Spanish Team

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Dr. F. Andreu Universidad de Valencia
Prof. A. Liñán Universidad Politécnica de Madrid
Dr. X. Cabré Universidad Politécnica de Catalunya
Prof. F. Bernis Universidad Nacional de Educación a Distancia
Dr. G. Galiano Universidad de Oviedo
Prof. J. Carrillo Universidad Complutense de Madrid
Dr. J. Hernández Universidad Autónoma de Madrid
Prof. M. Escobedo Universidad del País Vasco
Dr. V. Kourdyumov Universidad Politécnica de Madrid
Prof. J. Mazón Universidad de Valencia
Dr. S. I. Shmarev Universidad de Oviedo
Prof. J. M. Vega de Prada Universidad Politécnica de Madrid
Prof. C. Vázquez Universidad Politécnica de Madrid

Research experties

*analytic methods for nonlinear PDE's continuum mechanics and mathematical modelling

Research topics

*evolution of singularities and its relations to nonuniqueness for nonlinear degenerate equations

*qualitative behaviour of solutions of nonlinear second order elliptic and parabolic equations and systems

*free boundary problems and geometric evolutions problems

*analysis of fourth order degenerate parabolic equations and modelling of thin liquid films

*combustion

Young researches visiting the Spanish team

Vincent Millot, Paris VI
UCM, Predoc, 10/01/2002 to 12/31/2002 and October-November 2004
J.I. Díaz and **V. Millot,** Coulomb friction and oscillation: stabilization in finite time for a system of damped oscillators. CD-Rom Actas XVIII CEDYA / VIII CMA, Servicio de Publicaciones de la Univ. de Tarragona, 2003

K. Chaib, Toulouse
UCM, 3 March 2003-31 July 2003, 23 February 2004 – 3 April 2004

Chaib-Díaz, Some doubly nonlinear operators are subdifferentials in L^2
Belaud-Chaib-Diaz, Extinction in finite time for doubly nonlinear parabolic equations with weights

P. Begout (ParisVI)

"Universidad Complutense de Madrid" (March-July 2004)

Begoud-Díaz: On a Nonlinear Schrödinger Equation with a Localizing Effect

$$u_t = i\Delta u + (V_{\mathrm{Re}}(u) + iV_{\mathrm{Im}}(u))u,$$

this gives,

$$\begin{cases} -(u_2)_t = -\Delta u_1 - V_{\mathrm{Im}}(u)u_1 - V_{\mathrm{Re}}(u)u_2, \\ (u_1)_t = -\Delta u_2 - V_{\mathrm{Im}}(u)u_2 + V_{\mathrm{Re}}(u)u_1, \end{cases}$$

with $V_{\mathrm{Re}}(u) = -\mu_2|u|^{m-1}$ and $V_{\mathrm{Im}}(u) = \mu_1|u|^{m-1}$ The stationary case is

$$\begin{cases} -\Delta u_1 - V_{\mathrm{Im}}(u)u_1 - V_{\mathrm{Re}}(u)u_2 = 0, \\ -\Delta u_2 - V_{\mathrm{Im}}(u)u_2 + V_{\mathrm{Re}}(u)u_1 = 0. \end{cases}$$

$\mathrm{Re}(\mu) = \mu_1 < 0$ (and $\mathrm{Im}(\mu)$ as we want),

$\mathrm{Re}(\mu) = \mu_1 \geqslant 0$ and $\mathrm{Im}(\mu) = \mu_2 > 0$. Assume that $V_{\mathrm{Im}}(u) = 0$ (**WHICH IS GOOD WITH A POINT OF VIEW OF THE EXISTENCE FOR THE EVOLUTION EQUATION, IT MEANS THAT** $\mathrm{Re}(\mu) = 0$, **AND SO THEOREM 2.1.2 APPLY**). So we may write $V_{\mathrm{Re}}(u) = -\Phi(u) < 0$, since $\mu_2 > 0$. Assume further that, $V_{\mathrm{Re}}(u) = V_{\mathrm{Re}}(u_1) = -\Phi(u_1)$. In this case, we write (3.1.9) as

$$\begin{cases} -\Delta u_1 + \Phi(u_1)u_2 = 0, \\ -\Delta u_2 - \Phi(u_1)u_1 = 0. \end{cases} \tag{3.1.10}$$

Writing $\Psi(u_1) = \Phi(u_1)u_1$, we have with the second equation in (3.1.10), $u_1 = \Psi^{-1}(-\Delta u_2)$. With the first equation in (3.1.10), we obtain

$$-\Delta(\Psi^{-1}(-\Delta u_2)) + \Phi(\Psi^{-1}(-\Delta u_2))u_2 = 0.$$

J. Giacomoni, Toulouse
UCM, 1st April- 31 may 2004, 2 October-2 November 2004
Díaz-Giacomoni, On nonlinear elliptic equations with indefinite weight
Giacomoni-Hernández. In preparation

Yves Belaud, Tours 2003.
UCM, June 2003
Belaud-Díaz, FINITE EXTINCTION TIME FOR THE SOLUTIONS OF SOME PARABOLIC EQUATIONS WITH A SINGULAR TERM

M. Puel, ParisVI (Toulouse)
Barcelona and Madrid November-December 2003

Gabriela Litcanu, romanian
UCM, 01.12.2003 - 31.12.2003
Litcanu-Velázquez, In preparation

Posdocs 2004/2005

Myrto Sauvageot

Pascal Begoud
Giandomenico Orlandi

Spaniards young posdocs visiting other teams

Gerardo Oleaga (Leizpig)

Jose Ignacio Tello (Leiden, Haifa, Bratislava)

Other activities of young spaniards

M.B. Lerena (Creta)

A.M. Ramos (Bratislava)

(Paris, Leiden)

Moll (Leiden)

Joana Terra. Portugal (Paris): Cabré [Antonio Capela, M. Sanchón]

Seniors visiting Madrid

- A. Tesei
- M. Compte
- G. Shivashinski
- S. Kamin

Seniors joint researches

- Comte-Diaz
- Brezis-Díaz
- Diaz-Kamin-Rosenau
- Diaz-Tesei
- Liñán-Shivashinski
- Cabré-Lucia-Sanchón
- Flekinger-Hernández-De Thelin
- Hilhorst-Shmarev
- Galiano-van Duijn

Internal colaboration

- Díaz-Hernández
- Andreu-Caselles-Diaz-Mazon
- Diaz-Shmarev
- Diaz-Liñán (+Conca)
- Cabré-Diaz
- Diaz-J.I. Tello
- Hernández-Vega de Prada
- Liñán-Kurdyumov-Vázquez

Visits of seniors members

- Bath (Díaz) Paris (Díaz)
- Heindooven (Galiano), Paris (Shmarev)

Participation in meetings

Crete, Rome, Haiffa, Bratislava, Gaeta, Paris, Leiden

Task C
Front propagation in biological and chemical systems

Considerable progress has been made both in fundamental investigations of generic phenomena and in the application of the associated mathematical skills in providing insight into important specific applications, with the developments made in the former naturally having significant spin offs in terms of the latter.

With regard to applications, we highlight the following illustrative examples.

(i) Detailed mathematical investigations have been undertaken of the degradation of limestone by sulphur dioxide, important in understanding pollution-associated damage to both modern and historical buildings. These studies required the application of a wide range of mathematical tools, both analytical and numerical (such as degenerate-parabolic systems, sharp-interface limits, and finite element and finite difference approaches). Encouraging agreement has been obtained in initial comparisons with laboratory tests.

D. Aregba-Driollet, F. Diele, and R. Natalini. A mathematical model for the SO2 aggression to calcium carbonate stones: numerical approximation and asymptotic analysis, IAC Report 5/2003 N.6.

$$\mathrm{CaCO_3 + SO_2 + \frac{1}{2}O_2 + 2H_2O \rightarrow CaSO_4 \cdot 2H_2O + CO_2}.$$

$$\begin{cases} \partial_t(\varphi(c)s) - \nabla \cdot (\varphi(c)\nabla s) = -\varphi(c)sc, \\ \partial_t c = -\varphi(c)sc, \end{cases}$$

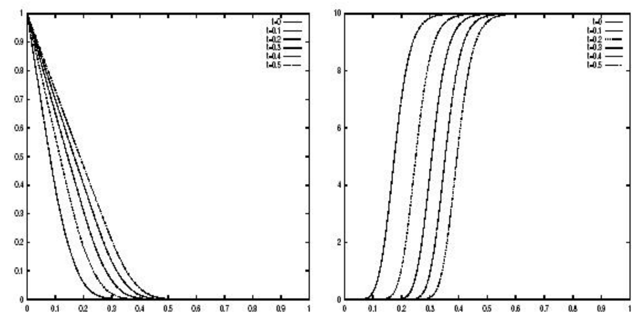

FIG. 6. *Solution from $t = 0$ to $t = 0.5$ with $A = 10000$. Left: SO_2 concentration; right: calcite density.*

(ii) Investigations of host tissue degradation by bacteria have been pursued, as part of a larger programme devoted to exploring new ways of treating antibiotic resistant infections.

Here a combination of formal and rigorous strategies (requiring expertise drawn from several different teams) has proved particularly efficacious.

$$\begin{aligned} 0 &= \partial_t u - \Delta u + u - w + \gamma k u(1-w), \\ 0 &= \partial_t w - k u(1-w), \end{aligned}$$

$$0 = \partial_t w - d\Delta w - k u(1-w),$$

D. Hilhorst, J. R. King, **M. Röger**. Mathematical analysis of a model describing tissue degradation by bacteria. Preprint.

D. Hilhorst, J. R. King, **M. Röger**. Travelling wave analysis of a model describing tissue degradation by bacteria. Preprint.

Other areas of application include to

- a wide variety of chemotactic phenomena, to
- nickel-titanium oxidation and its implications for biocompatibility,
- plant, sea-shell and tumour growth
- semiconductor device manufacture,
- biological invasion phenomena and competition-diffusion systems (e.g. D. Hilhorst, **G Karali**, H. Matano and K. Nakashima, Singular limit of a spatially inhomogeneous Lotka-Volterra competition-diffusion system. Preprint
- chemical autocatalysis **F. Dkhil**, E. Logak, and Y. Nishiura, Some analytical results o the Gray-Scott model. To appear in Asymptotic Analysis (2004).
- and polymerization.

An intriguing link has also been established between multiphase models for biological tissue and existing pseudoparabolic systems (J.R. King and **C. Cuesta**. Small and waiting time behaviour of a Darcy flow model with a dynamic pressure saturation relation. Preprin), reflecting one of the many synergies that are arising between the different Tasks.

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\left(u^{\alpha}\frac{\partial u}{\partial x} + u^{\beta}\frac{\partial^2 u}{\partial x \partial t}\right), \tag{1.1}$$

where α and β are positive constants, with initial condition $u(x,0) = u_0(x)$ with $u_0(x) = 0$ for $x \geq a$ which, for definiteness, we take to satisfy

$$u_0(x) \sim A(a-x)_+^p \quad \text{as } x \to a \tag{1.2}$$

for some positive constants A and p. Since we are concerned here with the local behaviour near the right-hand interface $x = a$, that near the left-hand one (if there is one) need to concern us.

Physical background to (1.1) is given in [18] and [9]; see also [10]. We summarise it here. The model equation for unsaturated flow in porous media with a dynamic capillary pressure relation has the general form

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\left(K(u)\frac{\partial}{\partial x}\left(-p_c(u) + L\frac{\partial u}{\partial t}\right)\right). \tag{1.3}$$

Fundamental mathematical studies
have included extensive investigations of
front propagation in homogenous and (especially) heterogeneous media, the
*'fast' nonlinear diffusion and of
*convergence to travelling waves in reaction-convection-diffusion systems
•latter treatments of pulsating fronts in periodic excitable media and of the
*fast-reaction limit of inhomogeneous Allen-Cahn equations (e.g. M. Alfaro, H. Garcke, D. Hilhorst and H. Matano, Singular limit of an inhomogeneous, anisotropic Allen-Cahn equation, in preparation)
• *Convective waves in coupled reaction-diffusion/Navier-Stokes systems,
*quenching and propagation in reaction-diffusion equations with heat loss H. Berestycki, F. Hamel, A. Kiselev, L. Ryzhik, Quenching and propagation in KPP reaction-diffusion equations with a heat loss. In preparation.
•soliton dynamics in the nonlinear Schroedinger equation have also been better understood, as have the important implications of non-local effects in reaction-diffusion systems (e.g. **F. Dkhil**, A. Stevens, Travelling wave speeds of nonlocally perturbed reaction diffusion equations. MPI MIS Preprint 1/04)
•Finally, exact solution techniques for degenerate-diffusion systems have also been developed.

The teams from France, Germany, Greece, Israel, Italy, Nederlands, Spain and the UK have been involved in Task C

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J.B. van den Berg, O. Baconneau, C.-M. Brauner and J. Hulshof, Multiplicity and stability of travelling wave solutions in a free boundary combustion-radiation problem, European J. Applied Math. **15**, 79-102 (2004)

M. Bertsch, **L. Giacomelli**, **G. Karali**. Thin-film equations with partial wetting energy: existence of weak solutions. Preprint Dipartimento Me.Mo.Mat. 4/2004.

D. Blanchard and **A. Porretta**, Stefan problems with nonlinear diffusion and convection, to appear in J. Diff. Equations.

R. Dal Passo, **L. Giacomelli**, G. Gruen. Waiting time phenomena for degenerate parabolic equations - a unifying approach. In "Geometric Analysis and Nonlinear Partial Differential Equations", S. Hildebrandt and H. Karcher Eds., Springer-Verlag Berlin Heidelberg New York (2003), 637-648.

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G. Galiano, A. Jüngel and **J. Velasco**, A parabolic cross-diffusion system for granular materials , SIAM Journal of Mathematical Analysis, Vol. 35, No. 3, pp. 561-578, 2003.

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F. Andreu, **S. Segura de León J. Toledo,** Quasilinear difusión equations with gradient terms and L^1-data. Nonlinear Analysis TMA 56 (2004), 1175-1209.

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J.I. Díaz, **M.B. Lerena** , J.F. Padial, J.M. Rakotoson,An evolution equation with a nonlocal term for the transient regime of a magnetically confined plasma in a Stellarator, *Journal of Differential Equations,* **198,** 2004, 321-355

S.N. Antontsev, J.I. Díaz, **H.B. de Oliveira,** Stopping a viscous fluid by a feedback dissipative field: thermal effects without phase changing, Proceeddings of the meetting dedicated to Professor V.A. Solonnikov on the occasion of his 70 th birthday

Donald G. Aronson, **Jan Bouwe van den Berg** and Josephus Hulshof. Parametric dependence of exponents and eigenvalues in focussing porous media flows, to appear.

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M. Escobedo, **S. Mischler,** J.J.L. Velazquez, Asymptotic description of Dirac ormation in kinetic equations por quantum particles. Preprint.

M. Winkler, Large time behavior and stability of equilibria in degenerate parabolic equations. Preprint.

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