number of monotone solutions and prove that if $\lambda \in (\lambda_1, \lambda_2)$, for suitable $\lambda_1 < \lambda_2$, then there are exactly two stationary solutions giving rise to a free boundary (i.e. generating two symmetric polar ice caps: North and South ones) where the solution with smaller polar ice caps is stable and the one with bigger ice caps is unstable. Here the goal is to extend the results of [3] to a one-dimensional p-Laplacian for $p > 2$ and in terms of the emmisivity ϵ. We recall that usually $\epsilon \leq 1$ (and that $\epsilon = 1$ corresponds to the perfect black body). We shall see that low values of the emmisivity ϵ leads to stable climate with a small polar ice cap and thus the adverse framework is (in some sense: at least in this hyper-simplified toy model) irreversible.

[3] S. Bensid and J.I. Diaz, On the exact number of monotone solutions of a simplified Budyko climate model and their different stability, Discrete and Continuous Dynamical Systems, Series B, 24, N 03, (2019), 1033-1047.


$x = 1$
$\overline{x}_{\mu,\lambda}$
$u^*_{\lambda,\mu}$
stable
$\underline{u}_{\lambda,\mu}$
$\overline{u}_{\lambda,\mu}$
stable
$\underline{x}_{\lambda,\mu}$
$x = 0$
μ
$-\underline{x}_{\lambda,\mu}$
$-\overline{x}_{\mu,\lambda}$
$x = -1$

In order to state the main results of our paper, we need some preparations. So, using the following change of variables

$$u(x) := \left(\frac{\lambda}{\epsilon}\right)^{\frac{1}{4}} v(Lx), \quad \text{where} \quad L = \epsilon\lambda^{\frac{2-p}{p-1}},$$

and assuming $u_0 = 0$, we obtain the following stationary problem

$$\begin{cases} -(\frac{\lambda L}{\epsilon})^{p-1}(|v'|^{p-2}v')' + \lambda v^4 = \lambda f(v) & \text{in} \ \ (0, L), \\ v'(0) = 0, \quad v(L) = 0. \end{cases} \tag{1.4}$$

Hence, the function v verifies

$$\begin{cases} -(|v'|^{p-2}v')' = g(v) & \text{in} \ \ (0, L), \\ v'(0) = 0, \quad v(L) = v_0. \end{cases} \tag{1.5}$$

where

$$g(v) := f(v) - v^4.$$

Multiplying equation (1.5) by v' and integrating, we have

$$\int_{v(x)}^{v(0)} ((v')^{p-1})'v' = \int_{v(x)}^{v(0)} g(v)v'.$$

So, we obtain

$$\frac{(p-1)}{p}(v')^p = G(v(0)) - G(v(x)),$$

where

$$G(v) = \int_0^v g(s)ds.$$

We define $v(0) = \xi$, we have

$$v'(x) = \left(\frac{p}{p-1}\right)^{1/p} (G(\xi) - G(v))^{1/p}$$

which implies that

$$\int_{v(x)}^{\xi} \frac{dr}{\left(\frac{p}{p-1}\right)^{1/p} (G(\xi) - G(r))^{1/p}} = x.$$

Notice that the above integral is improper with a integrable singularity for $r = \xi$ (since we are assuming $p > 2$).

In the paper [12], a similar change of variable is used to study the existence and multiplicity of solutions having a *dead core* of a one-dimensional eigenvalue type equation associated to a quasilinear operator with an strong absorption (this is radically different to our case). Moreover, another difference of problem (P) compared with the formulation considered in [12] is the presence of the discontinuous nonlinearity $H(u - \mu)$.
Define

$$\gamma(\xi) = \int_{0}^{\xi} \frac{dr}{\left(\frac{p}{p-1}\right)^{1/p} (G(\xi) - G(r))^{1/p}}. \tag{1.6}$$

[12] J. I. Díaz, J. Hernandez. Global bifurcation and continua of nonegative solutions for a quasilinear elliptic problem. Comptes Rendus Acad. Sci. Paris, 329, Série I, 587-592, 1999

Define

$$\gamma(\xi) = \int_0^\xi \frac{dr}{\left(\frac{p}{p-1}\right)^{1/p} (G(\xi) - G(r))^{1/p}}. \tag{1.6}$$

Then, L and ξ are related by the bifurcation equation

$$\gamma(\xi) = L. \tag{1.7}$$

The nonlinear character of the equation in problem (P) leads to the solvability of the bifurcation equation (1.6) in order to prove the existence and multiplicity of positive solutions of problem (P).
Taking $\xi = \|v\|_\infty$, then the relation

$$\gamma(\|v\|_\infty) = \epsilon\lambda^{\frac{p-2}{p-1}},$$

implies that

$$\gamma\left(\left(\frac{\lambda}{\epsilon}\right)^{\frac{-1}{4}} \|u_{\epsilon,\mu}\|_\infty\right) = \epsilon\lambda^{\frac{p-2}{p-1}},$$

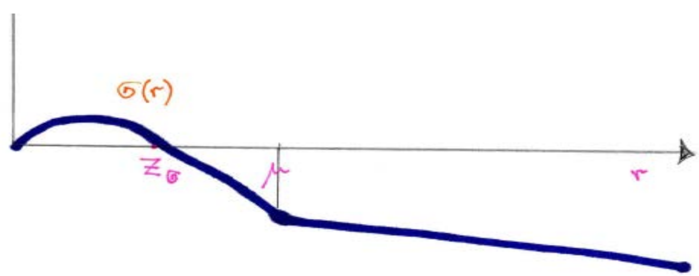
giving

$$\|u_{\epsilon,\mu}\|_\infty = \left(\frac{\lambda}{\epsilon}\right)^{\frac{1}{4}} \gamma^{-1}\left(\epsilon\lambda^{\frac{p-2}{p-1}}\right).$$

Let us recall that

$$G(r) = \int_0^r (f_0 + (1 - f_0)H(s - \mu) - s^4)ds$$

It is easy to see that G has a unique zero denoted by $z_G \neq 0$ and that $G(r) < 0$ if and only if $r > z_G$.

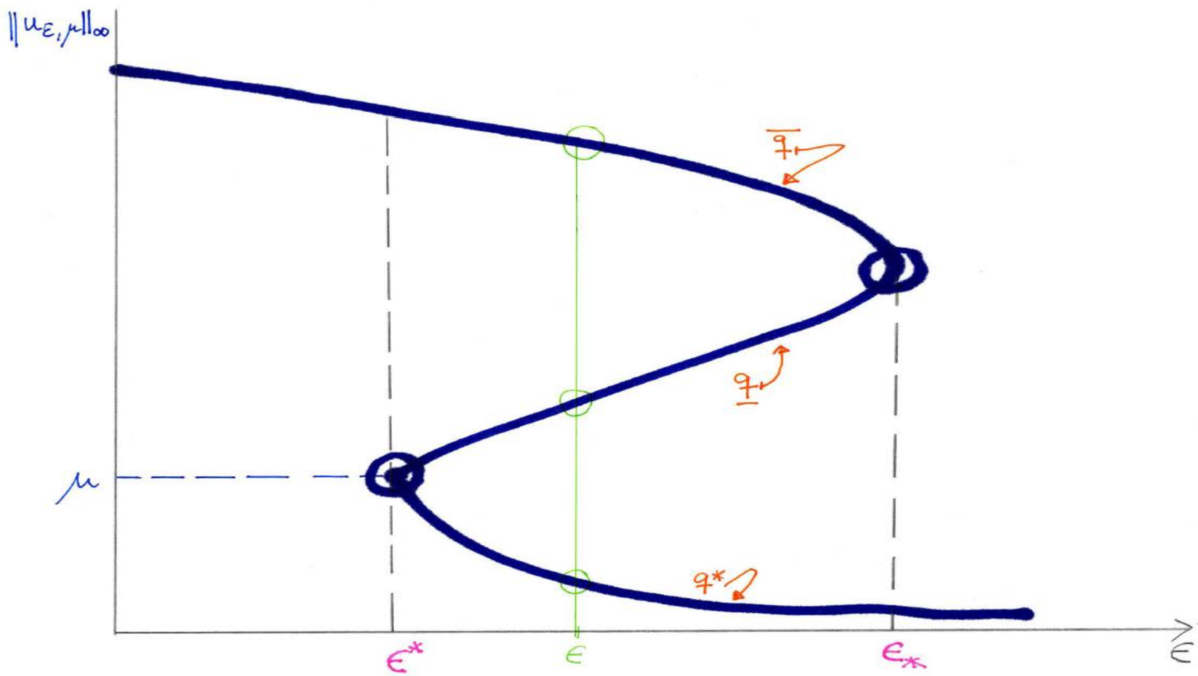


Our first result concerns the existence and multiplicity of solutions $u_{\epsilon,\mu}$ of problem (P). We have

Theorem 1.1 *Let*

$$\epsilon^* := \lambda^{-(\frac{p-2}{p-1})}(\frac{p-1}{p})^{1/p}\left[\int_0^{z_G}\frac{dr}{(-G(r))^{1/p}}\right].$$

Then there exists a $\epsilon_* \in (\epsilon^*, +\infty)$ *such that:*
1) If $\epsilon \in (\epsilon^*, +\infty)$, *there is a unique solution* $u^*_{\epsilon,\mu}$ *without free boundary of* (P). *Moreover, the curve* $q^*(\epsilon) := u^*_{\epsilon,\mu}(0)$ *is the decreasing part of the* $\epsilon-$*bifurcation diagram.*
2) If $\epsilon = \epsilon^*$, *there exists a unique solution* $u_{\epsilon^*,\mu}$ *giving rise to free boundary.*
3) If $\epsilon \in (\epsilon^*, \epsilon_*]$, *there exists a solutions* $\underline{u}_{\epsilon,\mu}$ *of* (P) *with a free boundary* $\underline{x}_{\epsilon,\mu}$ *such that* $\underline{q}(\epsilon)$ *is a increasing function of* ϵ.
4) If $\epsilon \in (0, \epsilon_*)$, *there exists a solution* $\overline{u}_{\epsilon,\mu}$ *of* (P) *with a free boundary* $\overline{x}_{\epsilon,\mu}$ *such that* $\mu < \|\overline{u}_{\epsilon,\mu}\|_\infty$ *and* $\|\underline{u}_{\epsilon,\mu}\|_\infty < \|\overline{u}_{\epsilon,\mu}\|_\infty$ *if* $\epsilon \in (\epsilon^*, \epsilon_*]$.



Now, to establish the stability results, we remark that the unique solvability of problem $PP^*(\epsilon, \beta, \tilde{u}_0)$ was proved in [16] once we assume

$$\tilde{u}_0 \text{ is non-degenerate near its free boundary.} \tag{1.8}$$

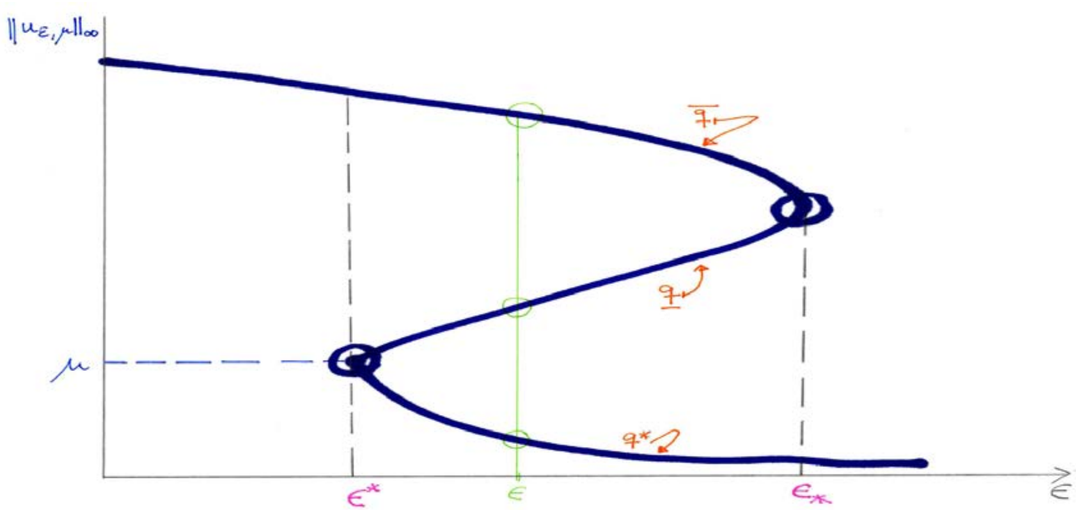
Hence, by denoting $u(t, x : \tilde{u}_0)$ the solution of problem $PP^*(\epsilon, \beta, \tilde{u}_0)$ and for $\epsilon \in (0, \epsilon_*)$, we can give the following theorem

Theorem 1.2 *Let $\tilde{u}_0 \in L^\infty(0,1)$ with $\tilde{u}_0 \geq 0$ a.e in $(0,1)$ satisfying (1.8). Let $\overline{u}_{\epsilon,\mu}$ be the monotone solution of (P) given in part 4) of the above Theorem.*

Assume that $\|\tilde{u}_0 - \overline{u}_{\epsilon,\mu}\|_{L^\infty}$ is sufficiently small and let $u(t, x : \tilde{u}_0)$ be the non-degenerate solution of $PP^*(\epsilon, \beta, \tilde{u}_0)$. Then

$$\|u(x, t : \tilde{u}_0) - \overline{u}_{\epsilon,\mu}(x)\|_{L^\infty(0,1)} < \varepsilon \qquad \text{for any } t \geq 0,$$

for some positive constant ε.



The result concerning the instability of decreasing part of bifurcation diagram is given in the following theorem

Theorem 1.3 *Let $\epsilon \in (\epsilon^*, \epsilon_*]$. For any $\varepsilon > 0$, there exists $\underline{\widetilde{u}}_0, \overline{\widetilde{u}}_0 \in L^\infty(0,1)$, non degenerate functions such that*

$$\underline{u}_{\epsilon,\mu} - \underline{\widetilde{u}}_0 \leq 0, \quad \widetilde{u}_{\epsilon,\mu} - \overline{u}_0 \geq 0,$$

and

$$||\underline{u}_{\epsilon,\mu} - \underline{\widetilde{u}}_0||_{L^\infty} < \varepsilon, \quad ||\underline{u}_{\epsilon,\mu} - \overline{\widetilde{u}}_0||_{L^\infty} < \varepsilon. \tag{1.9}$$

Moreover, if $u(t, x : \overline{\widetilde{u}}_0)$ (respectively $u(t, x : \underline{\widetilde{u}}_0)$) is the unique non degenerate solution of the corresponding problem $PP^*(\epsilon, \beta, \widetilde{u}_0)$, then

$$\frac{\partial u}{\partial t}(t, . : \overline{\widetilde{u}}_0) \geq 0 \quad a.e \quad t > 0 \tag{1.10}$$

$$u(t, . : \overline{\widetilde{u}}_0) \leq \overline{u}_{\epsilon,\mu}(.) \quad \forall t \in [0, +\infty) \tag{1.11}$$

$$u(t, . : \overline{\widetilde{u}}_0) \longrightarrow \overline{u}_{\epsilon,\mu}(.) \quad in \quad H^1(0,1) \quad when \quad t \to +\infty, \tag{1.12}$$

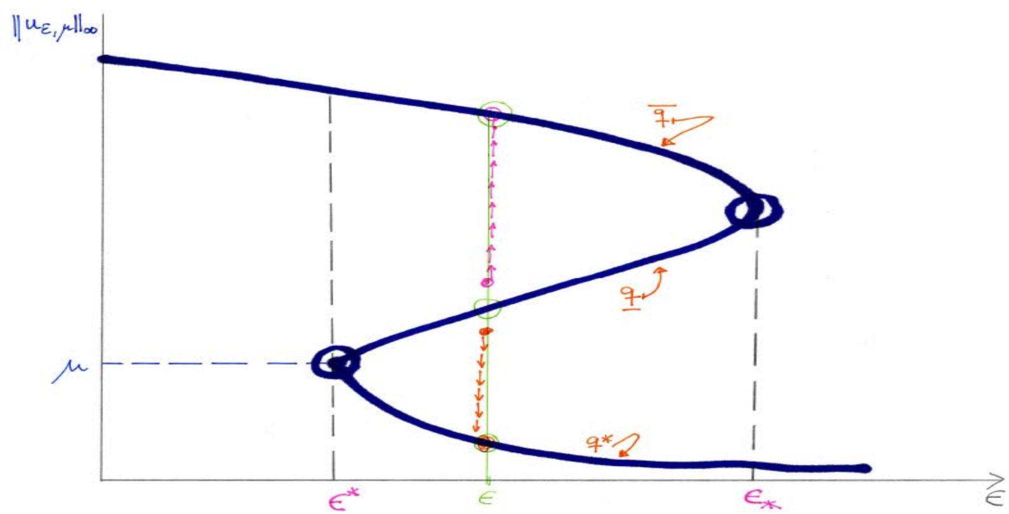
respectively

$$\frac{\partial u}{\partial t}(t, . : \underline{\widetilde{u}}_0) \leq 0 \quad a.e \quad t > 0 \tag{1.13}$$

$$u(t, . : \underline{\widetilde{u}}_0) \geq u^*_{\epsilon,\mu}(.) \quad \forall t \in [0, +\infty) \tag{1.14}$$

$$u(t, . : \underline{\widetilde{u}}_0) \longrightarrow u^*_{\epsilon,\mu}(.) \quad in \quad H^1(0,1) \quad when \quad t \to +\infty. \tag{1.15}$$

In particular, the stationary solution $\underline{u}_{\epsilon,\mu}(.)$ is unstable in H^1 (and so unstable also in L^∞).



2 Proof of Theorem 1.1: The S-reflected-shaped bifurcation diagram.

First, we begin the proof by considering the case without free boundary i.e when $f(u(x)) = f_0$ for $x \in (0,1)$. Hence, we consider the following problem

$$(P_0)\begin{cases} -(|u'|^{p-2}u')' = \lambda f_0 - \epsilon u^4 & \text{in } \ (0,1), \\ u'(0) = 0, u(1) = u_0, \end{cases}$$

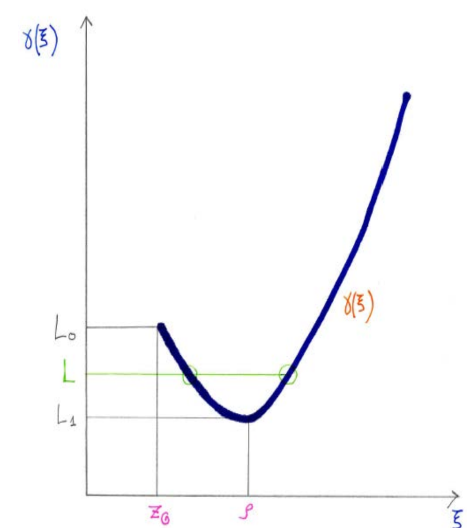
It is well known that the solution $u^*_{\epsilon,\mu}$ of (P_0) depend continuously on parameter ϵ, so, for a positive parameters $\overline{\epsilon}, \underline{\epsilon}$ such that $\overline{\epsilon} > \underline{\epsilon}$, we known by the maximum principle that

$$u_{\overline{\epsilon},\mu}(x) < u_{\underline{\epsilon},\mu}(x), \qquad \text{for} \quad x \in (0,1).$$

Then, we see easily that $q^*(\epsilon) := \|u^*_{\epsilon,\mu}\|_\infty$ is a continuous function and an decreasing curve. Remark that ϵ^* verifies $q^*(\epsilon^*) = \mu$ which is given in Theorem 1.1. This prove the statement 1) of Theorem 1.1

Now, we shall look the case with free boundary. For that, we weed the following proposition.

Proposition 2.1 *For $\rho \in (z_G, +\infty)$, let $L_0 = \gamma(z_G)$ and $L_1 = \gamma(\rho)$. Then,*
1) If $L < L_1$, then equation (1.6) has no roots.
2) If $L = L_1$, then equation (1.6) has one root.
3) If $L_1 < L \leq L_0$, then equation (1.6) has two roots.
4) If $L \in (L_0, +\infty)$ the equation (1.6) has one root.



Proof of Proposition 2.1

Let

$$\gamma(\xi) = \int_0^{\xi} \frac{dr}{\left(\frac{p}{p-1}\right)^{1/p} (G(\xi) - G(r))^{1/p}}.$$

It is a routine matter to show that the integral of γ has a point ξ as integrable singularity. Hence, for $\xi \in [z_G, +\infty)$, we study the behavior of function γ and we are interested by the case with free boundaries. (greater to μ.)

As in [12], we analyse the behavior of γ. Using the fact that $r = \tau\xi$, then we introduce the function

$$\Lambda(\xi) = \left(\frac{p}{p-1}\right)^{1/p} \gamma(\xi) = \xi \int_0^1 \frac{d\tau}{(F(\xi) - F(\tau\xi))^{1/p}},$$

where $F(t) = \int_0^t (s - \frac{s^5}{5}) ds$.

Hence,

$$\Lambda'(\xi) = \frac{\Lambda(\xi)}{\xi} - \frac{\xi}{p} \int_0^1 \frac{F'(\xi) - \tau F'(\tau\xi) d\tau}{(F(\xi) - F(\tau\xi))^{(p+1)/p}}$$

This integral is convergent and $\Lambda'(\xi) \in C(z_G, +\infty)$. If we set, $\theta(t) := pF(t) - tF'(t)$, then Λ' can be written as

$$\Lambda'(\xi) = \frac{1}{\xi p} \int_0^{\xi} \frac{(\theta(\xi) - \theta(r)) dr}{(F(\xi) - F(r))^{(p+1)/p}}$$

The same analysis used in [12], give that $\gamma'(\xi) = 0$ has a unique roots $\rho \in (z_G, +\infty)$.
Hence, if we denote by $L_1 := \gamma(z_G)$ and $L_0 := \gamma(\rho)$, then the function γ is decreasing in $[z_G, \rho)$ and increasing in $(\rho, +\infty)$. So, the shape of function γ give the conclusion of Proposition 2.$\square$.

To conclude the proof of Theorem 1.1, it suffice to use the previous change of variables.

$$\gamma\left(\left(\frac{\lambda}{\epsilon}\right)^{\frac{-1}{4}}\|u_{\epsilon,\mu}\|_\infty\right) = \epsilon\lambda^{\frac{2-p}{1-p}}. \tag{2.16}$$

Since $\gamma(\rho)$ is the minimum of function γ, then we deduce the existence of ϵ_* such that if $\epsilon < \epsilon_*$ the equation (2.14) has no solution (with free boundary) and for $\epsilon = \epsilon_*$, there is only one solution.
In other part, if $L_1 < \gamma < L_0$, then proposition 2 implies the existence of two roots corresponding to the upper and lower branch of solution $u_{\epsilon,\mu}$. Hence, by the continuity of solutions with respect to ϵ, we derive our bifurcation diagram. □

3 Proof of Theorem 1.2: Stability of decreasing part of bifurcation curve

The proof of Theorem 1.2 is similar to proof of Theorem 1.2 of [3] based on the two following propositions.

Proposition 3.1 *[3] Let the assumptions of Theorem 1.2 hold. Then, for some positive parameters θ and δ, there exist two continuous functions $\overline{\psi}_{\theta,\delta}(x:\epsilon,\mu), \underline{\psi}_{\theta,\delta}(x:\epsilon,\mu)$, which depend continuously of the parameters θ and δ, and $\eta = \eta(\theta,\delta) > 0$, such that*

$$\overline{u}_{\epsilon,\mu}(x) - \eta \leq \underline{\psi}_{\theta,\delta}(x:\epsilon,\mu) < \overline{u}_{\epsilon,\mu}(x) < \overline{\psi}_{\theta,\delta}(x:\epsilon,\mu) \leq \overline{u}_{\epsilon,\mu}(x) + \eta, \text{ for any } x \in [0,1]. \tag{3.17}$$

In particular,

$$\overline{\psi}_{\theta,\delta}(x:\epsilon,\mu), \underline{\psi}_{\theta,\delta}(x:\epsilon,\mu) \to \overline{u}_{\epsilon,\mu}(x), \text{ for any } x \in [0,1], \text{ if } \theta \to 0 \text{ and } \delta \to 0. \tag{3.18}$$

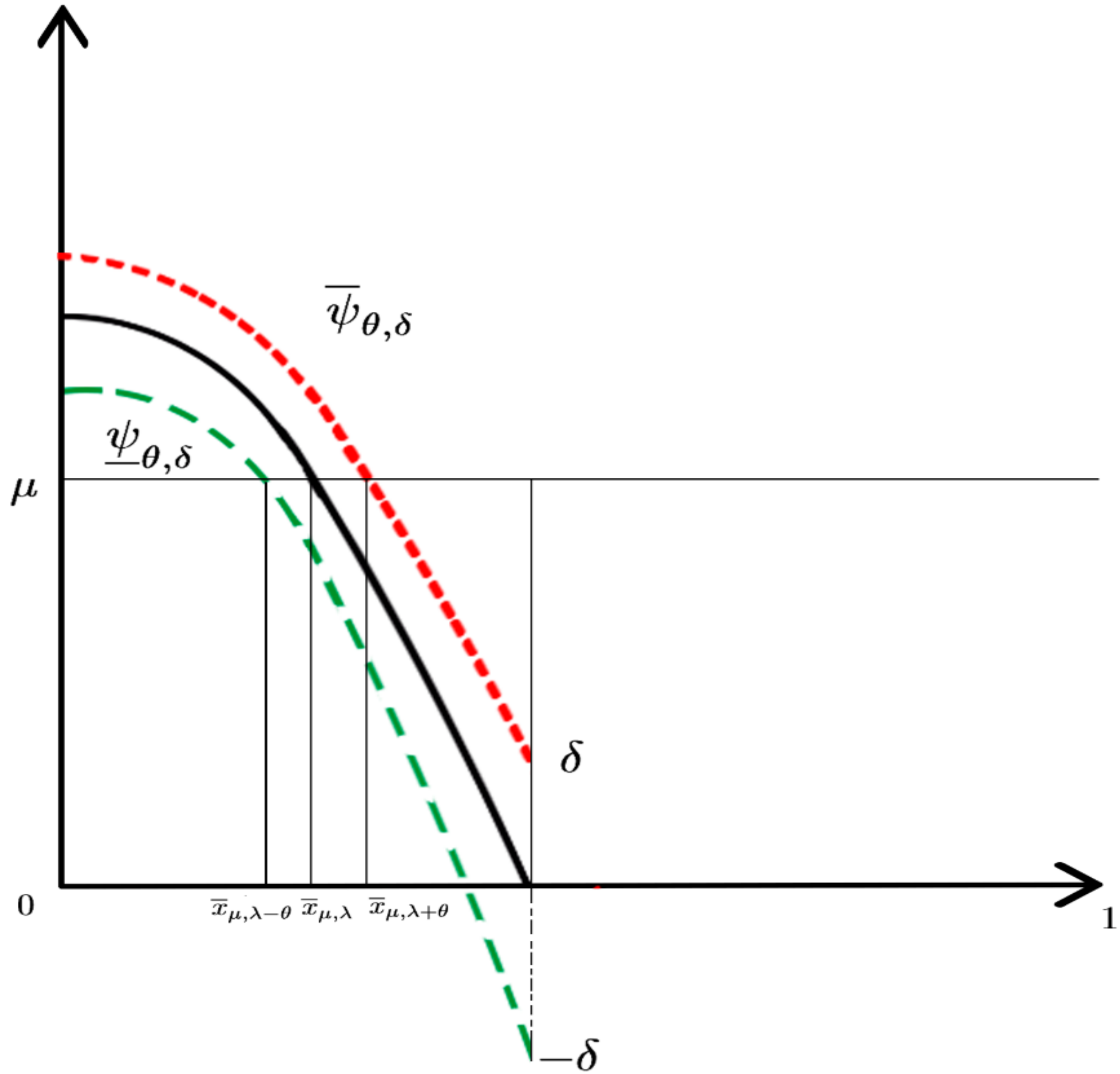
The main idea of the proof is to introduce the parameter $\theta > 0$, small enough, and consider $\overline{x}_{\lambda+\theta,\mu}$ the free boundary associated to the parameter $\lambda + \theta$. By continuity we know that there exists a small $\overline{h}(\theta) > 0$ such that $\overline{x}_{\lambda+\theta,\mu} = \overline{x}_{\lambda,\mu} + \overline{h}(\theta)$. Let us construct the upper barrier function $\overline{\psi}_{\theta,\delta}$ in the following way: $\overline{\psi}_{\theta,\delta}$ satisfies two different boundary value problems over different regions.

$$(\overline{P}_L) \begin{cases} -\left(\left|\overline{\psi}'_{\theta,\delta}\right|^{p-2} \overline{\psi}'_{\theta,\delta}\right)'(x) + g(\overline{\psi}_{\theta,\delta}) = \lambda + \theta & \text{in } (0, \overline{x}_{\lambda+\theta,\mu}), \\ \overline{\psi}'_{\theta,\delta}(0) = 0, \ \overline{\psi}_{\theta,\delta}(\overline{x}_{\lambda+\theta,\mu}) = \mu. \end{cases}$$

Moreover, for $\delta > 0$, small enough, we construct $\overline{\psi}_{\theta,\delta}$ on $(\overline{x}_{\lambda+\theta,\mu}, 1)$ such that

$$(\overline{P}_R) \begin{cases} -\left(\left|\overline{\psi}'_{\theta,\delta}\right|^{p-2} \overline{\psi}'_{\theta,\delta}\right)'(x) + g(\overline{\psi}_{\theta,\delta}) = (\lambda + \theta) f_0 & \text{in } (\overline{x}_{\lambda+\theta,\mu}, 1), \\ \overline{\psi}_{\theta,\delta}(\overline{x}_{\lambda+\theta,\mu}) = \mu, \ \overline{\psi}_{\theta,\delta}(1) = \delta. \end{cases}$$

In a similar way it is defined the function $\underline{\psi}_{\theta,\delta}(x : \epsilon, \mu)$ and thus we use the strong maximum principle (see [3]).


$\overline{\psi}_{\theta,\delta}$
$\underline{\psi}_{\theta,\delta}$
μ
δ
0
$\overline{x}_{\mu,\lambda-\theta}$ $\overline{x}_{\mu,\lambda}$ $\overline{x}_{\mu,\lambda+\theta}$
1
$-\delta$

Then, by defining the set $E_{\theta,\delta} := \{v \in L^\infty(0,1),\ \underline{\psi}_{\theta,\delta} < v < \overline{\psi}_{\theta,\delta}\}$, we prove that it is time-invariant:

Proposition 3.2 *For some positive parameters θ and δ, there exist $\eta = \eta(\theta,\delta) > 0$, such that if $\|u_0 - \overline{u}_{\lambda,\mu}\|_{L^\infty} < \eta$, then $u(t,.:u_0) \in E_{\theta,\delta}$ for any $t > 0$.*

To conclude that $\overline{u}_{\epsilon,\mu}$ is stable in $L^\infty(0,1)$, it suffices to combine Propositions 3.1 and 3.2 and to use that the invariant set $E_{\theta,\delta}$ containing the $L^\infty(0,1)-$neighborhood of $\overline{u}_{\epsilon,\mu}$ of radium $\eta = \eta(\theta,\delta)$. We invite the reader to the papers [3],[4] for more details.

4 Proof of Theorem 1.3: Instability of the increasing part of the bifurcation curve

A set of arguments quite similar to the one sused in [3] can be applied also here. We shall use a very special construction of auxiliary initial data which leads, after a suitable convergence, to solution $\underline{u}_{\epsilon,\mu}$. Let $\theta > 0$ small enough and take

$$\overline{\widetilde{u}}_0(x) := \underline{u}_{\epsilon+\theta,\mu}(x).$$

From the the proof of part 3) of Theorem 1.1, we have that $\theta > 0$ implies

$$\overline{\widetilde{u}}_0(x) > \underline{u}_{\epsilon,\mu}(x), \quad \forall x \in [0,1).$$

Moreover, to check condition (1.9) observe that it is clear that $\left\|\underline{u}_{\epsilon+\theta,\mu}\right\|_{L^\infty(0,1)}$ depends continuously with respect to θ. Indeed, this is exactly the continuity condition of the bifurcation curve given in Theorem 1.1. Notice, for instance, that the maximum point of $\underline{u}_{\epsilon+\theta,\mu}$ takes place at $x = 0$ and that, although at this point we only have a Neumann boundary condition, the fact that $\underline{u}_{\lambda+\theta,\mu}$ verifies

$$
\begin{cases} -(|\underline{u}'(x)|^{p-2}\underline{u}'(x))' + (\epsilon-\theta)\underline{u}^4(x) = \lambda, & \text{for } x \in (0, \underline{x}_{\epsilon+\theta,\mu}) \\ \underline{u}'_{\epsilon-\theta,\mu}(0) = 0, \ ,\underline{u}_{\epsilon+\theta,\mu}(\underline{x}_{\epsilon+\theta,\mu}) = \mu, \end{cases} \tag{4.19}
$$

implies the above mentioned continuous dependence as a by-product of the continuous dependence of solutions of problem (4.19) with respect to the $L^\infty(0,1)$ norm of the right hand function and the continuous dependence with respect to the own interval of definition (recall that we know that the continuity of the function γ implies that there exists a continuous function $\overline{h}(\theta) > 0$ such that $\underline{x}_{\epsilon+\theta,\mu} = \underline{x}_{\epsilon,\mu} + \overline{h}(\theta)$). Thus we have that, given $\theta > 0$ small enough, there exists $\varepsilon = \varepsilon(\theta) > 0$ such that

$$
0 < \overline{u}_0(x) - \underline{u}_{\epsilon,\mu}(x) \leq \varepsilon \quad \forall x \in [0,1), \tag{4.20}
$$

which shows (1.9).

We also need the following auxiliary result

Lemma 4.1 *If $u_s \in W^{2,\infty}(0,1)$ is non-degenerate and*

$$
(P_{u_s}) \begin{cases} -(|u_s'|^{p-2}u_s')' + \epsilon u_s^4 \geq \lambda\beta(u_s) & a.e \quad x \in (0,1), \\ u_s(0) = 0, \ u_s'(0) \geq 0, \end{cases}
$$

then the unique non-degenerate solution of $PP^*(\epsilon,\beta,u_s)$ satisfies that

$$
\frac{\partial u}{\partial t}(t,x) \leq 0 \quad a.e \quad x \in (0,1) \quad and\ a.e \quad t > 0. \tag{4.21}
$$

Now, to conclude the proof of Theorem 1.4, we remark that (1.7) is already shown in Lemma 4.1. To see (1.8), it is enough to check that $\overline{u}_{\epsilon,\mu}$ verifies $PP^*(\epsilon, \beta, \overline{u}_\varepsilon)$ and that

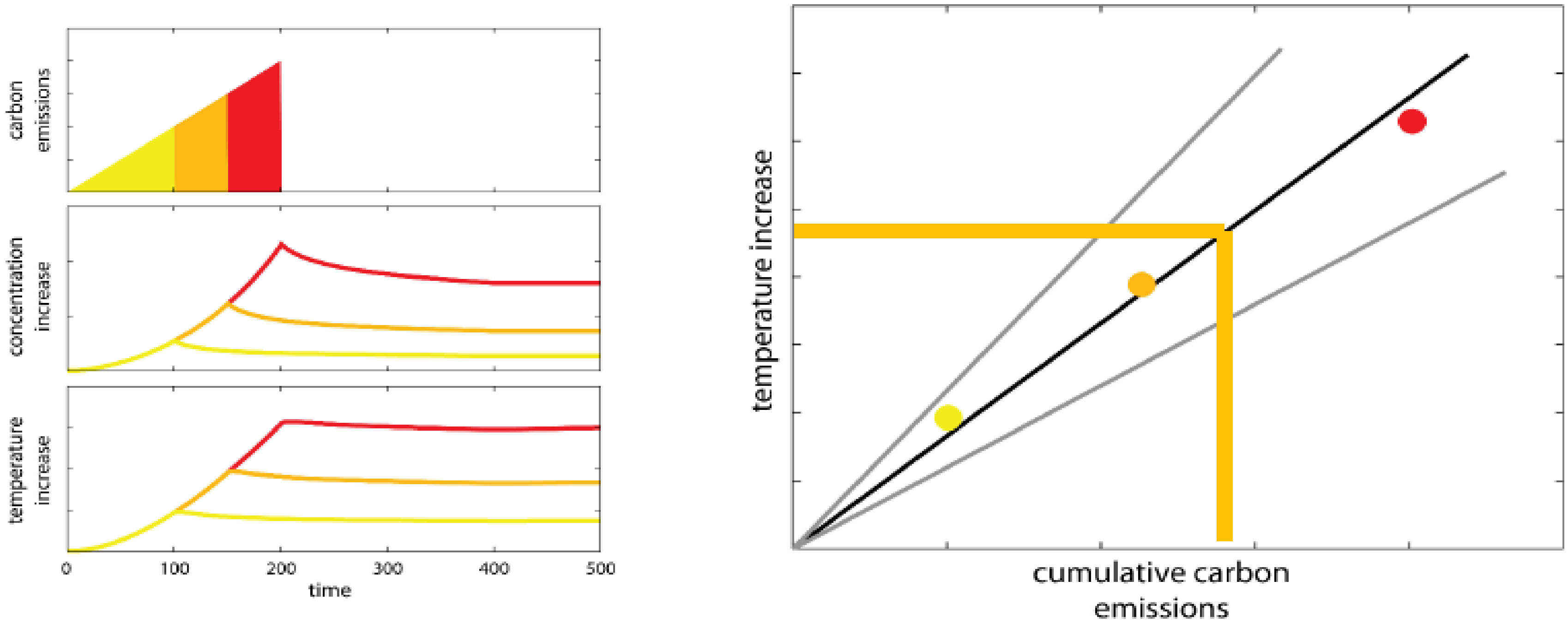
$$\underline{u}_{\epsilon+\theta,\mu}(x) = \overline{\widetilde{u}}_0(x) \leq \overline{u}_{\epsilon,\mu}(x) \text{ on } (0,1).$$

Thus, since $u(t : u_0)$ and $u_{\epsilon,\mu}(.)$ are non degenerate, using the comparison principle for non-degenerate solutions of $PP^*(\epsilon, \beta, v_0)$, we have that

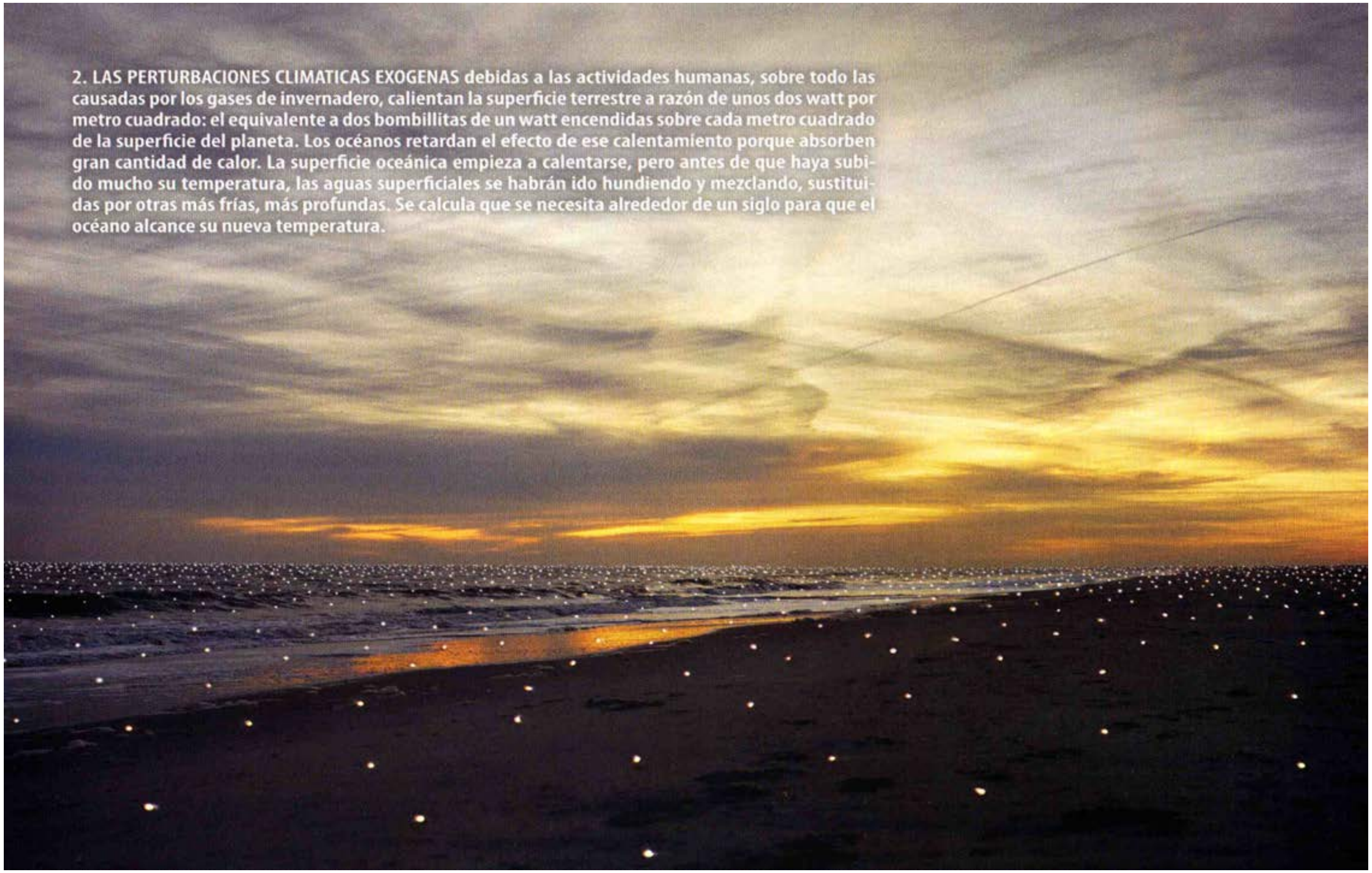
$$u(t : u_0) \leq \overline{u}_{\epsilon,\mu}, \qquad \forall t > 0 \text{ on } (0,1).$$

A similar conclusion can be obtained for the solution $u(t : \underline{u}_0)$ once we choose $\underline{u}_0(x) := \underline{u}_{\epsilon-\theta,\mu}(x)$.

Cumulative carbon determines warming



- Peak warming is approximately proportional to cumulative (total) emissions.
- Transient climate response to cumulative carbon emissions TCRE = Warming per 1000 PgC



Gracias por vuestra atención

Scientific findings regarding the causes of climate change and approaches to the related risks are fundamental to developing political counterstrategies, the statement says. The contribution of science to this global challenge has to be better integrated in politics and society. „We as universities have a special responsibility,“ stresses University President Professor Günter M. Ziegler. „Our task is not only to improve scientific knowledge and communicate this to society, but to also act in an exemplary way within our area of responsibility.“