Two singular quasilinear equations on problems raised by Newton and Euler

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An international conference
in honour of **Juan Luis Vázquez**
on the occasion of his 60th birthday.

El Escorial (Spain), June 26-29, 2007

Plan

1. **On the Newton partially flat minimal resistance body type problems**

Joint work with Myriam Comte (PMC ParisVI)

2. **The Euler tallest column**

Joint work with Myrto Sauvageot (PMC ParisVI)

A

Juan Luis

con admiración

3. **Six years that (perhaps) shook the Spanish Mathematics**

1. On the Newton partially flat minimal resistance body type problems

Seventy years prior to the derivation of the conservation laws for a nonviscous compressible fluid by L. Euler in 1755, I. Newton introduced, in 1685, one of the pioneering problems in the Calculus of Variations:

find the shape of a symmetrical revolution body moving in a fluid with minimal resistance to motion.

I. Newton, Philosophiae Naturalis Principia Mathematica. 1686
(Proposition XXXIV (Book II)

As a matter of fact, the problem was already suggested by Galileo in his famous Discursi in 1638 (for a detailed history see H. H. Goldstine, A History of the Calculus of Variations from the 17th through the 19th Century, Springer-Verlag, Heidelberg, 1980).

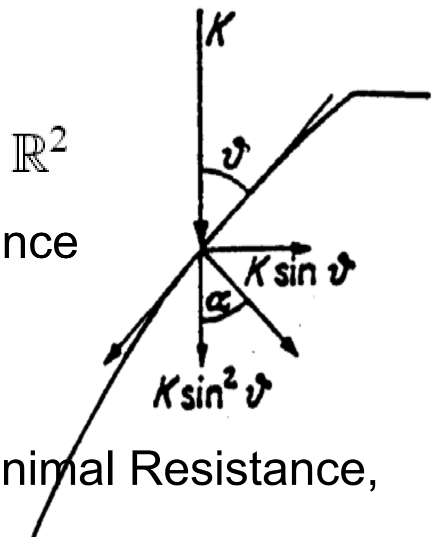
Newton was able to derive the resistance law of the body under the following assumptions

Firstly, he supposed that particles do not interact with each other, "a rare medium consisting of equal particles freely disposed at equal distances" and that each particle impacts the body at most once. Secondly, the impacts were assumed to be elastic and the resistance proportional to the impact angle.

If we write the problem in terms of a vertical flow, we can describe the body as

$$B = \{(x, z) : x \in \Omega,\ \ 0 \leq z \leq u(x)\},$$

with $u(x) = 0$ for $x \in \partial\Omega$ and for a given bottom set Ω in $\mathbb{R}^2$



In this framework it is not too difficult to show, see, for instance

E. Armanini, Sulla superficie di minima resitenza, Ann. mat. pura appl. (3) 4 (1900), pp. 131-148,

G. Buttazzo and B. Kawohl, On Newton's problem of Minimal Resistance, Mathematical Intelligencer, 15 (1993), pp. 7-12,

A. Wagner, A Remark on Newton's Resistance Formula, Z. Angew. math. Mech. ZAMM, 79 (1999), pp. 423-427,

that the total resistance of the body is proportional to the integral

$$I(u) = \int_\Omega \frac{1}{1 + |\nabla u(x)|^2}\, dx.$$

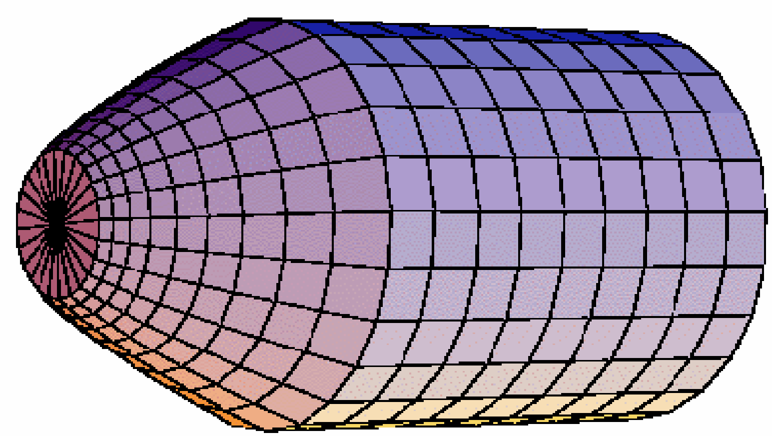
In the same historical book Newton also considered other resistance assumptions leading to different power expressions of the type

$$\int_\Omega \frac{1}{1 + |\nabla u(x)|^n}\, dx \qquad \text{with } n \geq 1.$$

In order to guarantee a single impact, it is common to assume the body to be concave. Nevertheless, some other profiles have been considered in the literature for the more general case in which any particle hitting the graph of u with vertical velocity does not hit again:

> M. Comte and T. Lachand-Robert, Newton's problem of the body of minimal resistance under a single-impact assumption, Calc. Var. 12, (2001), pp. 173-211 (see also T. Lachand-Robert and M. Peletier 2000, 2001,…).

It is worth mentioning that even though Newton's resistance model is only a crude approximation, it appears to provide good results in many contexts dealing, for instance, with a rarified gas in hypersonic aerodynamics. Many distinguished specialists in this area, von Karman, Ferrari, Lightill, and Sears have used this model (NASA report 1958)



In Newton's formulation one looks for a minimum of the functional (in the class of (suitably regular) functions satisfying two unilateral conditions

$$0 \leq u(x) \leq M \qquad \text{for } x \in \Omega.$$

Due to that fact, the associated Euler-Lagrange equations must be suitably understood. For instance, in terms of a variational inequality.

It can be shown that the Lagrange multiplier term associated with the unilateral condition $0 \leq u$ vanishes due the fact that the special form of the functionals leads to the concavity of any possible stationary point u(x) of that functional, and thus this unilateral condition is trivially satisfied once we assume the other (and crucial) unilateral condition $u \leq M$ (which, to the contrary, leads to a nonvanishing Lagrange multiplier term).

J. Eur. Math. Soc. 7, 395–411

JEMS

M. Comte · J. I. Díaz

On the Newton partially flat minimal resistance body type problems

It is worth mentioning that the integral in functionals is not globally convex in (although it is a convex function when $|\nabla u(x)| \geq \alpha$ for some suitable $\alpha > 0$)

Moreover it is not coercive (in fact it converges to zero when $|\nabla u(x)| \to +\infty$).

Those two facts arise quite often in many other special (but relevant) problems of the Calculus of Variations

B. Botteron and P. Marcellini, A general approach to the existence of minimizers of one-dimensional non-coercive integrals of the calculus of variations, Ann. Inst. Henri Poincaré, 8 (1991), pp. 197-223.

This motivates us to consider a general class of functionals (being invariant by symmetrical changes of coordinates) of the form

$$\int_\Omega F(|\nabla u(x)|)\, dx$$

with

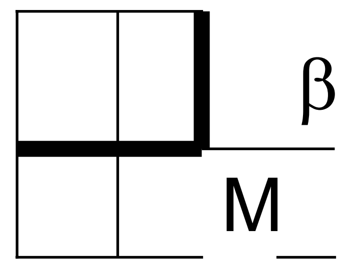
$$F(|\nabla u|) \to 0 \quad \text{as } |\nabla u| \to +\infty$$

Actually, we shall not deal with the associated minimization problem, but with the more general case of the Euler-Lagrange variational inequality satisfied by any stationary point u fulfilling the unilateral constraint u ≤M. So, given M>0, we shall consider a class of quasilinear obstacle problems which can be formulated as follows

$$(OP)\qquad \begin{cases} -\operatorname{div}(A(|\nabla u|)\nabla u) + \beta(u) \ni 0 & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

where β is the maximal monotone graph

$$\begin{cases} \beta(u) = \{0\} & \text{if } u < M, \\ \beta(M) = [0, +\infty), & \\ \beta(u) = \emptyset & \text{if } u > M, \end{cases}$$



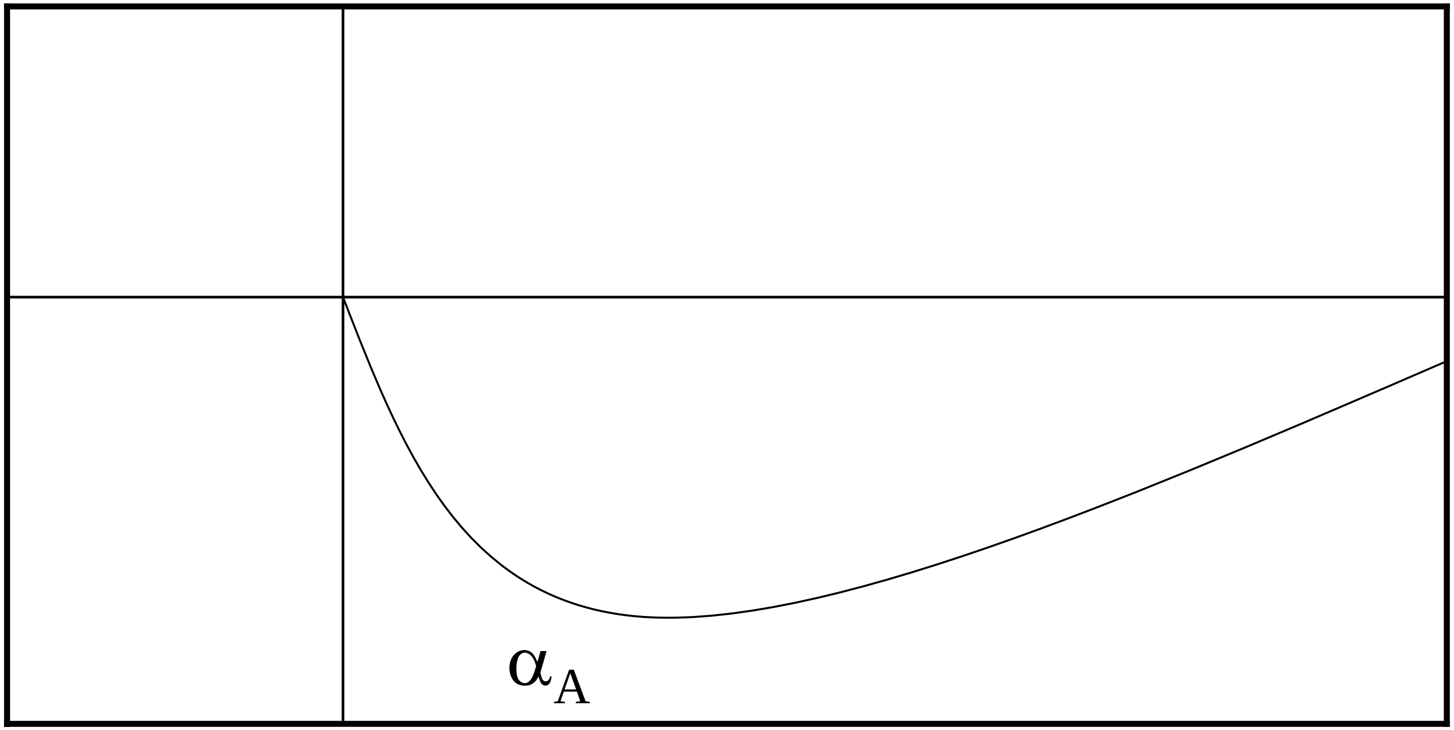
and $A \in C^1(0, +\infty)$ satisfies the following set of assumptions: there exists $\alpha_A \geq 0$ such that

the function $t \mapsto tA(t)$ is decreasing on $(0, \alpha_A)$ and increasing on $(\alpha_A, +\infty)$,

$$A < 0 \quad \text{on } [0, +\infty) \quad \text{and} \quad \lim_{t \to +\infty} tA(t) = 0,$$

$$\lim_{a \to +\infty} A(a) \int_{\alpha_A}^{a} \frac{d\tau}{\tau A(\tau)} < 1.$$

Other alternative assumptions on A are also considered in the paper.


α_A

Example In the classical Newton obstacle problem, we search

$$\operatorname*{Min}_{K} \int_{\Omega} F(|\nabla u|)\, dx, \qquad F(t) = (1+t^2)^{-1},$$

where $K = \{u \in H_0^1(\Omega) : u \leq M \text{ and } u \text{ concave}\}$. Thus, the associated Euler–Lagrange formulation (in terms of maximal monotone graphs) is given by

$$-\operatorname{div}\left(F'(|\nabla u|) \frac{\nabla u}{|\nabla u|} \right) + \beta(u) \ni 0.$$

In order to simplify the presentation for nonnegative functions $u(x)$, we notice that in the radial case and for nonincreasing functions $u = u(r)$, $r = |x|$, we have

$$-\operatorname{div}\left(F'(|\nabla u|) \frac{\nabla u}{|\nabla u|} \right) = r(F'(|u'(r)|))'.$$

In particular, we can identify the above Euler–Lagrange equation
by choosing

$$A(t)t = F'(t) = \frac{-2t}{(1+t^2)^2}$$

and so

$$A(t) = \frac{-2}{(1+t^2)^2}$$

It is easy to see that the function A satisfies the assumptions
In particular

$$\lim_{a\to+\infty} A(a)\int_{\alpha}^{a}\frac{d\tau}{\tau A(\tau)}=\frac{1}{4}.$$

In order to establish the existence of solutions for this type of noncoercive problems, several different additional conditions have been introduced in the literature: mainly the *concavity* of u :

G. Buttazzo, V. Ferone and B. Kawohl, Minimum Problems over Sets of Concave Functions and Related Questions, Math. Nachrichten, 173 (1993), pp. 71--89.

Here we shall deal with solutions of the obstacle problem (OP) in the class of functions such that

$$u\in H_0^1(\Omega)\quad\text{and}\quad|\nabla u(x)|\geq\alpha_A\quad\text{if } u(x)<M.$$

In the first part of our paper we first consider the radial case

Given $M > 0$, find $\rho \in (0, R)$ and u satisfying

$$\begin{cases} -\frac{1}{r}\frac{d}{dr}(rA(|u'|)u') + \beta(u) \ni 0 & \text{in } (0, R), \\ u(R) = 0, \quad -u'(R^-) > \alpha_A, \\ u(r) = M \quad \text{in } [0, \rho] \quad \text{and} \quad u(r) < M & \text{in } (\rho, R]. \end{cases}$$

Our study mainly focus on the properties of the coincidence set:

Theorem *Let $R > 0$ be given. Assume that A satisfies the conditions Then, for every $M > 0$, there exists $\rho_M \in (0, R)$ such that for any $m \in (\alpha_A, +\infty)$ there exists a solution $u(r) = u(r; m)$ of the obstacle problem satisfying*

(i) *$u \equiv M$ in $[0, \rho_m]$ for some $\rho_m \in [\rho_M, R)$,*

(ii) *$-u'(\rho_m) = m$,*

(iii) *u is strictly concave in (ρ_m, R) and $u \in W^{1,\infty}(0, R)$.*

Remark In the Newton case, $\alpha_A = 1/\sqrt{3}$, and the solution of the obstacle problem which corresponds to a minimum of energy is associated with $m = 1$:

Newton, I.: Philosophiae Naturalis Principia Mathematica. (1686)

Goldstine, H. H.: A History of the Calculus of Variations from the 17th through the 19th Century. Springer, Heidelberg (1980) Zbl 0452.49002 MR 0601774

Idea of the proof (*Hodograph formulation*). We have

$$rA(|u'(r)|)u'(r) = RA(|u'(R)|)u'(R).$$

Since the value of $u'(R^-)$ will play a crucial role, we introduce $a = -u'(R^-)$. As u is concave, a will be the maximal value of the function $-u'(r)$ in the interval (ρ, R). Now, the idea is to take a new parameter to solve the differential equation. We use $t = -u'(r)$ and observe that, since u is assumed to be concave decreasing, $t > 0$ and $-u'$ is bijective.

$$-rA(t)t = -RA(a)a$$

so that

$$r = \frac{RaA(a)}{tA(t)} = r(t).$$

That is,

$$u(t) = Ra - Ra\frac{A(a)}{A(t)} - RaA(a)\int_t^a \frac{d\tau}{\tau A(\tau)}.$$

Thus, the problem can be rephrased as follows: let $\rho \in (0, R)$ and $M \geq 0$ be given.

We want to find $a > \alpha$ such that there exists $t(a)$ with

$$\rho t(a) A(t(a)) = R a A(a)$$

and

$$M = Ra - Ra\frac{A(a)}{A(t)} - RaA(a)\int_t^a \frac{d\tau}{\tau A(\tau)}. \qquad (*)$$

We have the following theorem:

Theorem *Let $M > 0$ be a given number. there exists ρ_M such that* (*) *admits a solution if ρ belongs to (ρ_M, R), and none if $\rho \in (0, \rho_M)$.*

We also deal with the property of minimum solutions. We prove that although there is not a unique solution to problem there is an unique radial minimizer for the functional in the class of solutions of the associated problem.

Theorem *Let $M > 0$ be given. Then there exists a unique value of $m \in (\alpha_A, +\infty)$ such that the solution $u(r; m)$ given in Theorem is minimal among all solutions given in that theorem.*

Remarks.

In the Newton case, P. Guasoni (1996) established in the existence of a function in which is not radially symmetric, for which the value of the functional is smaller than any value arising from a radial function.

On the other hand, T. Lachand-Robert and É. Oudet (2005) found numerically another function leading to an even smaller value of the functional.
We conjecture that similarly, radial solutions are not minimizers either for the more general class of functions F considered in this paper.

Another consequence of our results is that they reveal some kind of optimality of the structure assumptions made in the regularity result by H. Brezis and D. Kinderlehrer.

Brezis, H., Kinderlehrer, D.: The smoothness of solutions to nonlinear variational inequalities. Indiana Univ. Math. J. **23**, 831–844 (1974) Zbl 0278.49011 MR 0361436

if the quasilinear operator A is "locally coercive", the solution of the associated obstacle problem fulfils $u \in W^{2,s}$ for every $1 < s < +\infty$. In contrast, we show here that the solutions of (OP) are not of class C^1.

In a second part of our paper we study the coincidence set (the flat region of the body) without any symmetry assumption.

In particular, we give answer to a question raised by Buttazo-Kawohl (1993).

2. The Euler tailer column

J. I. Díaz and M. Sauvageot: On the Euler best column: a singular non local quasilinear equation with a boundary blowing up flux condition, *CD-Rom Actas XIX CEDYA / IX CMA*, Univ. Carlos III, Madrid September, 2005 + Paper in preparation.

$$P(A,B)\begin{cases}\left[\dfrac{A(u)}{u_x^3}\right]_x + (B(u)+\Lambda)u = 0 \quad \text{in } (0,1),\\[2ex] u(0)=0,\ u'(1)=+\infty, \qquad \Lambda = 4k/\lambda\end{cases}$$

$$A(u)(x) = \left(\mu + \int_0^x u(t)^2 dt\right)^2$$

for some $\mu > 0$, and $\quad B(u)(x) = \int_x^1 \frac{1}{u_x(t)^2}\left(\mu + \int_0^x u(t)^2 dt\right) dt\,.$

Remark: Quasilinear operator like the p-Laplacian with p<0 (some similar facts with the Newton partially flat minimal resistance body problem)

Leonard Euler, Sur la force des colonnes, Académie Royale des Sciences et Belles Lettres, Berlin, 1757. Also in Leonhardi Euleri Opera Omnia, Scientiarum Naturalium Helveticae edenda curvaverunt F. Rudio, A. Krazer, P. Stackel. Lipsiae et Berolini, Typis et in aedibus B. G. Teubneri, 1911-.

Problem: For a given volume V of a given materiel, what is the height of the tallest possible column ? (not bending, not falling).

Solved previously by Euler for cylindric columns (after Leonardo da Vinci) and prismatic columns.

Question: Shape leading to the maximal height ?

Equation at loss of equilibrium: Bernoulli-Euler theory

$$\gamma A(z)^2 y_{zz} = \int_z^H \rho g A(z')[y(z') - y(z)] + P[y(H) - y(z)]$$

H = height of the column;

γ = parameter related to the elasticity;

P = load at the top of the column;

$A(z)$ = area at level $z \in [0, H]$;

$y(z)$ = lateral deflexion at level $z \in [0, H]$.

J. B. Keller and F. I. Niordson, The Tallest Column, Journal of Mathematics and Mechanics, 16, No.5, (1966), 433-446.

Rescaling and change of variables:

New parameters:

$$\lambda = \rho g H^4 / \alpha E V \qquad k = P/\rho g V \qquad \left(V = \int_0^H A(z)dz \right)$$

New variables:

$$x = z/H \in [0,1] \begin{cases} a(x) = H\,A(xH)/V & \int_0^1 a(x)dx = 1 \\ \eta(x) = y(xH)/H & u = \eta_x \end{cases}$$

Rescaled Euler's problem:

$$\left[a(x)^2 u_x(x)\right]_x + \lambda u(x) \left(\int_x^1 a(y)dy + k \right) = 0$$

$$u(0) = 0\,, \ \lim_{x \to 1} a(x)^2 u_x(x) = 0$$

$$Find\ \lambda_{max} = \max\{\lambda \,/\, \exists\,(a,u)\,,\ u \geq 0\,,\ u \not\equiv 0\}\,.$$

Reformulating Euler's best column problem :

$$\textit{Find } a \in W^{1,1}([0,1]),\ a \geq 0,\ \int_0^1 a(x)dx = 1$$
$$\textit{which maximizes } \lambda(a) :$$

$$\text{Given } a(x),\ \exists\,?\ \begin{cases} \lambda = \lambda(a) \\ u \geq 0\,,\ u \not\equiv 0 \end{cases} \quad \text{s.t.}$$

$$[E(a)] \qquad \begin{cases} \left[a(x)^2 u_x(x)\right]_x + \lambda u(x)\left(\int_x^1 a(y)dy + k\right) = 0 \\ \\ u(0) = 0\,,\ \lim_{x\to 1} a(x)^2 u_x(x) = 0 \end{cases}$$

Keller-Niordson assumption :

$\lambda(a)$ depend smoothly on a.

According to S.J. Cox & C.M. McCarthy (The Shape of the Tallest Column, Siam J. Math. Anal. 29, n° 3 (1998), 547-554) this property would fail in the unloaded case P=0, i.e. k=0

$\lambda(a)$ as a smallest eigenvalue

for the quadratic form $Q_a(u) = \int_0^1 a(x)|u_x(x)|^2 dx$

$$u \in W^{1,1}_{loc} \cap L^2([0,1])$$

on the space $L^2([0,1])$ with norm $||u||_a$:

$$||u||_a^2 = \int_0^1 |u(x)|^2 \left(\int_x^1 a(t)dt + k \right) dx\,.$$

Lemma 1. *Suppose that* $\exists u\,,\ R_a(u) = \lambda(a)$.

1/ There exists a unique minimizer $u(a)$ *s.t.* $u(a) \geq 0$, $u(a)_x \geq 0$, $||u(a)||_a = 1$.

2/ The maps $\alpha \to \lambda(\alpha)$ *and* $\alpha \to u(\alpha)$ *are smooth in a neighbourhood of* a.

$$\lambda_{max} = \sup_{a} \inf_{\substack{u \neq 0 \\ u(0)=0}} \frac{\int_0^1 a(x)^2 u_x(x)^2 dx}{\int_0^1 |u(x)|^2 \left(\int_x^1 a(t)dt + k \right) dx}.$$

Problem : given a, $\exists$? a minimizer u(a) for the Rayleigh quotient ?

Yes if $a > 0$ on $[0,1[$, $a_x \sim_{x \to 1} (1-x)^q$ with $q < -1$.

No if $a_x \sim_{x \to 1} (1-x)^2$ (unloaded case $k = 0$).

If yes, $\alpha \longrightarrow \lambda(\alpha)$ and $\alpha \longrightarrow u(\alpha)$ are smooth in a neighbourhood of a.

The optimal shape a corresponding to λ_{max} should satisfy

$$D\lambda(a) = 0 \ :$$

$$[DE(a)] \qquad \exists \mu > 0\,,\ a\, u_x^2 = \frac{\lambda}{2}\left(\int_0^x u(y)^2 dy + \mu\right)\,.$$

Eliminate a**:** $[E(a)] + [DE(a)] \Rightarrow$

$$P(A,B)\ \begin{cases} \left[\dfrac{A(u)}{u_x^3}\right]_x + (B(u)+\Lambda)u = 0 \quad \text{in } (0,1), \\ u(0)=0,\ u'(1)=+\infty, \qquad \Lambda = 4k/\lambda \end{cases}$$

$$A(u)(x) = \left(\mu + \int_0^x u(t)^2 dt\right)^2$$

for some $\mu > 0$, and $\quad B(u)(x) = \displaystyle\int_x^1 \frac{1}{u_x(t)^2}\left(\mu + \int_0^x u(t)^2 dt\right) dt\,.$

Conversely :

Let $u \in L^2([0,1])$ $u \geq 0$ a weak solution of $P(A,B)$.
Set

$$\lambda = 2\left(\int_0^1 u_x^{-2}\left(\mu + \int_0^x u^2(t)dt\right)dx\right)^{-1}$$

$$a(x) = \frac{\lambda}{2}u_x^{-2}\left(\mu + \int_0^x u(t)^2 dt\right)$$

Then (λ, u) satsisfy both Equations $[E(a)]$ and $[DE(a)]$.

Statement of the result

Theorem *Assume that*

$$\Lambda > \frac{9\pi^2}{128}.$$

Then Problem $P(A, B)$ *admits a solution* u *with* $u \in W^{1,p}(0,1)$ *for any* $p \in [1,3)$ *and such that,*

$$u_x(x) = O\left(\frac{1}{(1-x)^{1/3}}\right) \text{ near } x = 1.$$

In particular, $u \notin W^{1,3}(0,1)$.

To be proved by iteration. For given u_{n-1}, we define

$$A_n = A(u_{n-1}), \;\; B_n = B(u_{n-1}),$$

u_n constructed as a variational solution of

$$(P_n) \quad \begin{cases} \left[\dfrac{A_n}{u_x^3}\right]_x + (B_n + \Lambda)u = 0 & \text{in } (0,1), \\ u(0) = 0, \;\; u'(1) = +\infty, \end{cases}$$

Steps of the proof:

1/ Prove that the *singular* problem (P_n) has a well defined bounded variational solution.

2/ Show that under condition (3) on Λ, the iterated sequences $\{A_n\}$, $\{B_n\}$ and $\{u_n\}$ are equicontinuous.

3/ Prove that $\{u_n\}$ converges (in a suitable functional space) to a solution u of $P(A, B)$.

Solving the singular system

$$\begin{cases} \left[\dfrac{a(x)}{u_x^3}\right]_x + b(x)u(x) = 0 \quad \text{in } (0,1), \\ u(0) = 0, \; u'(1) = +\infty, \end{cases}$$

with $a, b \in C(0,1)$, $a(x) > \mu > 0$ and $b(x) > 0$.

Consider the functional $J : \mathcal{K} \rightarrow]0, +\infty]$ of the type

$$J(u) = \int_0^1 \frac{a(x)}{u_x(x)^2} dx + \int_0^1 b(x)u(x)^2 dx$$

on the convex cone

$$\mathcal{K} = \{ \, u \in W^{1,1}_{loc}(0,1), \; u \geq 0, \; u_x \geq 0, \; u(0) = 0 \, \}$$

Proposition

1/ There exists a unique minimizer u for J on $\mathcal{K}$.

2/ This minimizer u satisfies the equation

$$\frac{a(x)}{u_x(x)^3} = \int_x^1 b(t)u(t)dt$$

Consequently, the boundary flux blowing-up condition $\lim_{x\to 1} u'(x) = +\infty$ holds.

3/ u is a weak solution

Remark u is convex $(u_{xx} \geq 0)$ whenever $a(x)$ is assumed to be nondecreasing.

Equicontinuity of the iterated A_n, B_n and u_n.

Lemma *Assume* $\Lambda > \frac{9\pi^2}{128}$. *Then* $\{u_n\}$ *is bounded in* $L^2(0,1)$.

In fact,

$$u_n(x)^{1/3} u_{nx}(x) \le \frac{A_n(1)^{1/3}}{\Lambda^{1/3}} (1-x)^{-1/3}.$$

and, integrating between 0 and x

$$u_n(x)^{4/3} \le 2 \frac{A_n(1)^{1/3}}{\Lambda^{1/3}} \left[1 - (1-x)^{2/3}\right]$$

with $A_n(1) = (\mu + ||u_{n-1}||_2^2)^2 \;\Rightarrow ||u_n||_2^2 \le \alpha(\mu + ||u_{n-1}||_2^2)$

with $\alpha < 1$ iff $\Lambda > \frac{9\pi^2}{128}$. ■

Proposition *Assume* $\Lambda > \frac{9\pi^2}{128}$. *The sequence* $\{u_n^{4/3}\}$ *(and consequently the sequence* $\{u_n\}$*) is bounded and equicontinuous in* $C([0,1])$.

We prove then

Lemma *There exist* $d_1, d_2 > 0$ *s.t.*

$$\frac{d_1}{(1-x)^{1/3}} \leq u_{nx}(x) \leq \frac{d_2}{(1-x)^{1/3}}, \quad a.e. \ x \in (0,1)$$

Proposition *Assume* $\Lambda > \frac{9\pi^2}{128}$. *Then the sequences* $\{A_n\}$ *and* $\{B_n\}$ *are bounded and equicontinuous in* $C([0,1])$.

Complementary properties :

- u is convex
- $\exists \gamma > 0$ s.t. $\begin{cases} u(x) \sim \gamma(1-x)^{2/3} \\ u_x(x) \sim \frac{2\gamma}{3(1-x)^{1/3}} \end{cases}$ as $x \to 1$.

Coming back to the best column problem :

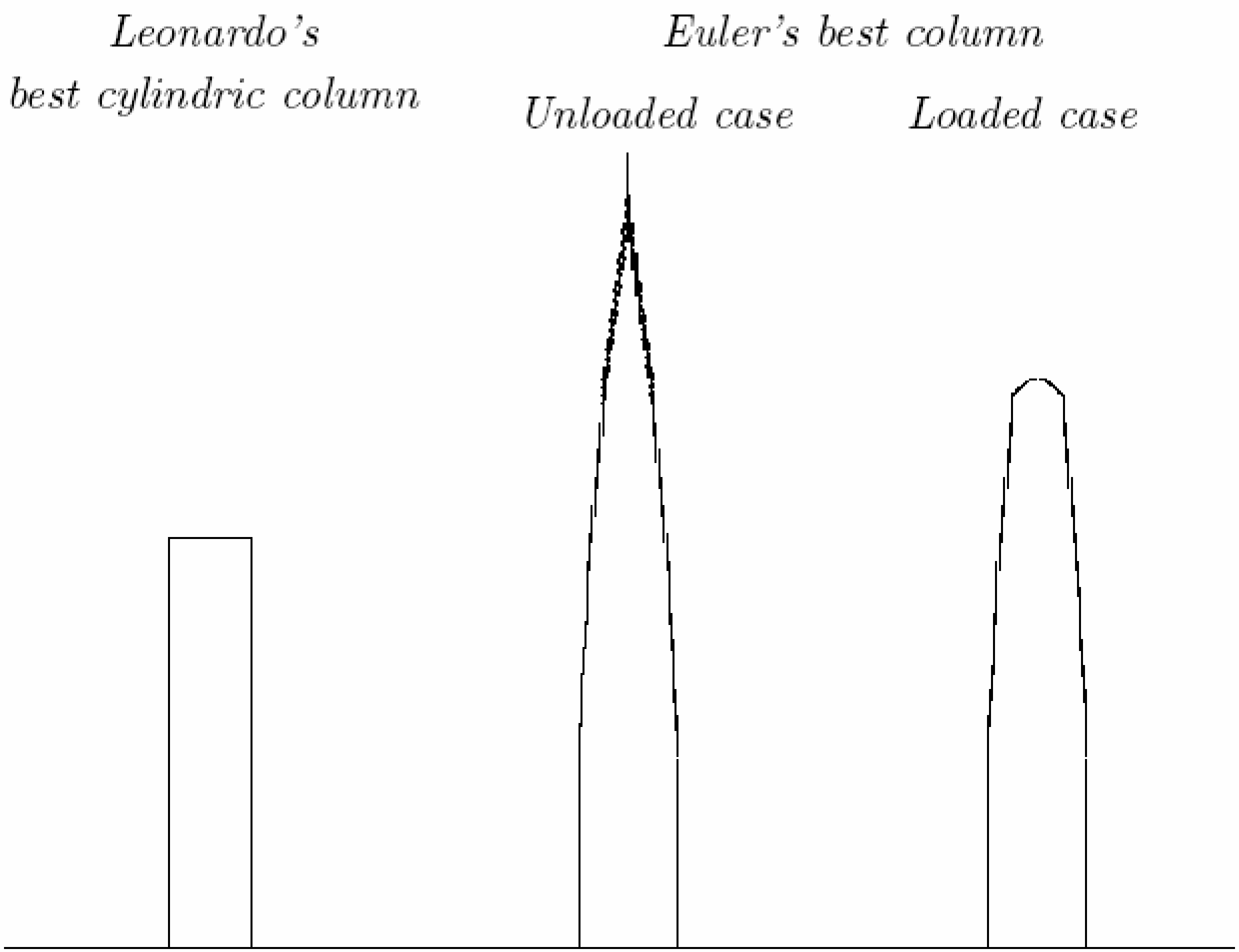
- Corresponding shape : $a(x) \sim_{x\to 1} c(1-x)^{2/3}$: round top
- The smallest eigenvalue problem for $\lambda(a)$ holds true.
- $$H = H_{cyl} \times \left(\frac{4}{B(u)(0)}\right)^{1/4}.$$

  (H_{cyl}=height of the best cylindric column).

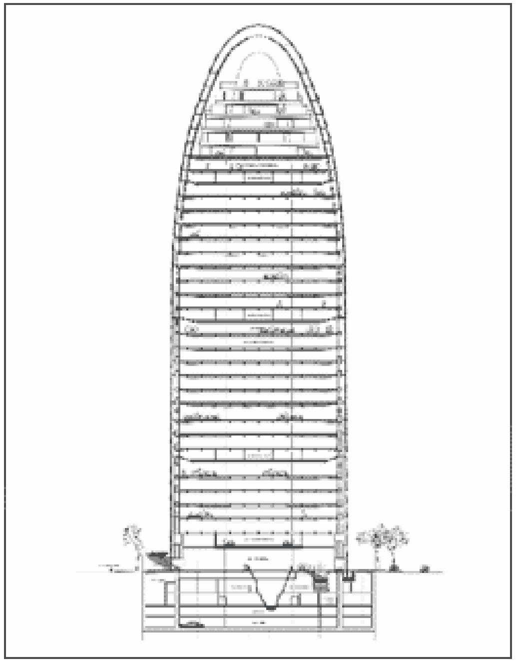
Numerical computation (Keller and Niordson):

Unloaded case : $H \sim 2 \times H_{cyl}$.

Infinite load : $H \sim 1.2 \times H_{cyl}$.



Agbar Tour, Barcelona 2004
Jean Nouvel



Some recent results:

A: On the uniqueness of solutions

Theorem

If the load P is large enough, then the solution to Problem $P(A, B)$ is unique.

B: Comparison of rearrangements (non radially symmetric case).

3. Six years that (perhaps) shook the Spanish Mathematics **1973-1979**

Mathematics, UCM, June 1973: A singular promotion

J,L,Vázquez (last years of Ingeniero de Telecomunicación, UPM)

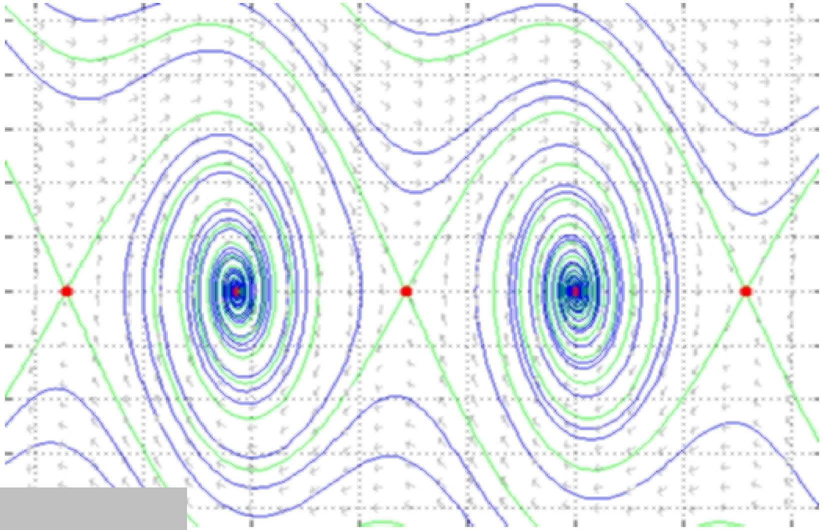
M. A. Herrero, S.J. Álvarez, JID,

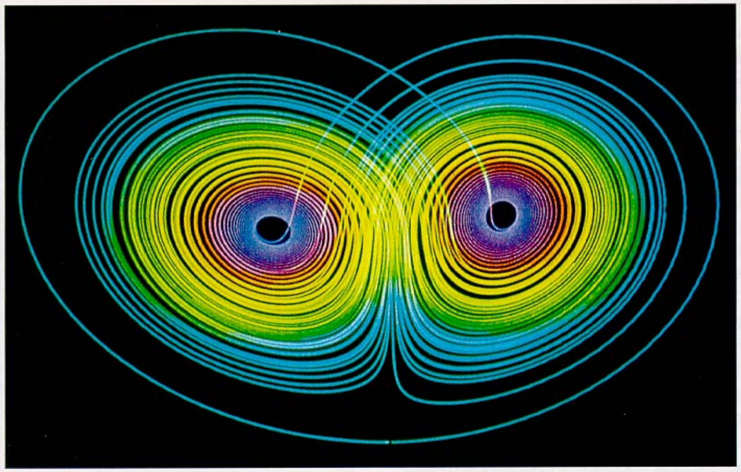
R. Moriyon, J.L. González-Llavona, J.M. Rodriguez Sanjurjo,
M. Rodriguez Artalejo, M.J. Ríos, P.Jimenez-Guerra,….

More than 20 Full and Associate Professors in different universities of Spain

A not so strange atractor:

the theory of nonlinear PDEs





El atractor de Lorenz. Ilustración de Chaos and Fractals. New Frontiers of Science. Edit. Springer-Verlag.

Two different initial paths:

JID: Partial Differential Equations

Alberto Dou

Departamento de Ecuaciones Funcionales,

Facultad de Matemáticas, UCM

J.L. Vázquez: Differential Topology

Enrique Outerelo

Departamento de Geometría y Topología,

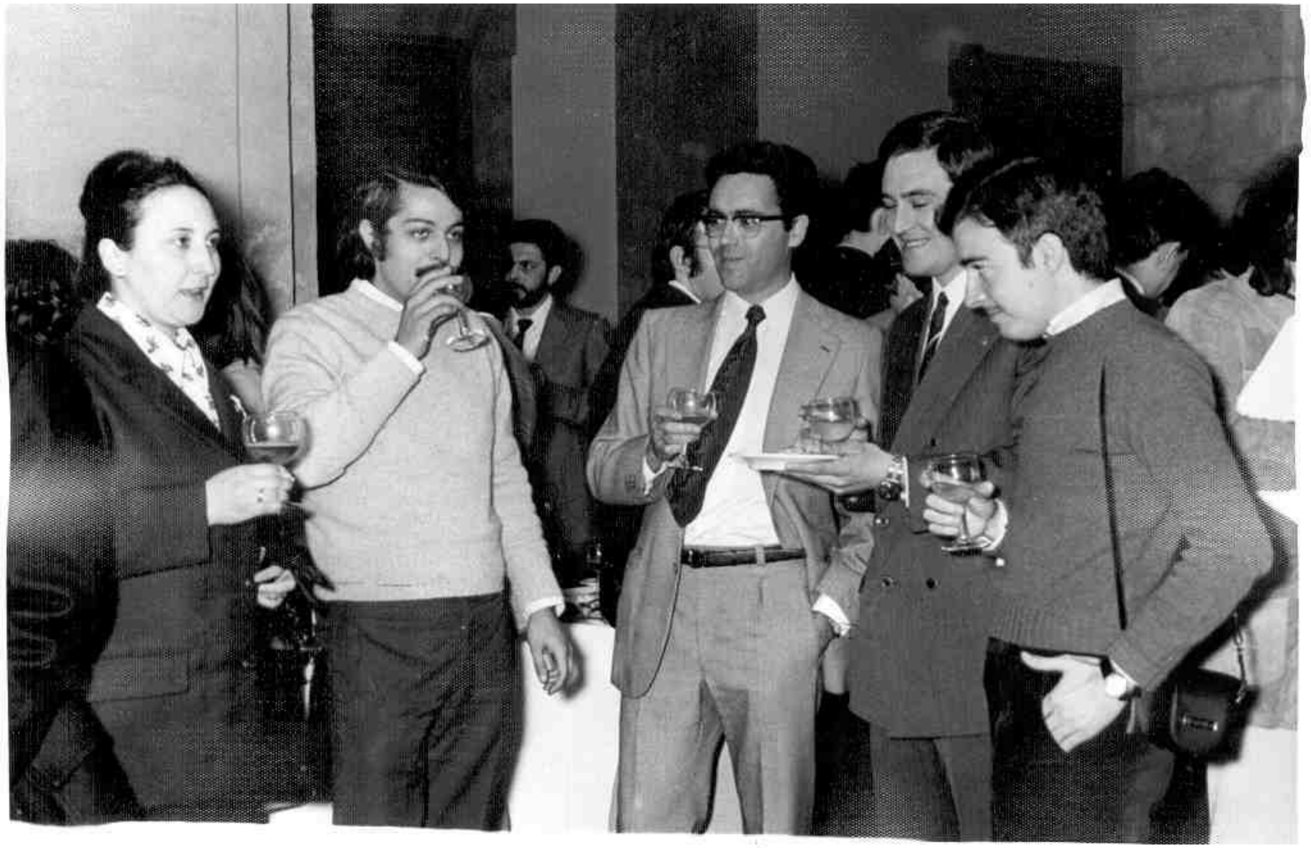
Facultad de Matemáticas, UCM

On the *Departamento de Ecuaciones Funcionales,*

Facultad de Matemáticas, UCM

PDE: Dou, J.L. Andrés Yebra, C. Fernández Pérez (other students of J.L. Lions)
A.Valle, M. Lobo

Harmonic Analysis: Guzmán, I. Peral, M Walias, T. Menarguez,....



J. I. Díaz
Soluciones con soporte compacto para ciertos problemas no lineales.
Tesis Doctoral. Departamento de Ecuaciones Funcionales, Facultad de Matemáticas, UCM, 16 de Octubre de 1976.

A. Dou (official Director) but: **Haïm Brezis** (and Philppe Benilan)

Role of the Director: adviser /Maitre/...

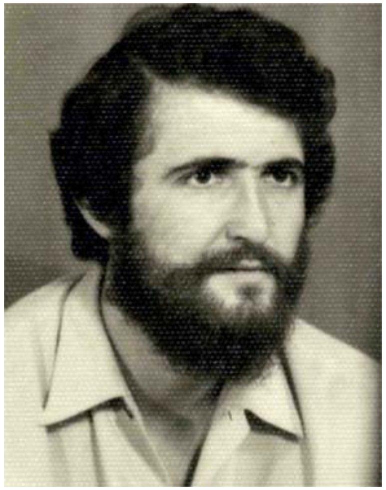
In my case: public knowledgement of the influence of A. Dou

H. Brezis (ENS,.....)

International meeting on nonlinear monotone operators. Brussels , Belgium (organized by the NATO), September, 11 al 30, 1975.

Postgraduate courses: 1977/1978, ...

Invitations to my friends and colleagues



J.L. Vázquez, M.A. Herrero, S.J. Alvarez,
G. Díaz, J. Carrillo,F. Bernis,...

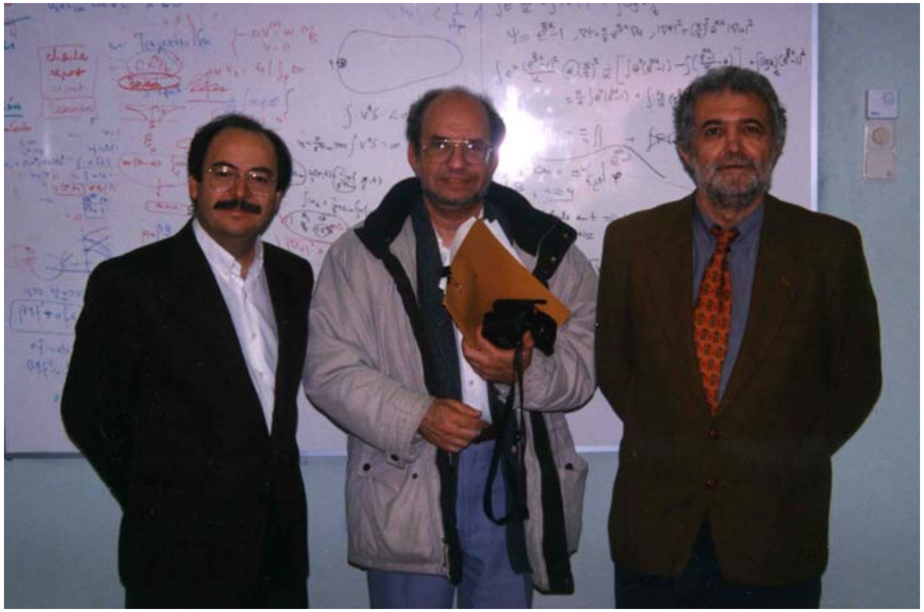
J.L. Vázquez words in CONTRIBUCIONES MATEMÁTICAS. Homenaje al profesor. Enrique **Outerelo** Domínguez. Publicaciones de la UCM, Madrid, 2004.

A comienzos de los años 1970, cuando yo era estudiante de Ciencias Matemáticas en la Universidad Complutense de Madrid, Enrique Outerelo reunió en un seminario a un grupo de jóvenes estudiantes, unos ya doctorandos y otros aún no licenciados, para contarnos las noticias que venían de California, del mítico IMPA del Brasil y de la URSS, a saber, la teoría de los Sistemas Dinámicos y las obras de Steve Smale, Mauricio Peixoto, Jacob Palis, Dmitry V. Anosov, Yakov G. Sinai y toda una pléyade de brillantes matemáticos. Nosotros, a quienes él había instilado el gusto de la topología abstracta, que para mí es aún hoy día una ciencia hermosa y distante, íbamos a sentir un gran cambio en nuestras vidas matemáticas que aún no presentíamos.

Hacia 1975 empecé a creer que la idea de Enrique de estudiar sistemas dinámicos abstractos era un tanto abstracta para mi gusto y mi pasado de ingeniero. En 1976 encontré la que al parecer era mi orientación en el mundo de las ecuaciones en derivadas parciales no lineales y en el Departamento de Ecuaciones Funcionales. Se cerraba una etapa y durante 3 años estudié ecuaciones elípticas no lineales que me condujeron a una tesis doctoral y un futuro en el mundo de las ecuaciones. Bien afincado en

J. L. Vazquez (sentences on the influence of JID in his career at Gaeta meeting, 2004): thanks Juan Luis !!!!

I introduced him to Haïm Brezis,…



and to Philpe Benilan, …

Journées d'Analyse non lineaire. Besançon, France, June, 6-9, 1977.

I CEDYA (Congreso de Ecuaciones Diferenciales y Aplicaciones), El Escorial (Madrid) (organized by A.Casal, J. I. Díaz, M.Lobo and J.Hernández), May, 29-31,1978.

Today:

Motivation for the creation of SEMA:

SeMA
Sociedad Española de Matemática Aplicada

Chinchón (1991), Comisión Gestora 1991: Valle, Díaz, Bermudez, Simó, Sanz-Serna, *Role of JID*: Sede oficial (UCM), First Secretary (Estatutos, Legal inscription,…)

Presidents:

D. Antonio Valle (1993-1994), Secretario JID

Jesús Ildefonso Díaz, de la Universidad Complutense de Madrid, en el período 1994-1995.

Juan Luis Vázquez Suárez, 1996-1998

End of the magic period: **1973-1979**

J.L. Vázquez. "Existencia, unicidad y propiedades de algunas ecuaciones en derivadas parciales semilineales". Univ. Complutense de Madrid, Febrero 1979.

M.A. Herrero. "Comportamiento de las soluciones de ciertos problemas no lineales sobre dominios no acotados". Univ. Complutense de Madrid, Febrero 1979.

G. Díaz. "Problemas en ecuaciones en derivadas parciales con no linealidades sobre operadores diferenciales de segundo orden". Univ. Complutense de Madrid, Junio 1980.

J. Carrillo. "Un problema no lineal con frontera libre". Univ. Complutense de Madrid, Enero 1981.

F. Bernis. "Compacidad del soporte en problemas convexos y no convexos de cuarto orden procedentes de modelos elásticos no lineales". Univ. Complutense de Madrid, Diciembre 1982.

…

Esta TESIS DOCTORAL fué presentada por D. Juan Luis Vázquez Suárez en la Facultad de Ciencias Matemáticas de la Universidad Complutense de Madrid, para la obtención del grado de DOCTOR EN CIENCIAS MATEMATICAS. Fué dirigida por el profesor de dicha Facultad Dr. D. Jesús Ildefonso Díaz Díaz.

Leida el 12 de Febrero de 1979 ante el Tribunal constituido por los Profesores:

Presidente: Dr. D. ALBERTO DOU MASDEXEXAS (Univ. Complutense Madrid)

Vocal: Dr. D. ANTONIO FERNANDEZ RAÑADA (Univ. Complutense, Madrid)

Vocal: Dr. D. ALFREDO MENDIZABAL ARACAMA (Univ. Politécnica, Madrid)

Vocal: Dr. D. FERNANDO BOMBAL GORDON (Univ. Complutense, Madrid)

Vocal Secretario: Dr. D. JESUS ILDEFONSO DIAZ DIAZ (Univ. Complutense, Madrid)

Vocal Invitado: Dr. D. HAIM BREZIS (Univ. Pierre et Marie Curie, Paris)

Obtuvo la calificación de SOBRESALIENTE CUM LAUDE.


Sir Isaac Newton (1642-1727)