On the Haïm Brezis pioneering contributions on the location of free boundaries

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Gaeta (Italy), May, 30th, 2004

1. Introduction

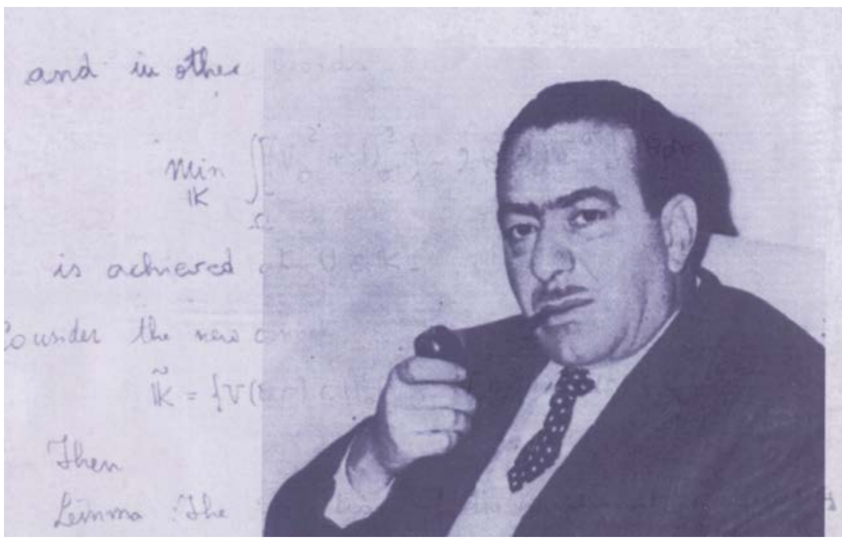
- Starting in the seventies, and simultaneously to his beautiful results on the existence and regularity of solutions of many nonlinear partial PDEs, Haïm Brezis produced a series of papers in which, in a pioneering way, he rigorously found new qualitative phenomena as, for instance, the compactness of the support of the solution of suitable problems posed on unbounded domains and more generally on the location of free boundaries (sometimes unexpected from the original formulation).

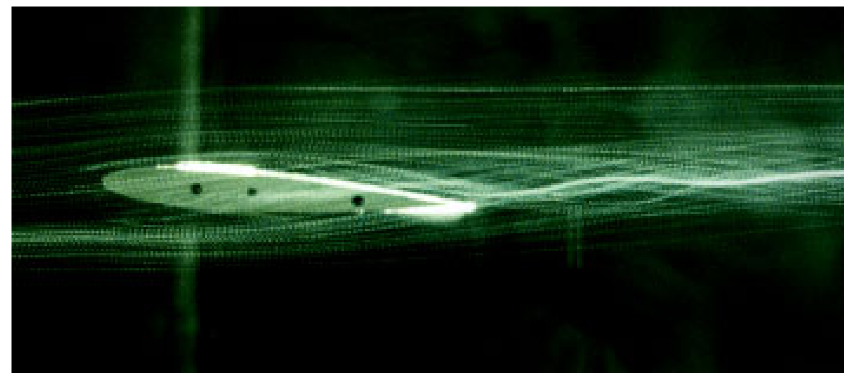
- In this lecture, we shall recall some of his results indicating their great impact in the literature which remains in continuous expansion thirty years later.

- Plan
  - The support of the solution of a Variational Inequality in Fluid Mechanics
  - The support of the solution of semilinear elliptic equations
  - Compact support properties and the abstract theory of montone operators

2. The support of the solution of a Variational Inequality in Fluid Mechanics

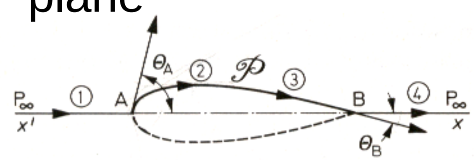
H. Brezis and G. Stampacchia,
Une nouvelle méthode pour l'étude d'écoulements stationnaires, *C.R. Acad. Sci.*, **276**, 1973, 129-132.
(Séance du 18 décembre 1972)





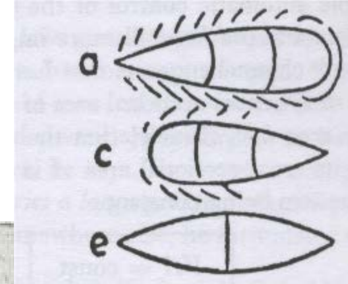
steady irrotational subsonic flow for a non viscous fluid,

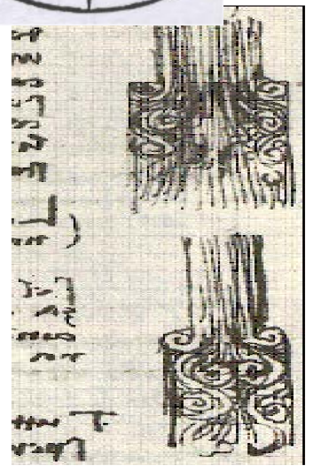
symmetric convex profile in the plane

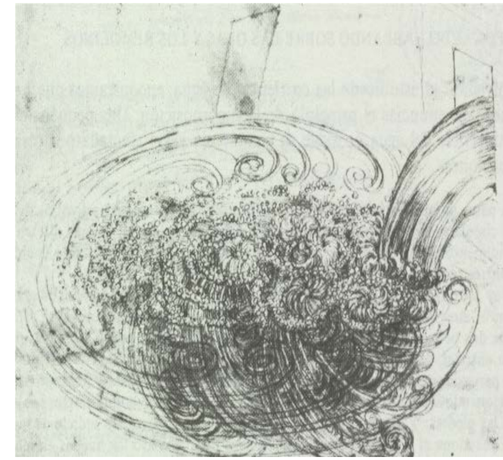


Previous works

Leonardo da Vinci (1452-1519)







P. Molenbroeck (1890), S.A. Chaplying (1902), J. Leray (1935), H. Bateman (1938), T. von Karman (1941), R. Courant and K.O. Friedrich (1948), L. Crocco (1951), L. Bers (1954), P. Germain (1954), M.J. Lighthill (1955), R. Finn and D. Gilbarg (1957), R. Finn and J. Serrin (1958),…

L. Bers, *Mathematical aspects of subsonic and transient gas dynamics*, Chapman and Hall, London, 1958.

J. Serrin: Mathematical Principles of Classical Fluid Mechanics, in *Handbuch der Physik*, **8,** Springer-Verlag,Berlin 1959, 125-263.

C.Ferrari and F. Tricomi, *Aerodinamica transonica*, Cremonese, Rome,1962.

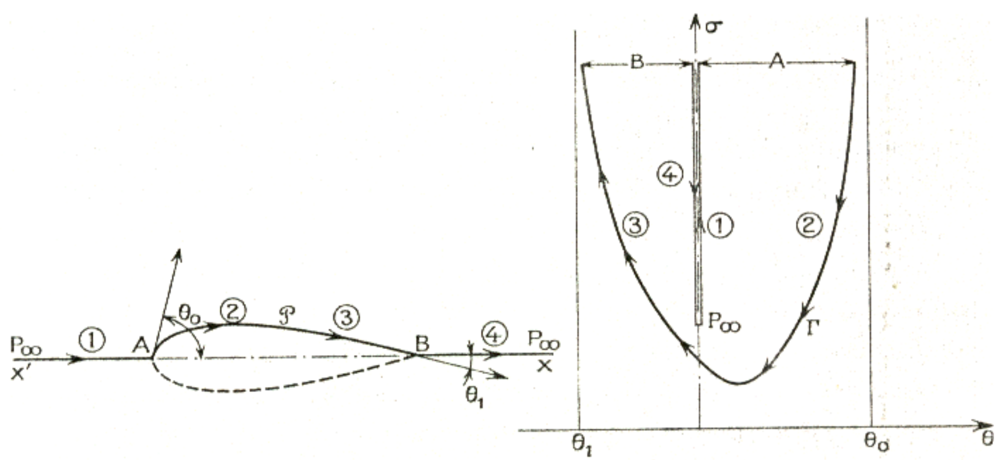
$\mathbf{q} = (u, v)$

$$div(\rho \mathbf{q}) = 0, \ rot(\mathbf{q}) = \mathbf{0}$$

$$\mathbf{q} \to \mathbf{q}_\infty \text{ as } |(x,y)| \to +\infty,$$

$\rho = h(q)$, h decreasing $q = |\mathbf{q}|$, ψ stream fuction $\psi_x = -\rho v \quad \psi_y = \rho u$

Hodograph trasformation $\mathcal{T} : (x, y) \to (u, v) \to (\theta, q)$ $tg\ \theta = \dfrac{v}{u}$



$$\sigma = \int_q^{q_c} \frac{h(\tau)}{\tau} d\tau \qquad a(q_c) = q_c$$

$$a^2(q) = -q\frac{h(q)}{h'(q)} \quad \text{local sound velocity}$$

$\partial \mathcal{P}_+$ transformed into the "free boundary" Γ

subsonic $\iff \Gamma \subset \{(\theta, \sigma) : \theta_1 < \theta < \theta_0, \ \sigma > 0\}$

$\psi = 0$ on Γ $\psi(0, \sigma) = 0$ for $\sigma \geq \sigma_\infty$ $\sigma_\infty = \displaystyle\int_{q_\infty}^{q_c} \frac{h(\tau)}{\tau} d\tau$

$\psi(\theta, \sigma) \to 0$ as $\theta \to +\infty$

$$\psi_{\sigma\sigma} + k\psi_{\theta\theta} = 0 \qquad \sigma > l(\theta),\, \theta_1 < \theta < \theta_0$$

$$k(q) = \frac{1}{h^2(q)}\left(1 - \frac{q^2}{a^2(q)}\right) = k(\sigma)$$

$$\psi(0,\sigma) = 0 \text{ for } \sigma \geq \sigma_\infty$$

$$\psi = 0 \text{ on } \Gamma \qquad \psi(\theta,\sigma) \to 0 \text{ as } \theta \to +\infty$$

New modelling

$$u(\theta,\sigma) = \int_{l(\theta)}^{\sigma} \frac{k(\tau)}{q(\tau)}\psi(\theta,\tau)d\tau \qquad \sigma > l(\theta),\, \theta_1 < \theta < \theta_0$$

$$\mathcal{D} = \{(\theta,\sigma) : \theta_1 < \theta < \theta_0,\ \sigma > l(\theta)\} \setminus \{(0,\sigma) : \sigma \geq \sigma_\infty\}$$

$$\begin{cases} \frac{1}{q^2}\left(\frac{q^2}{k}u_\sigma\right)_\sigma + u_{\theta\theta} + u = -R & \text{in } \mathcal{D}, \\ u = 0 & \text{on } \Gamma, \\ \nabla u = \mathbf{0} & \text{on } \Gamma, \\ u(0,\sigma) = \text{Constant} = H_{\mathcal{P}} & \sigma \geq \sigma_\infty \end{cases}$$

Global formulation

$$\Omega = \{(\theta,\sigma) : \theta_1 < \theta < \theta_0,\ \sigma > 0\}$$

$$V = \{v : qv \in L^2(\Omega),\ qv_\theta \in L^2(\Omega),\ \frac{q}{\sqrt{k}} v_\sigma \in L^2(\Omega) \text{ and } v = 0 \text{ on } \partial\Omega\}$$

$$K_H = \{v \in V : v \geq 0 \text{ on } \Omega \text{ and } u(0,\sigma) = H_P \text{ for } \sigma \geq \sigma_\infty\}$$

On introduit la forme bilinéaire

$$a(u, v) = \int_\Omega \left(\frac{1}{k} u_\sigma v_\sigma + u_\theta v_\theta - uv\right) q^2\, d\theta\, d\sigma$$

qui est coercive sur V à cause de l'hypothèse $\theta_0 - \theta_1 < \pi$.

Théorème. — *Si* $R \leq 0$ (*ce qui correspond à un profil* $\mathfrak{L}$ *convexe*), *on a* $u \in K_{H_p}$ *et*

(7) $$a(u, v - u) \geq \int R(v - u)\, q^2\, d\theta\, d\sigma \quad \textit{pour tout } v \in K_{H_p}.$$

Il est bien connu [*cf.* par exemple ([4])] que pour tout $H > 0$, il existe $u \in K_H$, unique solution de l'inéquation

$$a(u, v - u) \geq \int_\Omega R(v - u)\, q^2\, d\theta\, d\sigma;$$

on prouve (à l'aide du principe du maximum) que $u_\sigma \geq 0$.

Il reste enfin à déterminer la valeur de H, de manière à ce que la solution correspondante soit physiquement acceptable. A cet effet, on montre que $w(\theta) = \lim_{\sigma \to +\infty} u(\theta, \sigma)$ est la solution de l'inéquation variationnelle

$$w \in \mathcal{K}_H = \{ \zeta \in H_0^1(\theta_1, \theta_0), \zeta \geq 0 \text{ sur }]\theta_1, \theta_0[, \zeta(0) = H \}$$

et

$$\int_{\theta_1}^{\theta_0} w_\theta (\zeta_\theta - w_\theta) - w(\zeta - w)\, d\theta \geq \int_{\theta_1}^{\theta_0} R(\zeta - w)\, d\theta \quad \text{pour tout } \zeta \in \mathcal{K}_H.$$

Par ailleurs, il est nécessaire que $w > 0$ sur $]\theta_1, \theta_0[$ et que

$$w_\theta(\theta_0) = w_\theta(\theta_1) = 0.$$

La seule valeur admissible pour H est alors

$$H_p = -\int_0^{\theta_0} R(\tau) \sin \tau \, d\tau = -\int_0^{\theta_1} R(\tau) \sin \tau \, d\tau$$

qui représente la moitié de l'épaisseur maximum du profil $\mathcal{P}$.

Conclusion. — Étant donnés $\vec{q}_\infty$ et un profil $\mathcal{P}$ vérifiant les propriétés indiquées ci-dessus, on résoud l'inéquation variationnelle (7); on désigne par ω l'ensemble des points $[\theta, \sigma] \in \Omega$ où $u(\theta, \sigma) > 0$. Si $\overline{\omega} \cap \{ \sigma = 0 \} = \emptyset$ alors $\psi = (q/k)\, u_\sigma$ représente sur ω la fonction courant cherchée. On a ainsi réduit le problème envisagé à la résolution d'une inéquation variationnelle pour laquelle on connaît des méthodes numériques efficaces.

Ce travail a été effectué, en partie, pendant la visite du premier auteur à l'École Normale Supérieure de Pise; son séjour a été subventionné par l'Académie dei Lincei.

Sergei Vasilyevich Rachmaninov (1873-1943)

Rhapsody on a Theme of Paganini, Op.43

Pablo Picasso
Les Demoiselles d'Avignon. 1907

Domenicos Theotocopoulos
“El Greco”
The Visitation 1610-14

H. Brezis, Solutions à support compact d'inequations variationelles, Séminaire Leray, Collège de France, 1973-74, pp. III.1-III.6

H. Brezis Solutions of variational inequalities with compact support. Uspekhi Mat. Nauk. 1974. Vol 129. pp 103-108.

SOLUTIONS WITH COMPACT SUPPORT OF VARIATIONAL INEQUALITIES

Haim BREZIS

Let Ω be a smooth unbounded domain of $\mathbb{R}^n$ and let

$$L = -\sum_{i,j} a_{ij} \frac{\partial^2}{\partial x_i \partial x_j} + \sum_i a_i \frac{\partial}{\partial x_i} + a$$

be a second order elliptic operator. We assume that

(1) $a_{ij} \in C^1(\overline{\Omega}) \cap L^\infty(\Omega)$; $a_i, a \in L^\infty(\Omega)$.

(2) $\begin{cases} \text{for every } r > 0 \text{ there is } \alpha(r) > 0 \text{ such that} \\ \sum_{i,j} a_{ij}(x)\xi_i \xi_j \geq \alpha(r)|\xi|^2 \quad \text{for } x \in \Omega,\ |x| \leq r\ ; \xi \in \mathbb{R}^n \end{cases}$

(3) $a(x) \geq \delta > 0$ for $x \in \Omega$.

Let β be a maximal monotone graph in $\mathbb{R}^2$ such that $0 \in \beta(0)$; we denote the interval $\beta(0)$ by $[\gamma^-, \gamma^+]$.

<u>Question</u> : Under what conditions on f and φ does the problem

$$(4) \quad \begin{cases} Lu + \beta(u) \ni f & \text{on } \Omega \\ u = \varphi & \text{on } \partial\Omega \end{cases}$$

have a solution with compact support ?

It is clear that if (4) has a solution with compact support, then φ has also a compact support and $\gamma^- \leq f(x) \leq \gamma^+$ for $|x|$ large. These conditions are not sufficient, but as we shall see, they are "almost" sufficient. Our first result is the following:

<u>Theorem 1</u> : Assume,

(5) $\varphi \in C^2(\partial\Omega)$, φ has compact support, and $\beta^0(\varphi) \in L^\infty(\partial\Omega)$,

(6) $f \in L^\infty_{loc}(\overline{\Omega})$ and $\gamma^- < \liminf\limits_{|x|\to\infty} \text{ess } f(x) \leq \limsup\limits_{|x|\to\infty} \text{ess } f(x) < \gamma^+$.

Then (4) has a unique solution with compact support, $u \in W^{2,p}(\Omega)$ for all $p < +\infty$.

$$\varphi(x) = 0 \qquad \text{for } |x| \geq r_0$$

$$f(x) \leq \gamma^+ - \varepsilon \qquad \text{for } |x| \geq r_0$$

The function v is of the form $v(x) = G(|x|)$, where

$$G(t) = \begin{cases} -\frac{1}{2}\lambda(t^2 - r_0^2) + \frac{1}{2}\mu(R-r_0)^2 & \text{for } 0 \leq t \leq r_0 \\ \frac{1}{2}\mu(t-R)^2 & \text{for } r_0 \leq t \leq R \\ 0 & \text{for } t \geq R \end{cases}$$

and the constants $\lambda > 0$, $\mu > 0$, $R > r_0$, are to be determined.

<u>Corollary 1</u> : Given $f \in L^\infty_{loc}(\overline{\Omega})$ with $\limsup_{|x| \to \infty} \operatorname{ess} f(x) < 0$, there is a unique solution with compact support of the minimization problem

$$\min_{\substack{u \in H^1_0(\Omega) \\ u \geq 0 \text{ on } \Omega \\ \text{supp}\, u \text{ compact}}} \left\{ \int_\Omega \left(\frac{1}{2} \sum_i \left| \frac{\partial u}{\partial x_i} \right|^2 + \frac{1}{2} |u|^2 - f u \right) dx \right\}$$

Corollary 2 : Given $\varphi \in C^2(\partial\Omega)$ with compact support, there is a unique solution with compact support of the minimization problem

$$\underset{\substack{u \in H^1(\Omega) \cap L^1(\Omega) \\ u = \varphi \text{ on } \partial\Omega}}{\mathrm{Min}} \left\{ \int_\Omega \left(\frac{1}{2} \sum_i \left| \frac{\partial u}{\partial x_i} \right|^2 + \frac{1}{2} |u|^2 + |u| \right) dx \right\}$$

We just apply Theorem 1 respectively with

$$\beta(r) = \begin{cases} 0 & \text{for } r > 0 \\ (-\infty, 0] & \text{for } r = 0 \end{cases} \qquad \beta(r) = \begin{cases} +1 & \text{for } r > 0 \\ [-1, +1] & \text{for } r = 0 \\ -1 & \text{for } r < 0 \end{cases}$$

Remark : It has been shown by several authors that some nonlinear variational problems have a solution with compact support (see [1], [2], [3][(1)]). It would be of interest to unify these various results.

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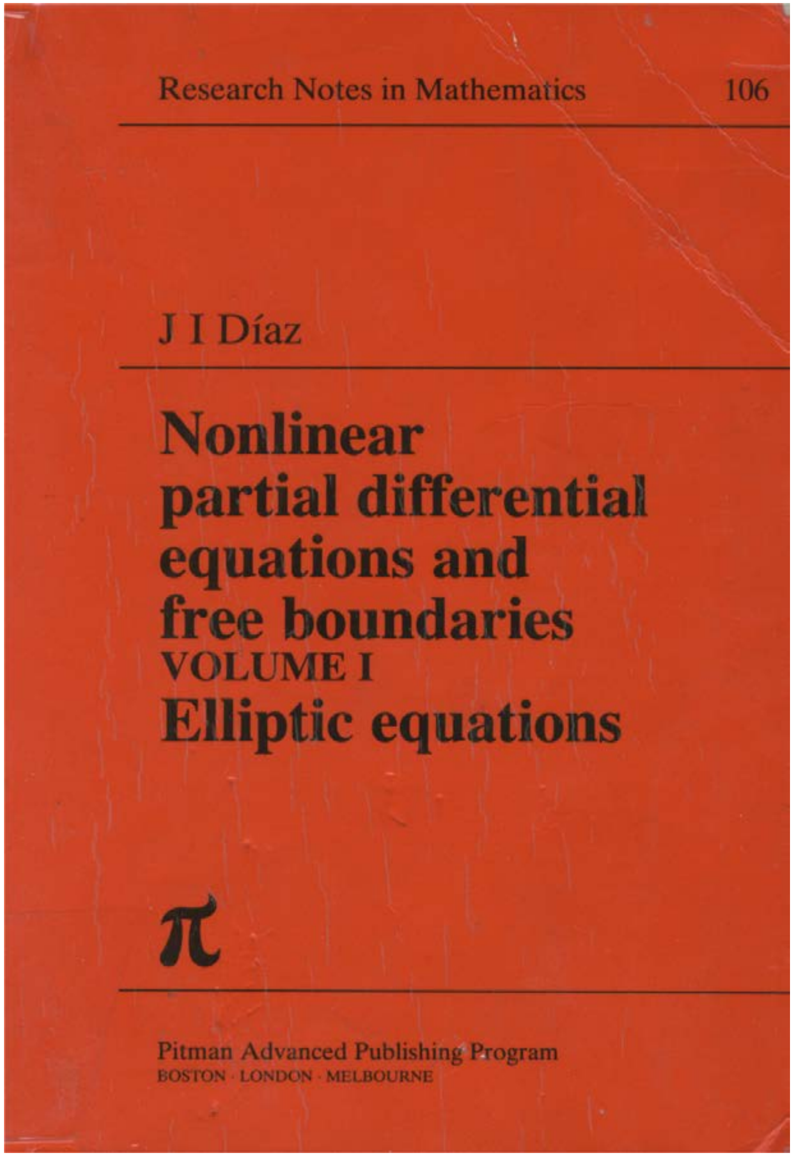
(1) .. A new result in that direction has been obtained very recently by M. Crandall.

He also knew the results on degenerate parabolic equations

Oleinik-Kalahnikov-Yui Lin (1958), Barenblatt, Aronson, Kalashnikov, Peletier,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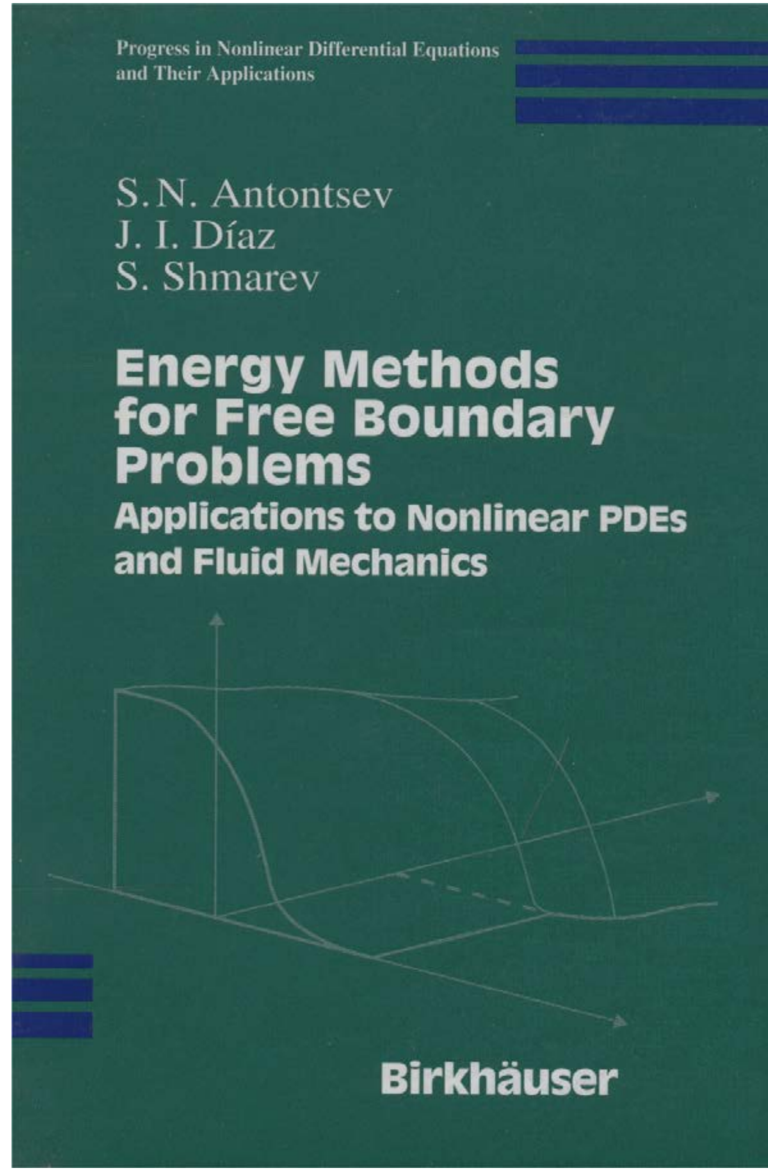
Progress in Nonlinear Differential Equations and Their Applications
S.N. Antontsev
J. I. Díaz
S. Shmarev
Energy Methods for Free Boundary Problems
Applications to Nonlinear PDEs and Fluid Mechanics
Birkhäuser

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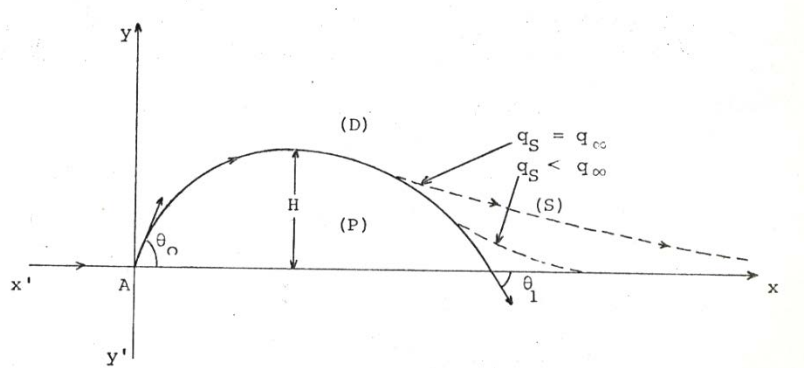

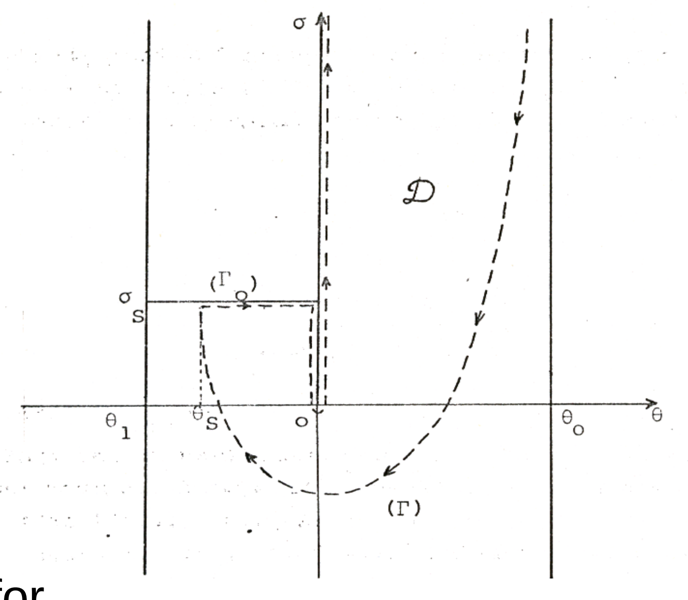

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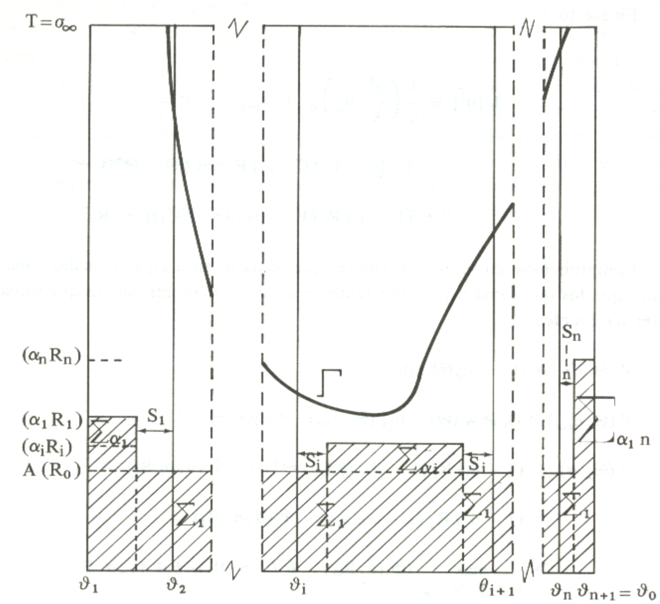

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Other problems for subsonic flows

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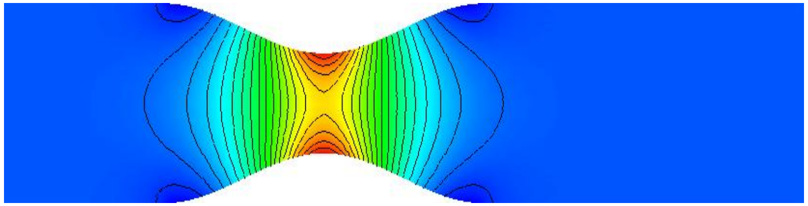
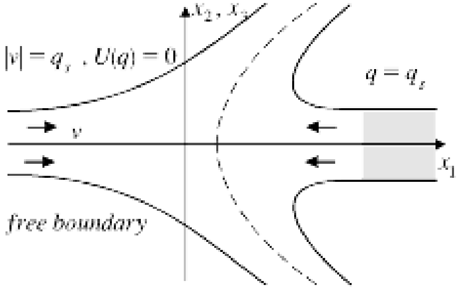


Different approach



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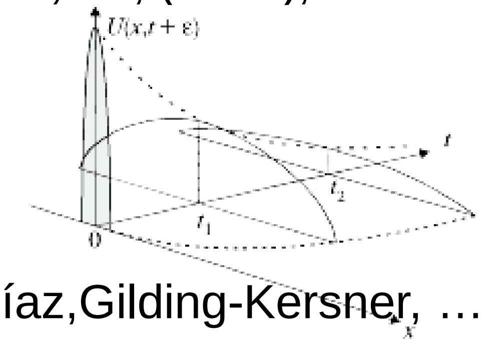
Property of support shrinking

Estimates on the support

Extinction in finite time



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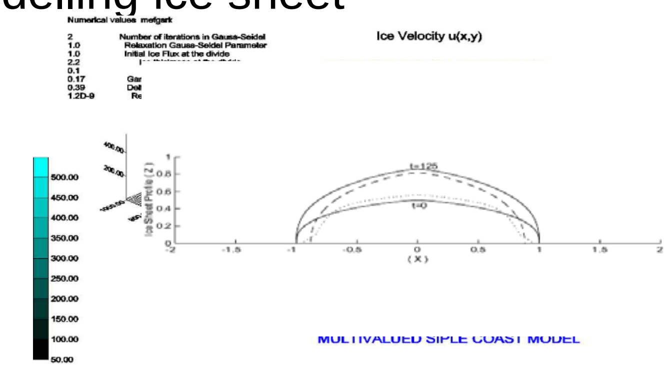
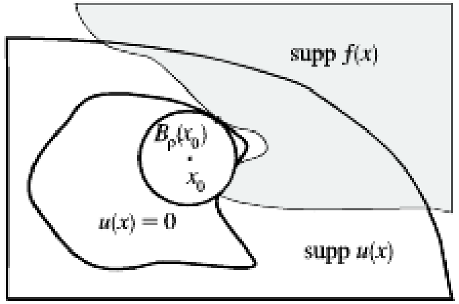
On a doubly nonlinear parabolic obstacle problem modelling ice sheet dynamics,*SIAM J. Appl. Math*., **63,** 2002, 683-707.

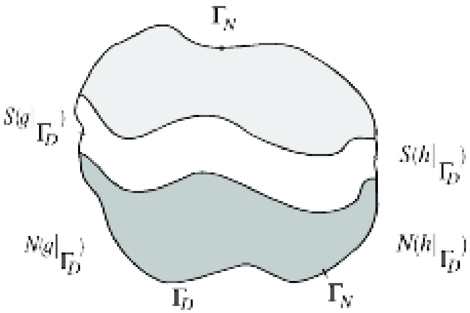
$$\begin{cases} h_t - \left(\frac{h^{n+2}}{n+2} |h_x|^{n-1} h_x - u_b h \right)_x + \beta(h) \ni a(t,x) & \text{in} \quad Q \\ h(t,x) = 0 & \text{on} \quad \Sigma \\ h(0,x) = h_0(x) & \text{on} \quad \Omega \end{cases}$$



First Order Hyperbolic Variational Inequalities: Bensoussan-J.L. Lions (linear operator), Diaz-Veron (balance laws),

A. Bensoussan, H. Brezis and A. Friedman Estimates on the free boundary for quasi variational inequalities**.** *Comm. PDEs* **2** (1977), no. 3, 297--321





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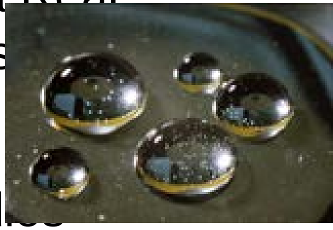

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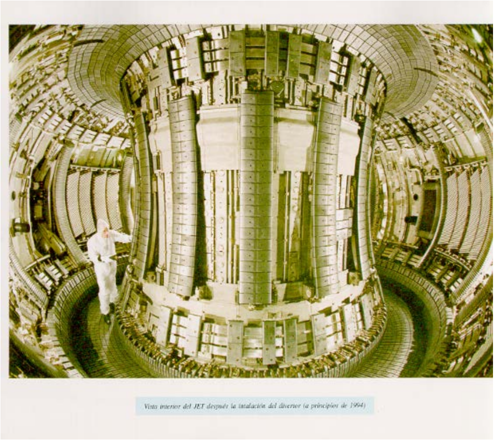
Other free boundary problems considerd by Haïm Brezis:

The dam problem:

Kinderlerer-Stampacchia, Pavia

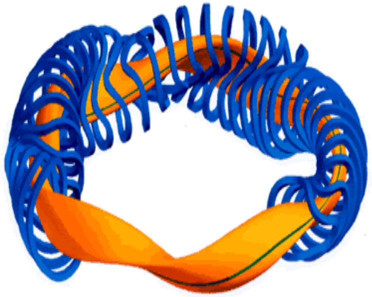
J. Carrillo and M. Chipot,…..

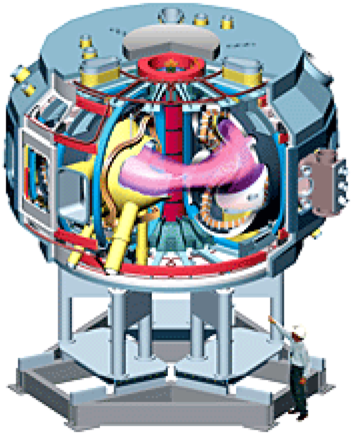


Plasma physics: Tokamaks

H. Beresticky-H. Brezis

Stellarators: Díaz-Rakotoson,…





3. The support of the solution of semilinear elliptic equations

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$$-div(|\nabla u|^{p-2}\,\nabla u)+g(u)=f(x)$$

$$\int_0^{\tau}\frac{ds}{\sqrt[p]{G(s)}}<+\infty,\ \ G(s)=\int_0^s g(r)dr$$

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J.I.Díaz, J.E.Saa, U.Thiel, Sobre la ecuación de curvatura media prescrita y otras ecuaciones cuasilineales elípticas con soluciones anulandose localmente. *Revista de la Unión Matemática Argentina*, **35**, 175-206, 1990

Bandle, Sperb and Stakgold, Kamin-Veron,…

P. Pucci, J. Serrin, H. Zou,…

P. Pucci and J. Serrin, The strong maximum principle revisted, *J. Diff. Equations, **196,** 2004, 1-66.*

Application to singularities:

Brezis and Nirenberg 1996: transformation $u=e^{-v}$ to study the singularity of v solution of

$$-\Delta v + |\nabla v|^2 = h^2(v)$$

for a suitable function $h^2(v)$.

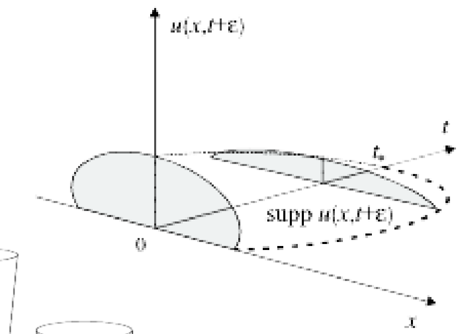
Parabolic equations

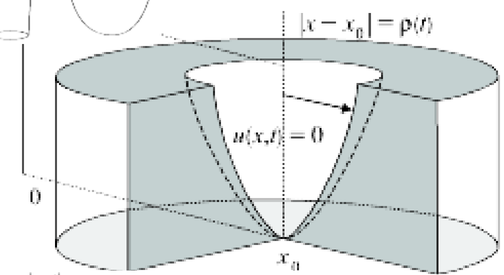
$$\frac{\partial \psi(u)}{\partial t} - \operatorname{div} \mathbf{A}(x,t,u,Du) + B(x,t,u,Du) + C(x,t,u) + \beta(u) \ni f(x,t),$$

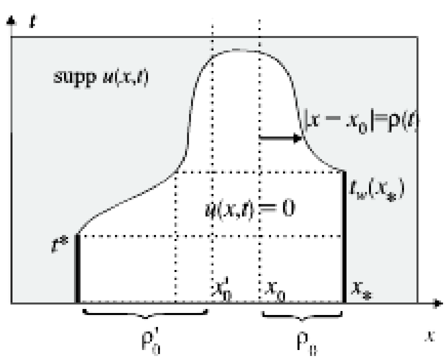
$$|\mathbf{A}(x,t,r,\mathbf{q})| \le C_1 |\mathbf{q}|^{p-1}, C_2 |\mathbf{q}|^p \le \mathbf{A}(x,t,r,\mathbf{q}) \cdot \mathbf{q},$$

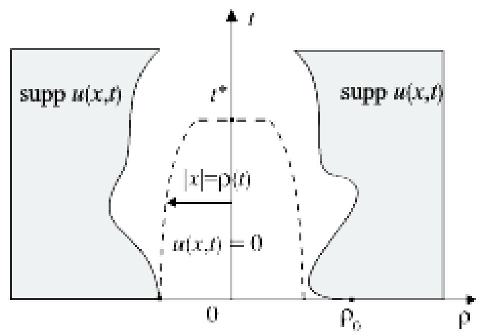
$$|B(x,t,r,\mathbf{q})| \le C_3 |r|^{\alpha} |\mathbf{q}|^{\beta},\ 0 \le C(x,t,r)\, r,$$

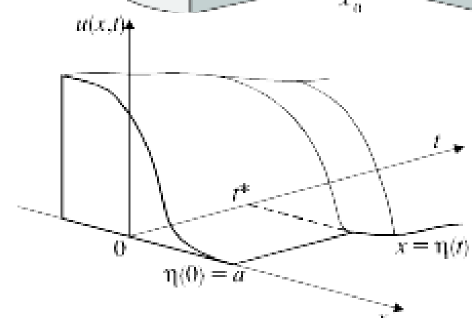
$$C_6 |r|^{\gamma+1} \le G(r) \le C_5 |r|^{\gamma+1}, \text{ where } G(r) = \psi(r)\, r - \int_0^r \psi(\tau)\, d\tau.$$











S.N. Antontsev, J.I. Díaz and S.I. Shmarev, *Energy Methods for Free Boundary Problems: Applications to Nonlinear PDEs and Fluid Mechanics*, Progress in Nonlinear Differential Equations and Their Applications, **48**, Birkhäuser, Boston, 2002.

The support of the solution of some elliptic systems

Minimum Action solutions of some Vector Field Equations (see Brezis and Lieb (1984)

Other systems and higher order equations: Bidaut-Veron, Bernis, Bertsch-Dal Passo, Shiskhov, Andreucci-Tedev, Cirmi,…

S.N. Antontsev, J.I. Díaz and S.I. Shmarev, *Energy Methods for Free Boundary Problems: Applications to Nonlinear PDEs and Fluid Mechanics*, Progress in Nonlinear Differential Equations and Their Applications, **48**, Birkhäuser, Boston, 2002.

$$\Omega = (0,\infty)\times(0,L),$$
$$\mathbf{u}(\mathbf{x}) = (u(\mathbf{x}), v(\mathbf{x}))\quad \mathbf{x}=(x,y)\in\Omega,$$

$$-\nu\Delta\mathbf{u} + (\mathbf{u}\cdot\nabla)\mathbf{u} = \mathbf{f} - \nabla p \quad \text{in } \Omega,$$
$$\operatorname{div}\mathbf{u} = 0 \quad \text{in } \Omega,$$
$$\mathbf{u}(0,y) = \mathbf{u}_*(y),\ \ y\in(0,L)$$
$$|\mathbf{u}(x,y)| \to 0,\ \text{as } x\to\infty \text{ and } \ y\in(0,L).$$
$$\mathbf{u}(x,0) = \mathbf{u}(x,L) = \mathbf{0},\ x\in(0,\infty).$$

Example: $\mathbf{f}(\mathbf{x},\mathbf{u}) = -\delta\chi_{\mathbf{f}}(\mathbf{x})(|u|^{\sigma-1}u,0)$

Antontsev-Díaz- Oliverira (2002,…)

5. Compact support properties and the abstract theory of montone operators

5.1. Degenerate type parabolic equations

A.Berger, H. Brezis and J. C. W. Rogers A numerical method for solving the probler

$$u_t - \Delta f(u) = 0.$$

RAIRO Anal. Numér. **13** (1979), no. 4, 297--312.

L. Alvarez, and J.I. Díaz: The waiting time property for parabolic problems trough the nondiffusion of the support for the stationary problems, *Rev. R. Acad. Cien.Serie A Matem (RACSAM)* **97**, 2003, 83-88.

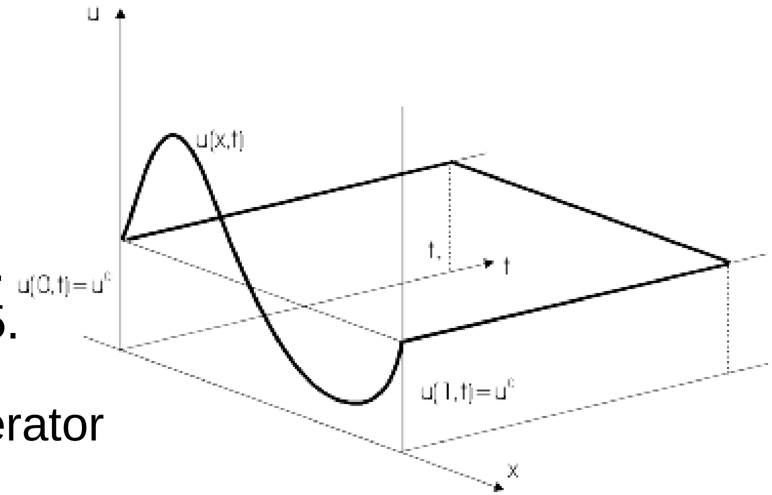
$$\begin{cases} \dfrac{\partial u}{\partial t} = \Delta(|u|^{m-1}u) \text{ and } |u| \leq M & \text{in } \Omega \times (0,+\infty), \\ u(x,0) = u_0(x) & \text{on } \Omega, \end{cases}$$

$$u_n(x) \leq C_n \left|x - x_0\right|^{\frac{2}{m-1}}$$

$$\frac{u_n - u_{n-1}}{\tau} = \Delta u_n^m \quad \text{and } |u_n| \leq M \quad \text{in} \quad \Omega.$$

$$-\Delta w + \frac{1}{\tau} w^{1/m} = f(x) \text{ in } \Omega,$$

5.1. Abstract results on finite extinction time



H. Brezis, Monotone operators, nonlinear semigroups and applications. *Proceedings of the International Congress of Mathematicians (Vancouver, B. C., 1974), Vol. 2,* pp. 249--255. *Canad. Math. Congress, Montreal, Que.,* 1975.

X=H Hilbert space, A maximal monotone operator

$$\begin{cases} \frac{du}{dt}(t) + Au(t) \ni f(t) \quad \text{in } X \\ u(0) = u_0 \end{cases}$$

$$B(f(t), \epsilon) \subset A(0) \text{ for some } \epsilon > 0 \text{ and a.e. } t \geq t_0$$

X Banach space, A m-accretive operator

J.I.Díaz. Anulación de soluciones para operadores acretivos en espacios de Banach. Aplicaciones a ciertos problemas parabólicos no lineales. *Rev. Real. Acad. Ciencias Exactas, Físicas y Naturales de Madrid*, Tomo LXXIV, 865-880, 1980.

Application to

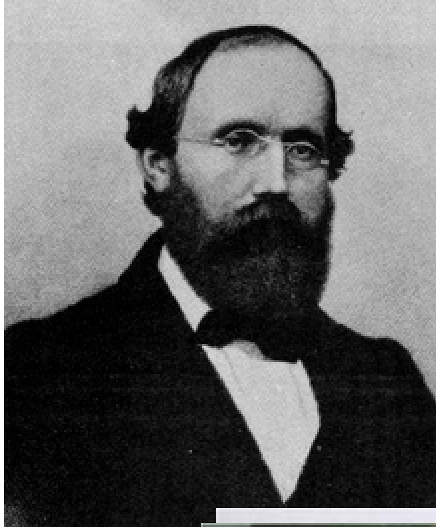
$$\begin{cases} u_t - \nu \Delta u - g\,\mathrm{div}\left(\frac{Du}{|Du|}\right) = c & \text{in } Q, \\ u = 0 & \text{on } \Sigma, \\ u(0,x) = u_0(x) & \text{on } \Omega, \end{cases}$$

E. C. Bingham, in 1922 (non-Newtonian fluids) [also in Image Processing [Chan et al, SIAM Journal on Scientific Computing, 20, 1999]

F.Andreu, V. Caselles, J.I. Díaz, J.M. Mazón, Some Qualitative properties for the Total Variation, *Journal of Functional Analysis,* **188,** 516-547, 2002

B. Riemann: Über die Fläche vom Kleinsten Inhalt bei gegebener Begrenzung,

Abh. Königl. Ges.d. Wiss. Göttingen, Mathem. Cl. ***13****, 3-52 (1867)*




N=3
φ=4
Yes

Ciclo de conferencias

Profesor HAIM BREZIS

Université Pierre et Marie Curie & Rutgers University
Academia Francesa de Ciencias

"The Ginzburg-Landau model, between Physics, Analysis and Topology"

Miércoles 12 de abril de 2000
12:30 horas
Dpto. de Matemática Aplicada, Aula 210
Facultad de Matemáticas
Universidad Complutense de Madrid

"My vision of Mathematics"

Miércoles 12 de abril de 2000
19:00 horas
Real Academia de Ciencias
C/ Valverde, 22
Madrid

"Lifting in Sobolev Spaces"

Jueves 13 de abril de 2000
15:00 horas
Dpto. de Matemáticas. Facultad de Ciencias
Módulo C-XV, Seminario 520
Universidad Autónoma de Madrid

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