Modélisation

(climat, stellarators, milieux continus, ..)

équations aux dérivées partielles et leur contrôle

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Real Academia de Ciencias

1. Introduction

Message de Pierre Suquet ,
Délégué pour les prix de la section des Sciences Mécaniques et Informatiques du 11/9/2015:

… une brève conférence en rapport avec le sujet de votre prix.
…. devrait vous permettre de présenter quelques éléments de votre parcours scientifique et vos travaux les plus marquants.

Grande Prix Jacques-Louis Lions

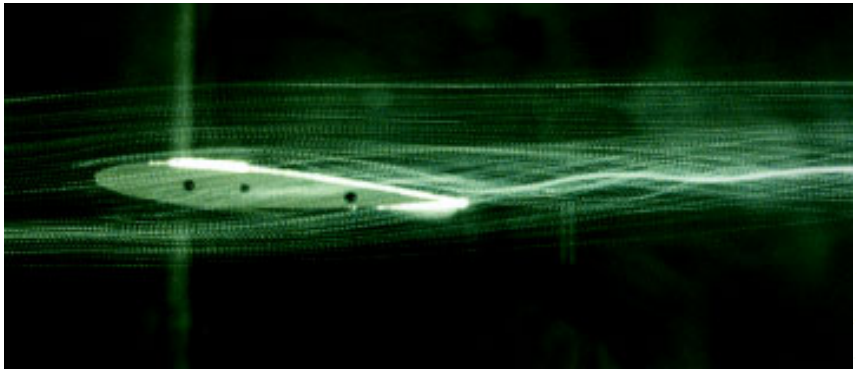
Ce prix récompense un scientifique pour un ensemble de travaux [de très grande valeur] en mathématiques appliquées, effectués en France ou en étroite relation avec un laboratoire français, dans les domaines dans lesquels Jacques-Louis Lions a travaillé: équations aux dérivées partielles, théorie du contrôle, analyse numérique, calcul scientifique et leurs applications.

- Plan
  - 2. Free boundary problems
  - 3. Climate and environment
  - 4. Stellerators: magnetic confinement of a fusion plasma
  - 5. Control of nonlinear PDEs
  - 6. Many other subjects in my relation with France

2. Free boundary problems: solutions of quasilinear elliptic and parabolic equations with compact support

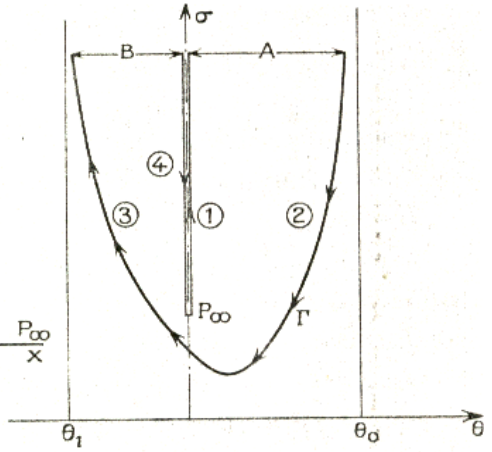
Pionering works: a classical problem of the Fluid Mechanics under a new view:

H. Brezis and G. Stampacchia: Une nouvelle méthode pour l'étude d'écoulements stationnaires, *C.R. Acad. Sci.,* **276**, 1973, 129-132. (Séance du 18 décembre 1972)



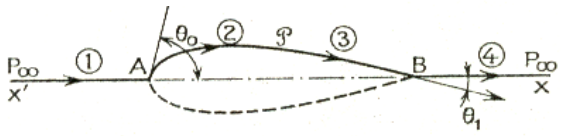
steady irrotational subsonic flow for a non viscous fluid,

symmetric convex profile in the plane



Hodograph trasformation $\mathcal{T} : (x, y) \to (u, v) \to (\theta, q)$

$$\begin{cases} \frac{1}{q^2}\left(\frac{q^2}{k}u_\sigma\right)_\sigma + u_{\theta\theta} + u = -R & \text{in } \mathcal{D}, \\ u = 0 & \text{on } \Gamma, \\ \nabla u = \mathbf{0} & \text{on } \Gamma, \\ u(0,\sigma) = \text{Constant} = H_{\mathcal{P}} & \sigma \geq \sigma_\infty \end{cases}$$



Formulation as a Variational Inequality (Lions-Stampacchia 1967)

$$K_H = \{v \in V : v \geq 0 \text{ on } \Omega \text{ and } u(0,\sigma) = H_{\mathcal{P}} \text{ for } \sigma \geq \sigma_\infty\}$$

H. Brezis and G. Duvaut, Écoulements avec sillages autour d'un profil symmétrique sans incidence, C.R. Acad.Sci., 276, 1973, pp. 875-878.

H. Brezis, Solutions à support compact d'inequations variationnelles, Séminaire Leray, Collège de France, 1973-74, pp. III.1-III.6

H. Brezis Solutions of variational inequalities with compact support. Uspekhi Mat. Nauk. 1974. Vol 129. pp 103-108.

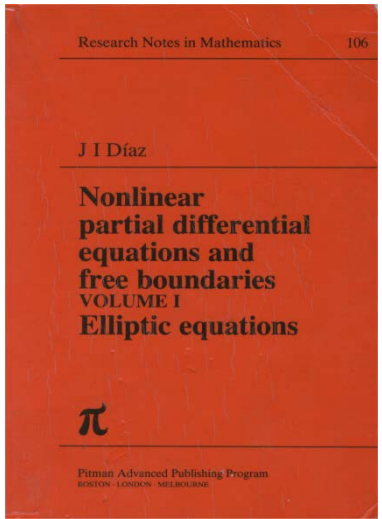
Pionering references on parabolic quasilinear equations (porous media): Oleinik-Kalahnikov-Yui Lin (1958), Barenblatt, Aronson, Kalashnikov, Peletier, …

JID:1976 Thesis (Paris VI, UCM): Adviser H. Brezis

$$\frac{\partial \psi(u)}{\partial t} - \operatorname{div} \mathbf{A}(x,t,u,D\,u) + B(x,t,u,D\,u) + C(x,t,u) + \beta(u) \ni f(x,t),$$

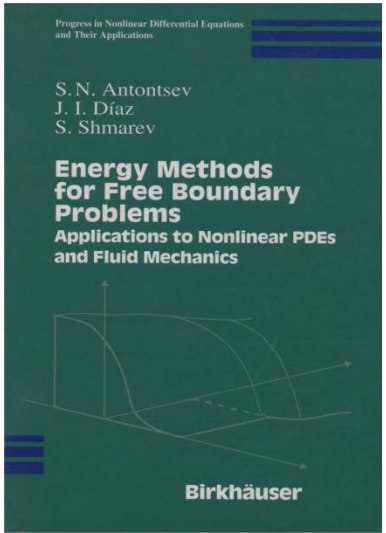
Method of "local super and subsolutions"

1985 Pitman



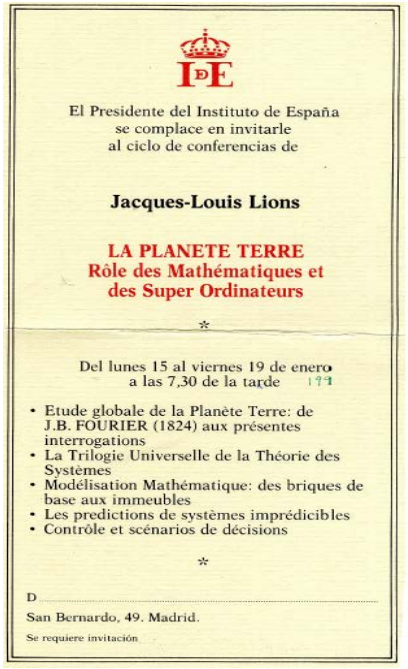
Method of "local integral energy"

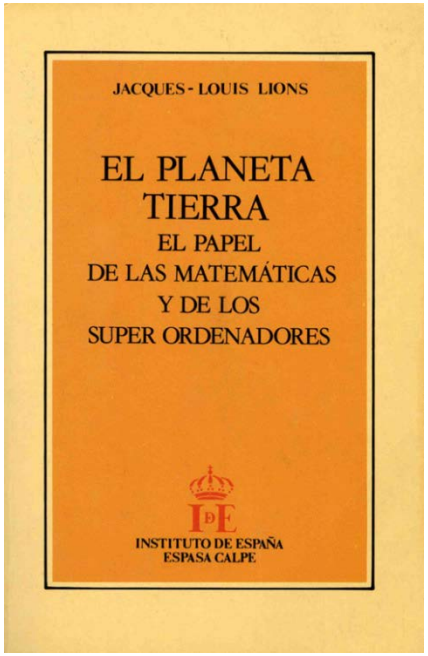
2002 Birkhäuser



http://www. **mat.ucm.es/~jidiaz**

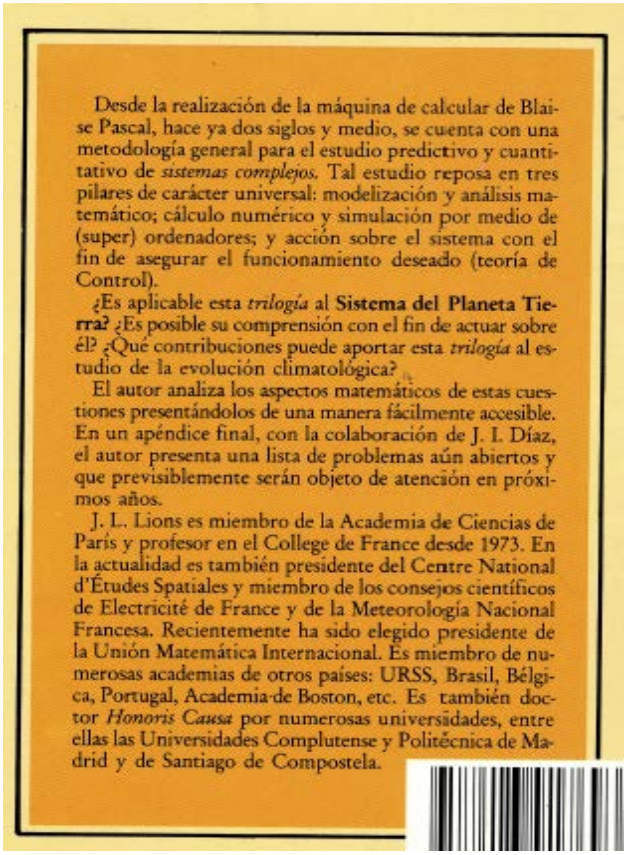
3. Climate and environment





PARIS2015
UN CLIMATE CHANGE CONFERENCE
COP21·CMP11

Appendix (and Spanish translation)


Desde la realización de la máquina de calcular de Blaise Pascal, hace ya dos siglos y medio, se cuenta con una metodología general para el estudio predictivo y cuantitativo de *sistemas complejos*. Tal estudio reposa en tres pilares de carácter universal: modelización y análisis matemático; cálculo numérico y simulación por medio de (super) ordenadores; y acción sobre el sistema con el fin de asegurar el funcionamiento deseado (teoría de Control).

¿Es aplicable esta *trilogía* al **Sistema del Planeta Tierra?** ¿Es posible su comprensión con el fin de actuar sobre él? ¿Qué contribuciones puede aportar esta *trilogía* al estudio de la evolución climatológica?

El autor analiza los aspectos matemáticos de estas cuestiones presentándolos de una manera fácilmente accesible. En un apéndice final, con la colaboración de J. I. Díaz, el autor presenta una lista de problemas aún abiertos y que previsiblemente serán objeto de atención en próximos años.

J. L. Lions es miembro de la Academia de Ciencias de París y profesor en el College de France desde 1973. En la actualidad es también presidente del Centre National d'Études Spatiales y miembro de los consejos científicos de Electricité de France y de la Meteorología Nacional Francesa. Recientemente ha sido elegido presidente de la Unión Matemática Internacional. Es miembro de numerosas academias de otros países: URSS, Brasil, Bélgica, Portugal, Academia de Boston, etc. Es también doctor *Honoris Causa* por numerosas universidades, entre ellas las Universidades Complutense y Politécnica de Madrid y de Santiago de Compostela.

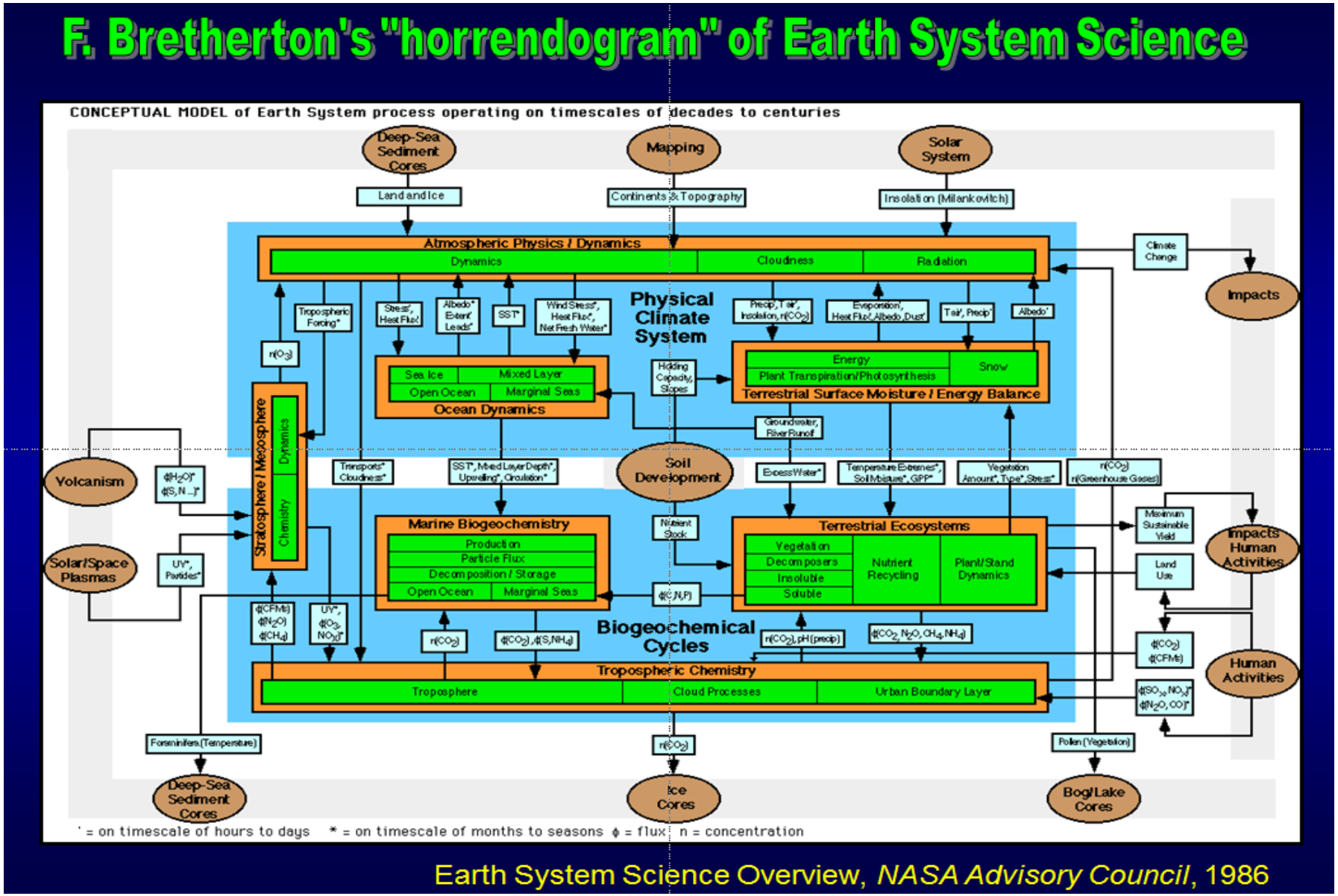
J.I. Díaz and **J.-L. Lions,** *Matemáticas, superordenadores y control para el planeta Tierra***.** Unpublished. *Contract with the UCM and Oxford Univ. Press, 2002.*

Mentioned in **Lions J.-L.,** Some Remarks on the Mathematical Modelling of Planet Earth System**,** *Atti dei Convegni Lincei, Accademia Nazionale dei Lincei***, 158**, 2000, 73-93**.**

Díaz, J.I., **Lions, J.-L.**, eds., ***Mathematics, Climate and Environment***, Research Notes in Applied Mathematics 27, Masson, Paris, 1993.

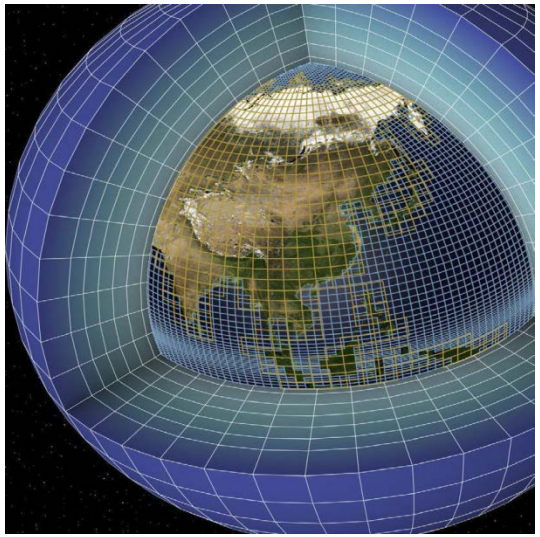
Díaz, J.I., **Lions, J.-L.**, eds., ***Environment, Economics and Their Mathematical Models***, Research Notes in Applied Mathematics 35, Masson, Paris, 1994.

Diffusive energy balance models versus “horrendogram” of Earth System Science



Earth System Science Overview, *NASA Advisory Council*, 1986

Last paper on the subject:



PROCEEDINGS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

Multiple solutions and numerical analysis to the dynamic and stationary models coupling a delayed energy balance model involving latent heat and discontinuous albedo with a deep ocean

J. I. Díaz, A. Hidalgo and L. Tello

Proc. R. Soc. A 2014 470, 20140376, published 27 August 2014

$$\frac{\partial \gamma(U)}{\partial t} - \operatorname{div}(\nabla U) + w\frac{\partial U}{\partial z} \ni 0 \quad \text{in } (0,T)\times\Omega,$$

$$\frac{\partial u}{\partial t} - \operatorname{div}(|\nabla_{\mathcal{M}} u|^{p-2}\nabla_{\mathcal{M}} u) + \frac{\partial U}{\partial n} + F(x,\nabla_{\mathcal{M}} u) + G(t,x,u,u(t-\tau))$$

$$\ni \frac{1}{\rho c} QS(t,x)\beta(u) + f(t,x) \quad \text{in } (0,T)\times\mathcal{M},$$

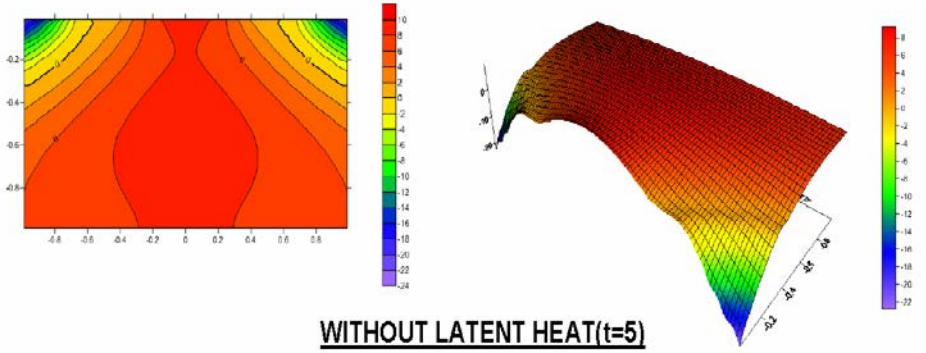
$$U|_{[0,T]\times\mathcal{M}} = u,$$

$$\hat{F}(x,\nabla_N U) + \frac{\partial U}{\partial z} = 0 \quad \text{in } (0,T)\times N,$$

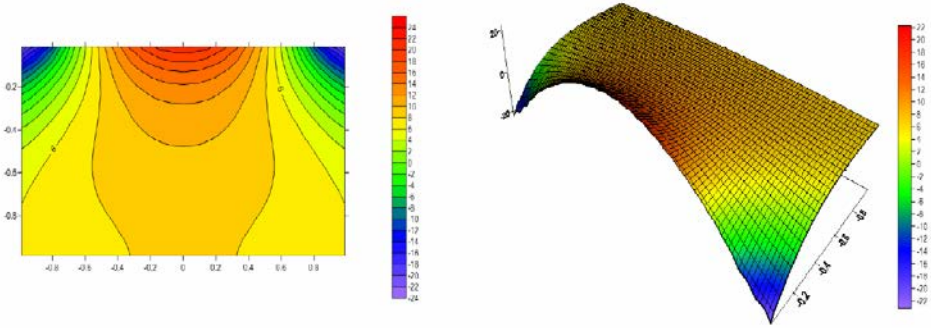
$$U(0,x,z) = U_0(x,z) \quad \text{in } \Omega,$$

$$u(s,\cdot) = u_0(s,\cdot) \quad \text{on } [-\tau,0]\times\mathcal{M}.$$

WITH LATENT HEAT (t=5)

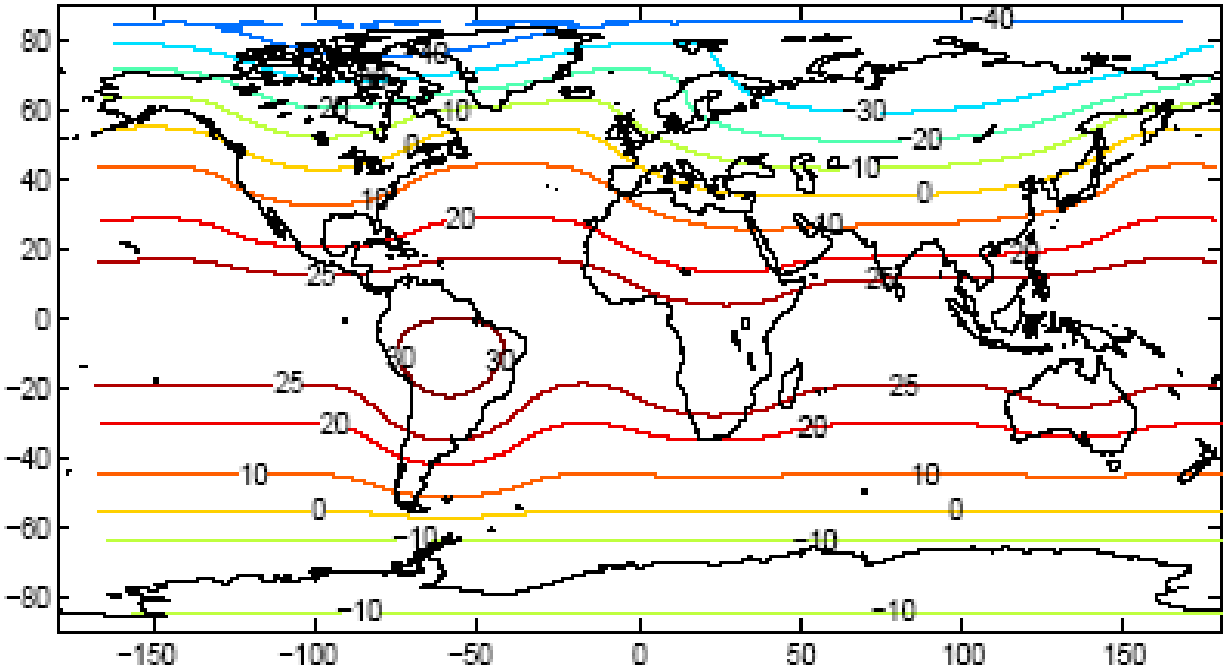


WITHOUT LATENT HEAT(t=5)



R. Bermejo, J. Carpio, J.I. Díaz, P. Galán

A **finite element** algorithm of a nonlinear diffusive climate energy balance model, *Pure and Applied Geophysics*, **165**, 2008, 1025-1048.



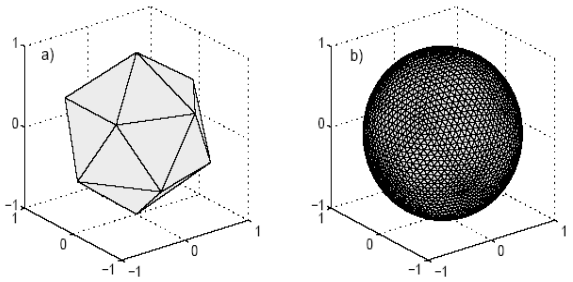


Figure 1: Initial Icosahedron and mesh after 4 refinements.

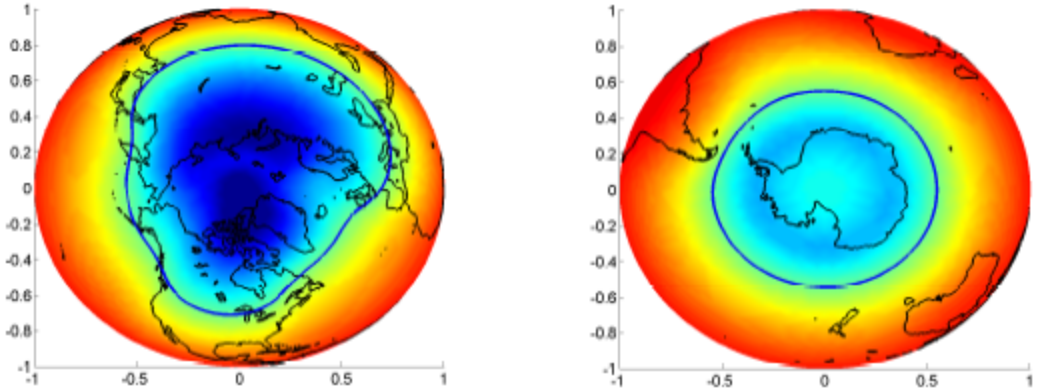


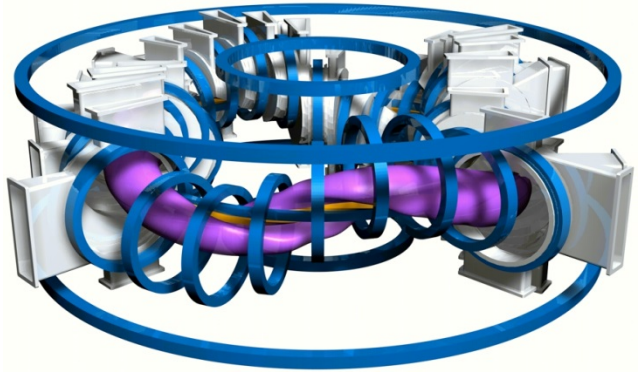
Figure 4: -2^oC January snow line. Left: northern hemisphere; right: southern hemisphere .

Joint “Académie des Sciences-Real Academia de Ciencias” workshop: "Environnement et Mathématiques», Paris, May 2002.

Special Issue of the Rev. R. Acad. Cien.Serie A Matem, **RACSAM** 96, nº 3, 2003 (**H. Brezis** and J. I. Díaz eds.)

4. Stellarators: Magnetic confinement of a fusion plasma

Stellarators avoid the disadvantages of a large current flowing in a tokamak plasma: plasma discharges lasting up to 30 minutes (shorter periods on tokamaks)



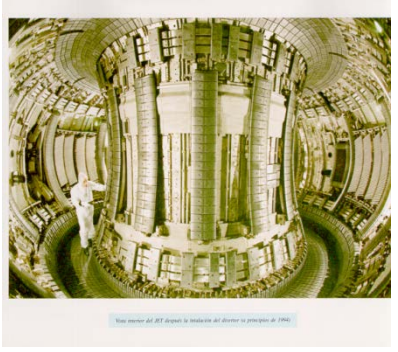
Heliac Flexible TJ-II: The largest currently operational European stellarator, located at CIEMAT (Madrid)

***C**entro de **I**nvestigaciones **E**nergéticas, **M**edioambientales y **T**ecnológicas [CEA, France]*

Operating since 1998

- April 1991: Telephone call from Spanish Ministry of Industry.
  - Enormous EURATOM grant to Spain (CIEMAT) [½(TJ-II budget].
  - No imitation of any other Stellarator in the world.
  - Needed of a mathematical model.

Mathematical modelling under toroidal symmetry for Tokamaks:
R. Temam (1975), H. Berestycki and H. Brezis (1980),…



Scientific adviser of CIEMAT (1991-1996)

Internal EURATOM-CIEMAT reports: 1991,1992,1993,1994

3 Ph.D. Thesis, 11 articles, 1 bock as editor, …

C. R. Acad. Sci. Paris, t. 317, Série I, p. 353-358, 1993

Équations aux dérivées partielles/*Partial Differential Equations*

On a two-dimensional stationary free boundary problem arising in the confinement of a plasma in a Stellarator

Jesus Ildefonso DIAZ and Jean-Michel RAKOTOSON

Averaging process, special coordinates system, ideal MHD, nonlocal nonlinear free boundary problem of Grad-Safranov type,…

To find: (u, F) $u\colon \Omega \to \mathbb{R}$, $F : \mathbb{R} \to \mathbb{R}^+ \cup \{0\}$ such that $F(s) = F_v$ for any $s \leq 0$ and satisfying

$$(\mathcal{P}) \begin{cases} -\Delta u + \beta(u\chi_\omega(x)) \ni a(x) F(u(x)) + \frac{1}{2}\left(F(u(x))^2\right)' + b(x)\, p'(u(x)) & \text{in } \Omega, \\ u - \gamma \in H_0^1(\Omega), \\ \int\limits_{\{x:u(x)>s\}} \frac{1}{2}\left(F(u(x))^2\right)' + b(x)\, p'(u(x))\, dx = j(s_+, \|u_+\|_{L^\infty(\Omega)}) \\ \text{for any } s \in \left[\operatorname{ess\,inf}_\Omega u, \operatorname{ess\,sup}_\Omega u\right], \end{cases}$$

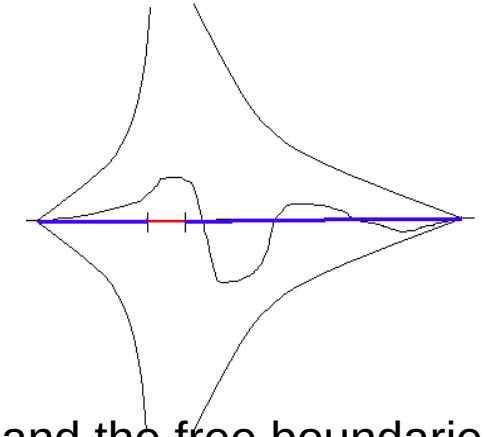
5. Control of nonlinear PDEs

C. R. Acad. Sci. Paris, t. 312, Série I, p. 519-522, 1991

Contrôle optimal/*Optimal Control*

Sur la contrôlabilité approchée des inéquations variationnelles et d'autres problèmes paraboliques non linéaires

Jesus Ildefonso Diaz



"Obstruction phenomenon", controllability of solutions and the free boundaries, von Neumman conjecture on climate modification, …

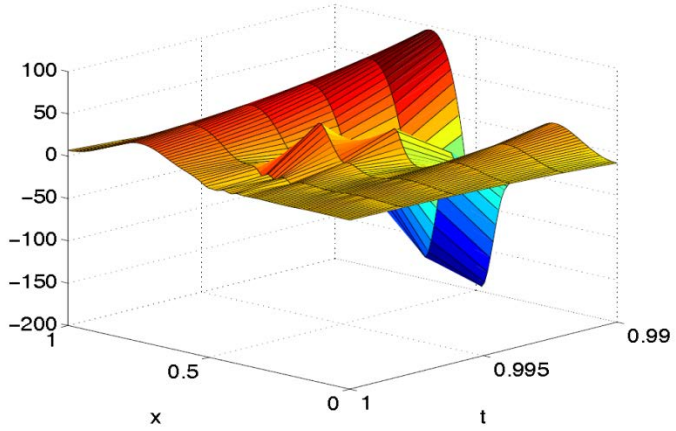
2 Ph.D. Thesis, 21 articles, plenary lecturer 16th IFIP "System Modelling and Optimization" July 5-9, 1993; Compiegne (France), …

J. I. Díaz and **J.L. Lions**. **Sur la contrôlabilté approchée de problèmes paraboliques avec phénomènes d'explosion**, Comptes Rendus Acad. Sci. Paris, t. 327, Série I, 173-177, 1998.

J. I. Díaz and **J. L. Lions**. **On the approximate controllability for some explosive parabolic problems**, *International Series of Numerical Mathematics*, Vol. 133, Birkhäuser Verlag, Basel, pp. 115-132, 1999.

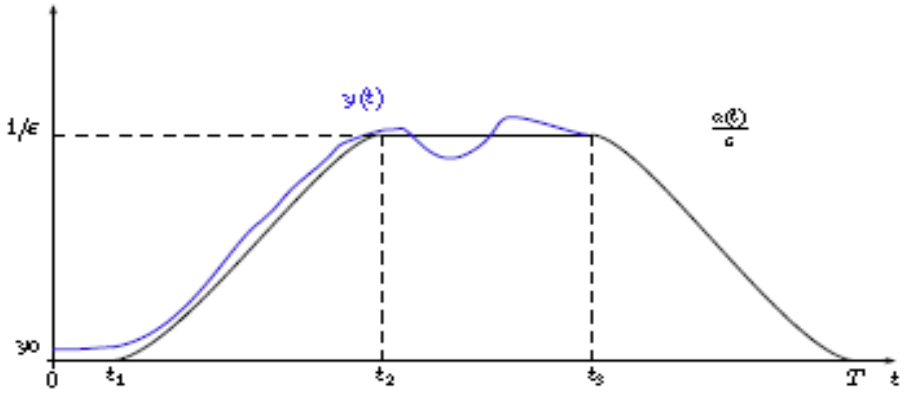
J. I. Díaz and **J. L. Lions**. **On the Approximate Controllability of Stackelberg-Nash Strategies**. In, Ocean Circulation and Pollution Control. A Mathematical and Numerical Inquiry, (J. I. Díaz ed.). Lecture Notes, EMS Volume, Proceedings of the Diderot Videoconference Amsterdam-Madrid-Venice, Lecture-Notes, Springer Verlag 2003, 17-28.

Numerical experiences on the control for non-Lipschitz semilinear problems



J.-M. Coron, J. I. Díaz, A. Drici and T. Mingazzini, "Global Null Controllability of the 1-Dimensional Nonlinear Slow Diffusion Equation", Chinese Annals of Mathematics, 34B(3), 2013, 333–344 (special issue devoted to J.L. Lions)

Non differentiable operators, beyond linearization: return method,…



6. Many other subjects in my relation with France

Some of my joint papers with other French colleagues:

- Ph. Benilan (Paris VI and Besançon): Nonlinear diffusion
- L. Veron (Tours): Free boundaries for conservation laws and energy methods
- P.L. Lions (Paris Dauphine, Collège de France): Steiner symmetrization
- F. Murat (CRNS-Paris VI): Renormailzed solutions
- J.M. Morel (ENS-Cachan): Singular nonlinear elliptic equations
- R. Dautray (CEA): Climate and environment
- M. Compte (Paris VI): Newton best obstacle problem
- E. Sanchez-Palencia (Paris VI): Shells, beyond distribution theory, …
- R. Glowinski (Paris VI, Houston): Bingham fluids
- P. Bégout (Toulouse): Schrödinger equations with a free boundary
- …

http://www. mat.ucm.es/~jidiaz

Inspiration in French research structures created by me in Spain:

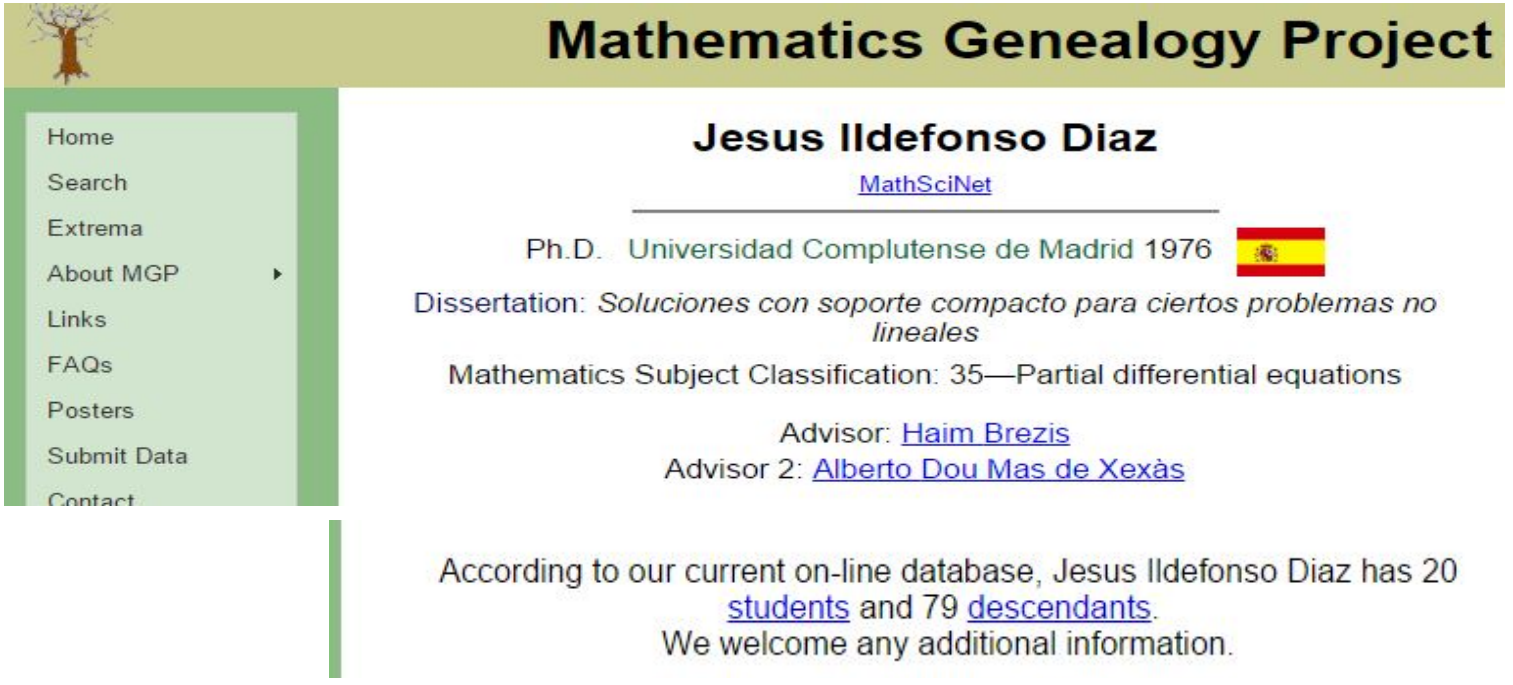
CEDYA (1978: now in its 24 edition):…..CANUM

SEMA (1991): SMAI

RACSAM (2000): CRAS,…

IMI (2005): INRIA (to a small scale)

Several French Ph.D. students and descendents



Merci
pour le Prix
et
pour votre attention