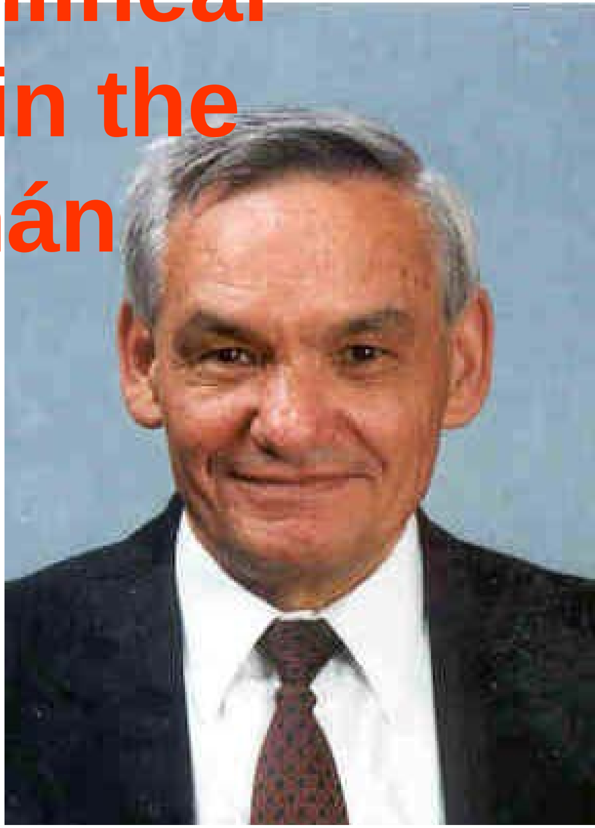
On some of the many nonlinear mathematical problems in the oeuvre of Amable Liñán



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Granada, September 18, 2004

1. CATALYSIS: FRONTS AND HOMOGENIZATION

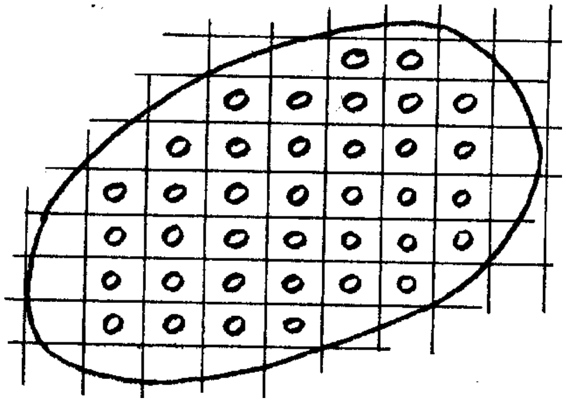
I Congreso de Ecuaciones Diferenciales y Aplicaciones CEDYA. El Escorial, May 29th-31th, 1978).

$$\begin{cases} \dfrac{\partial u}{\partial t} = \Delta u - \alpha u^p e^{\gamma(v-1)/v} & \text{in } \Omega \times (0,\infty), \\ \dfrac{\partial v}{\partial t} = \Delta v + \beta u^p e^{\gamma(v-1)/v} & \text{in } \Omega \times (0,\infty), \\ \dfrac{\partial u}{\partial n} = \sigma(1-u), \ \dfrac{\partial v}{\partial n} = v(1-v) & \text{on } \partial\Omega \times (0,\infty). \\ u(x,0) = u_i(x), \ v(x,0) = v_i(x) & \text{on } \Omega, \end{cases}$$

Liñán and Vega (1979) on the "formation of the dead core" typical of reactions of low order ($p \in [0,1)$)

Díaz-Hernández (1984, 1987,...)

Homogenization process related to the overall modelling in presence of a double spatial scale



Sánchez-Palencia, Bensoussan-Lions-Papanicolau,...

C. Conca, J.I. Díaz, A. Liñan and C. Timofte, Homogenization in Chemical Reactive Flows through Porous Media, Electr. J. Diff. Eqns. 2004, 1-22 (2004).

$$\begin{cases} -D_f \Delta u^\varepsilon = f \quad \text{in } \Omega^\varepsilon, \\ -D_f \dfrac{\partial u^\varepsilon}{\partial \nu} = a\varepsilon g(u^\varepsilon) \quad \text{on } S^\varepsilon, \\ u^\varepsilon = 0 \quad \text{on } \partial\Omega. \end{cases}$$

$$g(v) = \frac{\alpha v}{1+\beta v}, \quad \alpha, \beta > 0 \quad \text{(Langmuir kinetics)}$$

$$g(v) = |v|^{p-1} v, \quad 0 < p < 1 \quad \text{(Freundlich kinetics)}$$

$$\begin{cases} -\sum_{i,j=1}^{n} a_{ij}^{0} \frac{\partial^2 u}{\partial x_i \partial x_j} + a \frac{|T|}{|Y \setminus T|} g(u) = f & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega. \end{cases}$$

$$a_{ij}^{0} = \frac{1}{|Y|} \int_Y \left(a_{ij} + a_{ik} \frac{\partial \chi_j}{\partial y_k} \right) dy, \qquad \begin{cases} -\mathrm{div}(AD(y_j + \chi_j)) = 0 & \text{in } Y, \\ \chi_j - Y \text{ periodic.} \end{cases}$$

2. THE p-LAPLACIAN IN FLUID DYNAMICS

$$\Delta_p u = \mathrm{div}(|\nabla u|^{p-2} \nabla u), \quad 1 < p < \infty.$$

$$\begin{cases} \frac{\partial u}{\partial t} = div\left(|\nabla u|^{p-2} \nabla u \right) & \text{in} \quad Q = (0, \infty) \times \Omega, \\ u(t, x) = 0 & \text{on} \quad S = (0, \infty) \times \partial\Omega, \\ u(0, x) = u_0(x) & \text{in} \quad x \in \Omega,, \end{cases}$$

In Diaz-Herrero (1978,1981) we proved that if

$p>2$ then there is **finite speed of propagation** (i.e., if $supp(u_0)\subset B(0,r)\subset$, then $supp(u(t))$ is a compact set for any $t>0$ but if $1<p\leq 2$, $u_0\geq 0$, $u_0\neq 0$, then $u(t)>0$ or $u(t)=0$ in for all $t>0$ **(strong maximum principle)**

finite time extinction of the solutions of when $((2N)/(N+2))\leq p<2$, $N\geq 2$ Bamberger (1977), and, for $1<p<((2N)/(N+1))$, Herrero-Vázquez (1981) (see also Antontsev-Diaz-Shmarev 2001).

Díaz and Liñán (1989) : discharge of a turbulent and perfect gas in a pipeline occupying the interval (0,L) and with a section of diameter D very small in comparison with L (hydraulic approximation)

$$\begin{cases} \frac{\partial \rho}{\partial t}+\frac{\partial(\rho u)}{\partial x}=0 \\ \rho\frac{\partial u}{\partial t}+\rho u\frac{\partial u}{\partial x}=-\frac{\partial p}{\partial x}-\frac{1}{2}\rho\left|u\right|u \\ (\frac{\partial u}{\partial t}+u\frac{\partial}{\partial x})(\frac{\gamma}{\gamma-1}T+\frac{1}{2}u^2)-\frac{\partial p}{\partial t}=-\frac{1}{2}\left|u\right|(\frac{\gamma}{\gamma-1}T-\frac{1}{2}u^2) \\ \frac{p}{\rho}=T. \end{cases}$$

$$\left(\frac{\partial u}{\partial t} + u\frac{\partial}{\partial x}\right)(Ln(p/\rho^{\gamma}) = -\frac{|u|}{2}\frac{\gamma}{T}(T - 1 - \frac{\gamma - 1}{2\gamma}u^2)$$

$$\begin{cases} u(x,0) = 0,\ T(0,x) = 1,\ p(x,0) = 1 & x \in (0,1), \\ u(0,t) = 0 & t > 0, \\ \frac{u(1,t)}{\sqrt{\gamma T(1,t)}} < 1 & \text{if } p(1,t) = p_a, \\ \frac{u(1,t)}{\sqrt{\gamma T(1,t)}} = 1 & \text{if } p(1,t) > p_a. \end{cases}$$

Two different steps in the discharge:

in the first one (very short, of the order 1/f) all the terms at the entalpia equation are of the same order but the auxiliary conditions can be simplified allowing a local study made by using the Riemann invariants

In the second step, when t>>1/f, we show that the second and fourth equation can be simplified, by neglecting lower order terms and using some suitable variable scales) to

$$0 = -\frac{\partial p}{\partial x} - \frac{1}{2}\rho\,|u|\,u \text{ and } \frac{p}{\rho} = T = 1.$$

$$\begin{cases} \frac{\partial p}{\partial t} - \frac{\partial}{\partial x}\left(\left|\frac{\partial p^2}{\partial x}\right|^{1/2} sign\left(\frac{\partial p^2}{\partial x}\right)\right) = 0 & t>0,\ x\in(0,1), \\ \frac{\partial p}{\partial x}(0,t)=0,\ p(1,t)=p_a & t>0, \\ p(x,0)=1 & x\in(0,1). \end{cases}$$

making $p^2 - p_a^2 := w \quad \psi(w) = (w + p_a^m)^{1/m}$

$$\begin{cases} \frac{\partial \psi(w)}{\partial t} - \Delta_q w = 0 & t>0,\ x\in(0,1), \\ \frac{\partial w}{\partial x}(0,t)=0,\ w(1,t)=0 & t>0, \\ w(x,0)=w_0 & x\in(0,1), \end{cases}$$

m=2 and q=3/2 nevertheless other interesting cases are m=7/4 and q=11/7 (case of very polished pipes) and m=1 and q=3/2 (laminar regime)

Results

a) finite extinction property if p_a >0 and q<2 (for any m>0 arbitrary) either p_a =0 and m(q-1)<1.

b) if, by the contrary, we assume p_a ≥0 and m(q-1)=1 we show

$$e^{-\lambda_1(t+\tau_1)}v(x) \le w(x,t) \le e^{-\lambda_1(t+\tau_2)}v(x),$$

$$\begin{cases} -\Delta_q v = \lambda v^{1/m} & x \in (0,1), \\ \frac{\partial v}{\partial x}(0) = 0, \ v(1) = 0. \end{cases}$$

Andreu, Caselles, Díaz, Mazón (2002)

$$\frac{\partial u}{\partial t} = \operatorname{div}\left(\frac{Du}{|Du|}\right)$$

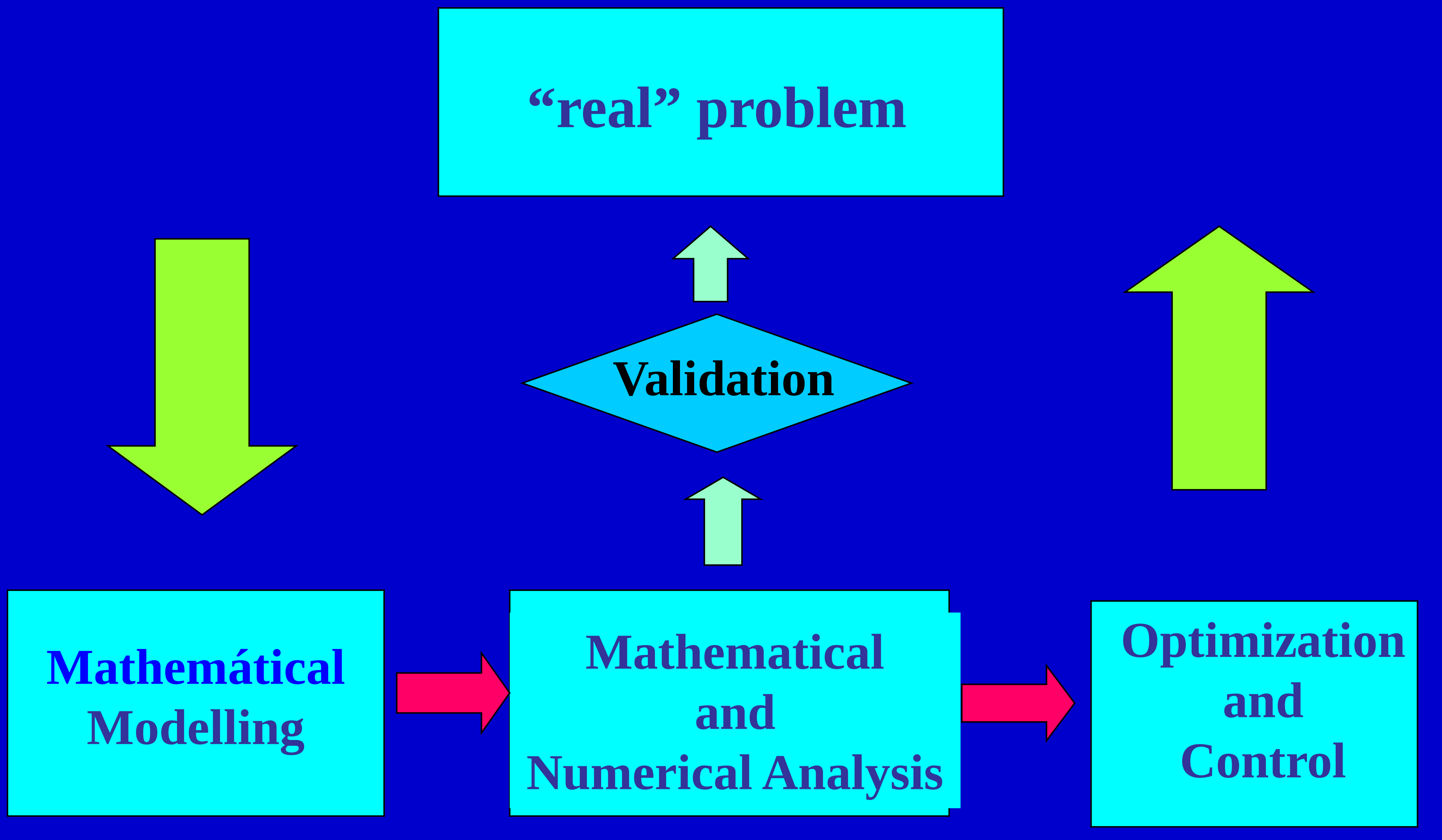
3. A source of problems in lubrication

A.Liñán "Problemas matemáticos de lubricación hydrodinámica" held at the Seminario de Matemática Aplicada (UCM) on April 14th 1986

Uniqueness of (weak) solutions: thesis by S.J. Álvarez at the UCM (Alvarez and Carrillo (1994)). The evolution problem in Álvarez, Carrillo and Díaz (1990).

Propagated very fast to specialists of the universities of Santiago de Compostela (Bermudez de Castro, doctoral dissertation of C. Vazquez (1994)) and Vigo (Durany),

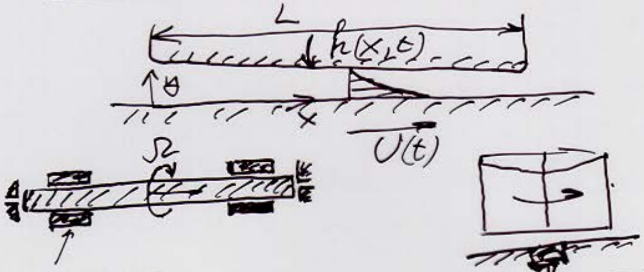
Postgraduate course "Introducción a la Mecánica de Fluidos", lectured jointly with Liñán and me since 1996 (the first two years also in collaboration with M.G. Velard
...



Modelling

TEORIA DE LA LUBRICACIÓN FLUIDODINÁMICA (REYNOLDS)

LUBRICACIÓN HIDRODINÁMICA



COJINETE CILÍNDRICO

COJINETE DE EMPUJE

ZAPATA BIDIMENSIONAL

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

$$U_c/L = V_c/h_0$$

$$\rho\left\{\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right\}=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial x^2}\right)$$

$$\rho U_c/t_o \quad \rho U_c^2/L \quad \delta_x P/L \quad \mu U_c/h_0^2 \quad \mu U_c/L^2$$

$$\rho\left\{\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right\}=-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial x^2}\right)$$

$$\rho V_c/t_o \quad \rho U_c V_c/L \quad \delta_y P/h_0 \quad \mu \frac{V_c}{h_0^2} \quad \mu\frac{V_c}{L^2}$$

$y=0$: $u=U(t)$, $v=0$

$y=h(x,t)$: $u=0$, $v=\frac{\partial h}{\partial t}$

$$\frac{\delta_y P}{\delta_x P}\sim\left(\frac{h_o}{L}\right)^2$$

$U(t) \nearrow U_c,\ \searrow t_o$: $h(x,t) \nearrow h_o, t_o, L$

$h_o/L \ll 1$, $h_o^2/\nu t_o \ll 1$; $h_o^2/\nu\frac{L}{U_c} \ll 1$

HIPOTÉSIS DE LA TEORIA DE LA LUBRICACIÓN

$h_o/L \ll 1$ (CAPAS DELGADAS)

$h_o^2/\nu t_o \ll 1$ (ACELERACION LOCAL DESPREC.)

$\frac{Uh_o}{\nu}\frac{h_o}{L} \ll 1$ (ACELERACIÓN CONVECTIVA DESPRECIABLE)

ECUACIONES (DE STOKES) SIMPLIFICADAS

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

$$0=-\frac{\partial p}{\partial x}+\mu\frac{\partial^2 u}{\partial y^2}$$

$$0=\frac{\partial p}{\partial y} \rightarrow p=p(x,t)\ ?$$

$y=0$: $u=U(t)$, $v=0$

$y=h(x,t)$: $u=0$, $v=\partial h/\partial t$

$$u = \underbrace{U\left(1-\frac{y}{h}\right)}_{\text{Coutte}}+\underbrace{\frac{1}{2\mu}\frac{\partial p}{\partial x}y(y-h)}_{\text{Poiseuille}}$$

$$v=-\int_0^y\frac{\partial u}{\partial x}dy \rightarrow \frac{\partial h}{\partial t}=-\int_0^h\frac{\partial u}{\partial x}dy$$

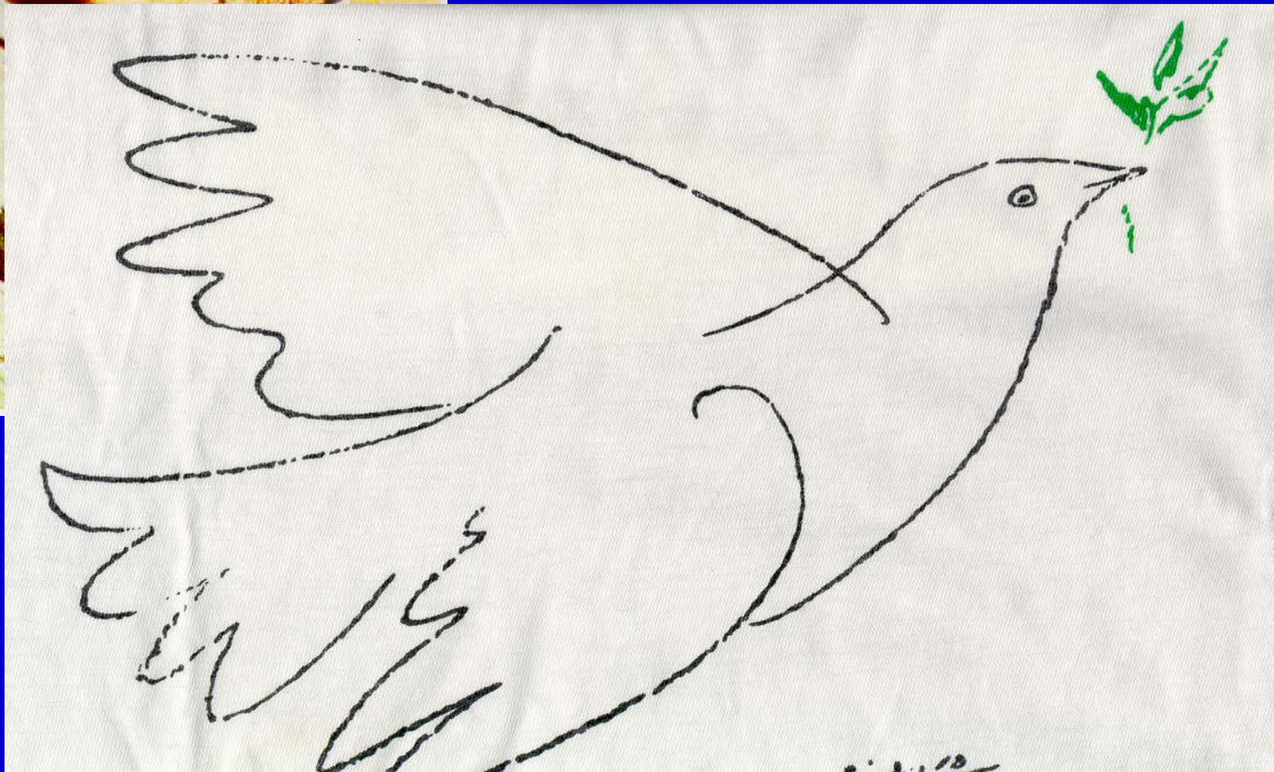
$$\boxed{\frac{\partial h}{\partial t}+\frac{\partial}{\partial x}\int_0^h u\,dy=0} \qquad q_x=\int_0^h u\,dy$$

$$q_x=\frac{Uh}{2}-\frac{h^3}{12\mu}\frac{\partial p}{\partial x}$$

$$\frac{\partial h}{\partial t}+\frac{\partial}{\partial x}\left(\frac{Uh}{2}-\frac{h^3}{12\mu}\frac{\partial p}{\partial x}\right)=0$$

(ECUACIÓN DE REYNOLDS DE LA LUBRICACIÓN)

Modelling: similar points in Art



4. Asymptotics in Coulomb friction type problems

$$m x_{tt} + \mu \left| x_t \right|^{\alpha-1} x_t + kx = 0,$$

$\alpha \in (0,1)$ and $\mu, k > 0.$

$$x_{tt} + \left| x_t \right|^{\alpha-1} x_t + x = 0$$

rescaling $\tilde{x}(\tilde{t}) = \beta^{1/(\alpha-1)} x(\lambda \tilde{t})$

$\lambda = \frac{\sqrt{m}}{\sqrt{k}}$ and $\beta = \frac{\mu}{k^{(2-\alpha)/2} m^{\alpha/2}}.$

$\alpha \to 0$ corresponds to the Coulomb friction equation $x_{tt} + sign(x_t) + x \ni 0$

$\alpha \to 1$ linear damping equation $x_{tt} + x_t + x = 0.$

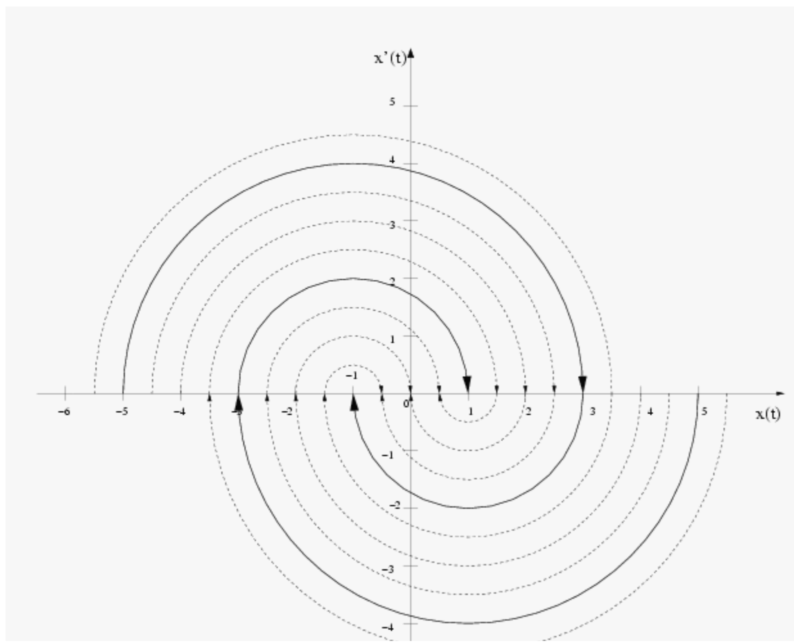
$$P_\alpha \begin{cases} x_{tt} + \left| x_t \right|^{\alpha-1} x_t + x = 0 & t > 0, \\ x(0) = x_0, \; x_t(0) = v_0 \end{cases}$$

The asymptotic behavior, for $t \to \infty$, of solutions of the limit problems P_0 and P_1 is well known (see, for instance, Jordan and Smith [49]). In the first case the decay is exponential. In the second one it is easy to see that "given x_0 and v_0 there exist a finite time $T = T(x_0, v_0)$ and a number $\zeta \in [-1,1]$ such that $x(t) \equiv \zeta$ for any $t \geq T(x_0, v_0)$". For problem P_α it is well-known that $(x(t), x_t(t)) \to (0,0)$ as $t \to \infty$ (see, e.g. Haraux [41]).

- The damped oscillator

$$(DO)\quad \begin{cases} \ddot{x}(t) + x(t) + \beta(\dot{x}(t)) \ni 0, \\ x(0) = x_0, \\ \dot{x}(0) = y_0. \end{cases}$$

By matching orbits of $\ddot{x} + x = \pm 1$, we get the phase plane



A particle take a time π to cross half a circle in the phase plane. Thus, a particle take a finite time to reach the equilibrium position.

Main goal of D-Liñán (2001,2002), the generic asymptotic behavior above described for the limit case P_0 is only exceptional for the sublinear case $\alpha \in (0,1)$ since the generic orbits $(x(t), x_t(t))$ decay to $(0,0)$ in a infinite time and only two uniparametric families of them decay to $(0,0)$ in a finite time: in other words, when $\alpha \to 0$ the exceptional behavior becomes generic.

planar system
$$\begin{cases} x_t = y \\ y_t = -x - |y|^{\alpha-1} y \end{cases}$$

$$y_x = \frac{-x - |y|^{\alpha-1} y}{y}$$

the plane phase is antisymmetric

$$(x^2 + y^2)_t = 2\,|y|^{\alpha+1}.$$

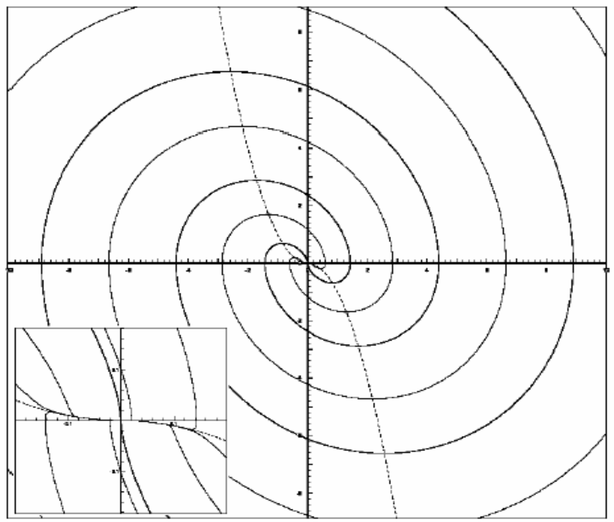
$(1/x, 1/y)$ satisfy a system which has the point $(0,0)$ as a spiral unstable critical point.

We proved that there are two modes of approach to the origin and so that the origin $(0,0)$ is a node for the system

The lines of zero slope $\quad -x = |y|^{\alpha-1} y$

convergence to $(0,0)$ is only possible through the regions

$\{(x,y) : x > 0, y < -x^{1/\alpha}\} \cup \{(x,y) : x < 0,\ y > (-x)^{1/\alpha}\}$



"ordinary" mode small effects of the inertia

Let $-y = \tilde{y} > 0$. $\tilde{y}\tilde{y}_x = -x + \tilde{y}^{\alpha}.$ line of zero slope is $\tilde{y} = x^{1/\alpha}$

we search for orbits obeying, for $0 < x << 1$, $\tilde{y} = x^{1/\alpha} + z(x)$ for some function $z(x)$.

condition $0 < z(x) << x^{1/\alpha}$, "linearized form" $\frac{1}{\alpha}x^{(\frac{1}{\alpha}-1)}z + x^{\frac{1}{\alpha}}z_x - \alpha x^{(1-\frac{1}{\alpha})}z = 0.$

$$z(x) \sim C \exp\{-[\alpha^2/2(1-\alpha)]x^{-\frac{2(1-\alpha)}{\alpha}}\}$$

C an arbitrary constant (which explain the name of "ordinary" orbits).

close to the origin, $\tilde{y} \sim x^{1/\alpha} \sim -\frac{dx}{dt}$ $\frac{dx}{dt} = -x^{1/\alpha}$ $x(t) \sim [\frac{\alpha}{(1-\alpha)(t+t_1)}]^{\alpha/(1-\alpha)}$

Some different orbits approaching the origin can be found

large values of $|y|$ compared with $|x|^{1/\alpha}$.

$$\tilde{y}\tilde{y}_x = \tilde{y}^{\alpha}$$

$$\tilde{y}(x) = -\{(2-\alpha)x\}^{1/(2-\alpha)}.$$

it involves no arbitrary constant. which justifies the term of "extraordinary" orbit.

$$-\frac{dx}{dt} = [(2-\alpha)x]^{1/(2-\alpha)}$$

$$x(t) = \frac{1}{(2-\alpha)}\left[\frac{(2-\alpha)(1-\alpha)}{2\alpha}(t_0 - t)_+\right]^{(2-\alpha)/(1-\alpha)}$$

$$v_0 \sim \pm[(2-\alpha)\,|x_0|]^{1/(2-\alpha)}$$

exceptional orbits separatrix curve in the phase plane.

Further results Amann-D (2004), J.L. Vázquez (2004), D-Millot (2004), ...

5. Last collaboration: Turbulent regimes at the Real Academia de Ciencias, ...

UN MATEMÁTICO PUBLICA UN ARTÍCULO EN UNA REVISTA CIENTÍFICA PARA DEMOSTRAR LA EXISTENCIA DEL SER SUPREMO

Dios más dios son cuatro

ARCADI ESPADA

1 Abstract

No hay noticia de que en los más de 150 años de la *Revista de la Real Academia de Ciencias Exactas, Físicas y Naturales* tres académicos hubiesen retirado su nombre del encabezamiento para no legitimar con su firma uno de los artículos publicados. Ha sucedido en el último número (volumen 97, número 1. 2003). El artículo, de 13 páginas, lo firma Baltasar Rodríguez-Salinas (1925), catedrático ya jubilado de Análisis Matemático de la Universidad Complutense de Madrid. Los tres académicos disidentes son: Ildefonso Díaz y Miguel de Guzmán, matemáticos, y el ingeniero aeronáutico Amable Liñán, premio Príncipe de Asturias.

El artículo se titula *Sobre los big bangs y el principio y el final de los tiempos del Universo* y presenta 17 teoremas y un corolario. Su intención es doblemente ambiciosa. En primer lugar, probar que el universo tiene un número finito de elementos: "Se prueba que el Universo Físico es un conjunto finito, aunque según la teoría cuántica pareciese infinito, de donde resulta la primera demostración de que U ha tenido un principio A y tendrá un final Z; y se compara los resultados con las cinco vías de Santo Tomás [los cinco argumentos de la *Summa Teologica* que aluden a la divinidad]". ...

El Dios todopoderoso del techo de la Capilla Sixtina, en el momento de crear al hombre, según la visión de Miguel Ángel. AP

párrafos de ese documento, muy crítico con el desarrollo de la ciencia en España, dice: "Sólo la producción de ciencia de calidad puede equilibrar los indicadores, hacer más competitiva una economía basada en el conocimiento y ...

dente. "Cuando estaba en la Universidad, ya me hacían la vida imposible éstos. No han leído el artículo, lo único que pasa es que no han leído el artículo". El matemático alude a la continuidad que Sobre los *big bangs*...

Baltasar Rodríguez-Salinas, catedrático jubilado de Análisis Matemático, ha escrito un artículo de 13

cionó una cinta sobre el *Cosmos* de Carl Sagan en la que se cita a Aristarco de Samos (...). Y, seguidamente, a los pocos días, Paula nos trajo dos recordatorios del funeral de una hermana suya con la imagen del Cristo de la...".

Organization ...

Workshop in Honor of Amable Liñán by his 80th Birthday

“Simplicity, Rigor and Relevance in Human Life”