Nonlinear Partial Differential Equations and Applications

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1. Introduction

http://www. mat.ucm.es/~jidiaz

Proyecto MTM2017-85449-P
Ecuaciones en derivadas parciales singulares

Proyecto: PID2020-112517GB-I00
Problemas no lineales en Ciencias Naturales y Ciencias Sociales

Grupo UCM: MOMAT (co-director Ángel Manuel Ramos del Olmo)

Modelos **Mat**emáticos en Ciencia y Tecnología: Desarrollo, Análisis, Simulación Numérica y Control

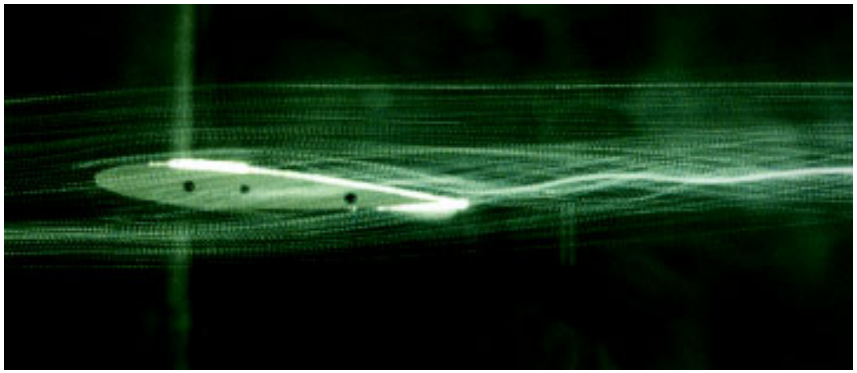
https://www.ucm.es/momat

- Plan
  - 2. Free boundary problems
  - 3. Climate and environment
  - 4. Control of nonlinear PDEs
  - 5. Homogeneization to the critical scale
  - 6. Many other research directions,…

2. Free boundary problems: solutions of quasilinear elliptic and parabolic equations with compact support

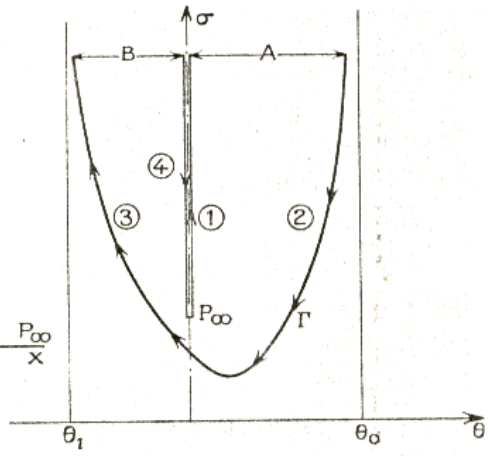
Pionering work: a classical problem of the Fluid Mechanics under a new view:

H. Brezis and G. Stampacchia: Une nouvelle méthode pour l'étude d'écoulements stationnaires, *C.R. Acad. Sci.,* **276**, 1973, 129-132.



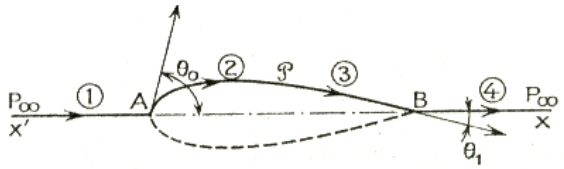
steady irrotational subsonic flow for a non viscous fluid,

symmetric convex profile in the plane



Hodograph trasformation $\mathcal{T} : (x, y) \to (u, v) \to (\theta, q)$

$$\begin{cases} \frac{1}{q^2}\left(\frac{q^2}{k}u_\sigma\right)_\sigma + u_{\theta\theta} + u = -R & \text{in } \mathcal{D}, \\ u = 0 & \text{on } \Gamma, \\ \nabla u = \mathbf{0} & \text{on } \Gamma, \\ u(0,\sigma) = \text{Constant} = H_{\mathcal{P}} & \sigma \geq \sigma_\infty \end{cases}$$



Formulation as a Variational Inequality (Lions-Stampacchia 1967)

$$K_H = \{v \in V : v \geq 0 \text{ on } \Omega \text{ and } u(0,\sigma) = H_{\mathcal{P}} \text{ for } \sigma \geq \sigma_\infty\}$$

A. Dou, J. I. Díaz. Sobre flujos subsónicos alrededor de un obstáculo simétrico. Collectanea Mathematica, Nº 3, 142-160, 1983

H. Brezis, Solutions à support compact d'inequations variationnelles, Séminaire Leray, Collège de France, 1973-74, pp. III.1-III.6

H. Brezis Solutions of variational inequalities with compact support. Uspekhi Mat. Nauk. 1974. Vol 129. pp 103-108.

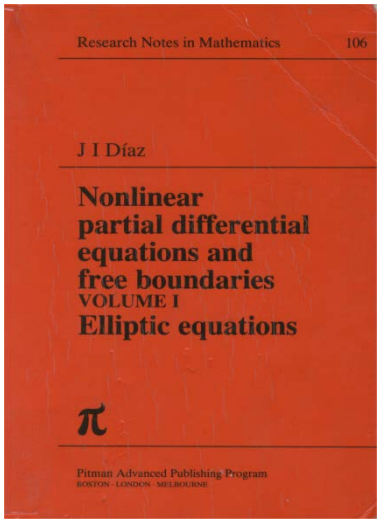
Pionering references on parabolic quasilinear equations (porous media): Oleinik-Kalahnikov-Yui Lin (1958), Barenblatt, Aronson, Kalashnikov, Peletier, …

JID:1976 Thesis (Paris VI, UCM): Adviser H. Brezis

$$\frac{\partial \psi(u)}{\partial t} - \operatorname{div} \mathbf{A}(x,t,u,D\,u) + B(x,t,u,D\,u) + C(x,t,u) + \beta(u) \ni f(x,t),$$

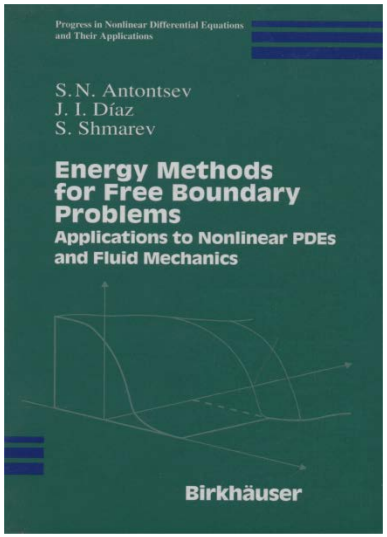
Method of "local super and subsolutions"

1985 Pitman



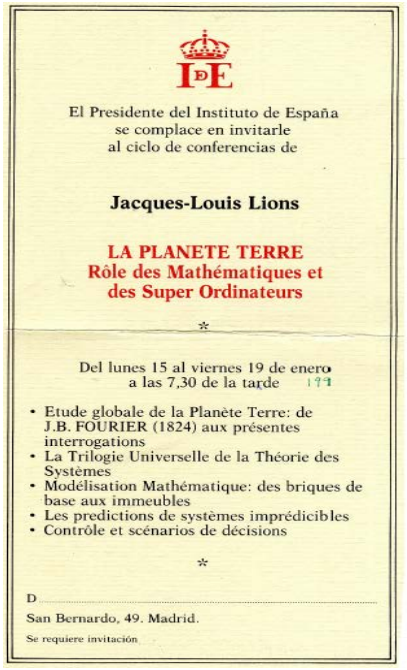
Method of "local integral energy"

2002 Birkhäuser



A concrete open problem: Free boundary for nonlinear (and linear !!) Schrödinger eqs.

3. Climate and environment



El Presidente del Instituto de España
se complace en invitarle
al ciclo de conferencias de

Jacques-Louis Lions

LA PLANETE TERRE
Rôle des Mathématiques et
des Super Ordinateurs

*

Del lunes 15 al viernes 19 de enero
a las 7,30 de la tarde

- Etude globale de la Planète Terre: de J.B. FOURIER (1824) aux présentes interrogations
- La Trilogie Universelle de la Théorie des Systèmes
- Modélisation Mathématique: des briques de base aux immeubles
- Les predictions de systèmes imprédicibles
- Contrôle et scénarios de décisions

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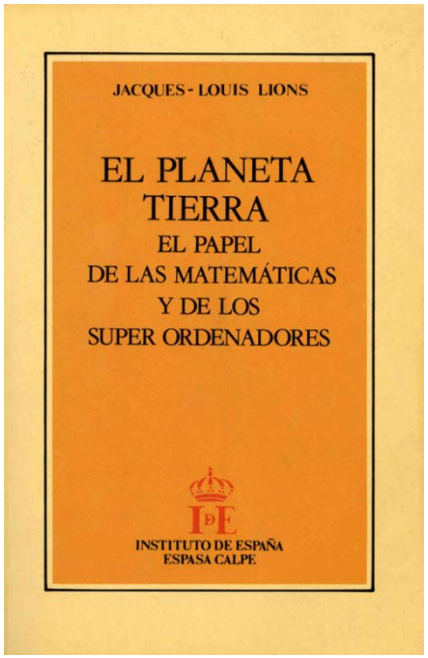
D.....

San Bernardo, 49. Madrid.

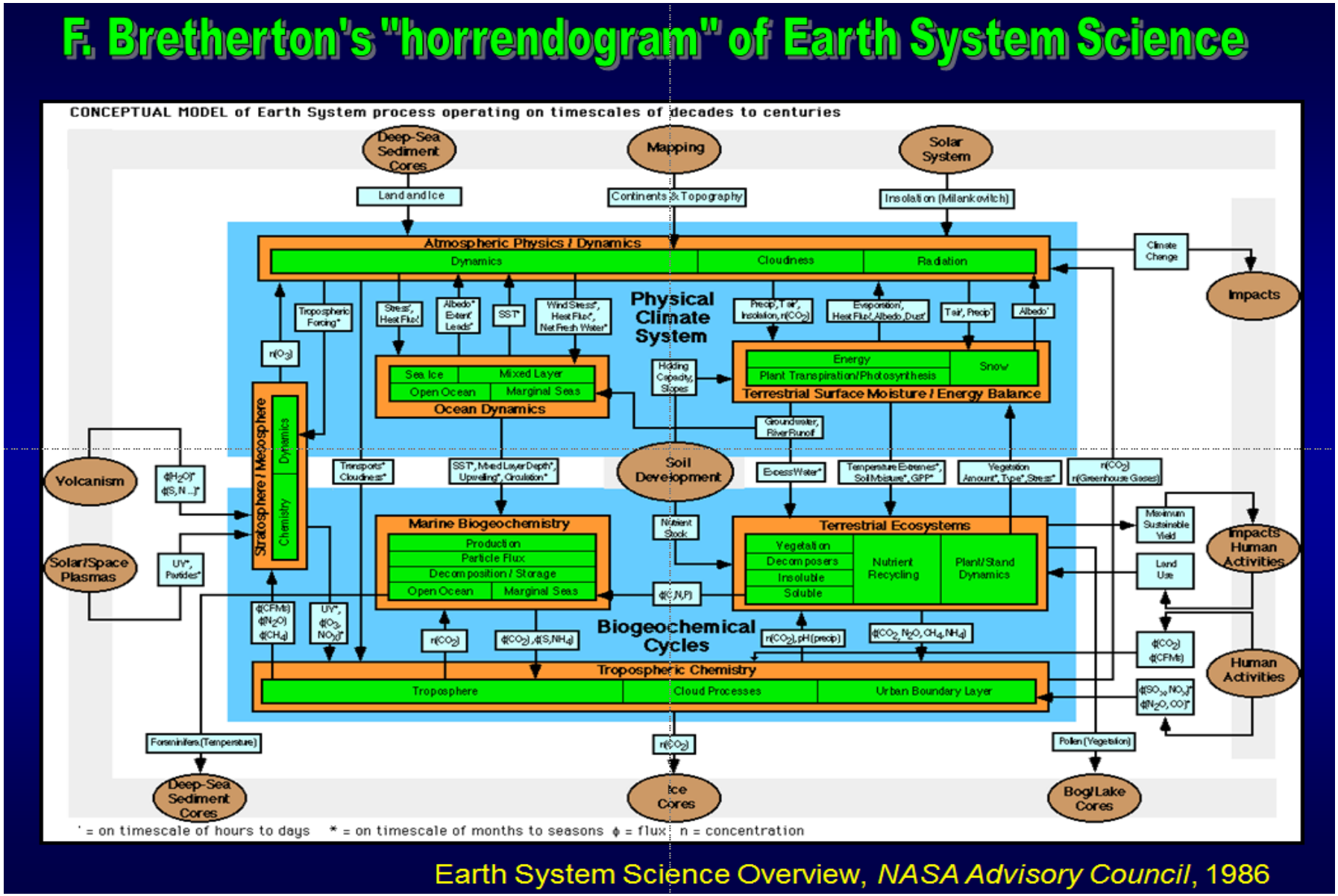
Se requiere invitación



Díaz, J.I., **Lions, J.-L.**, eds., ***Mathematics, Climate and Environment***, Research Notes in Applied Mathematics 27, Masson, Paris, 1993.

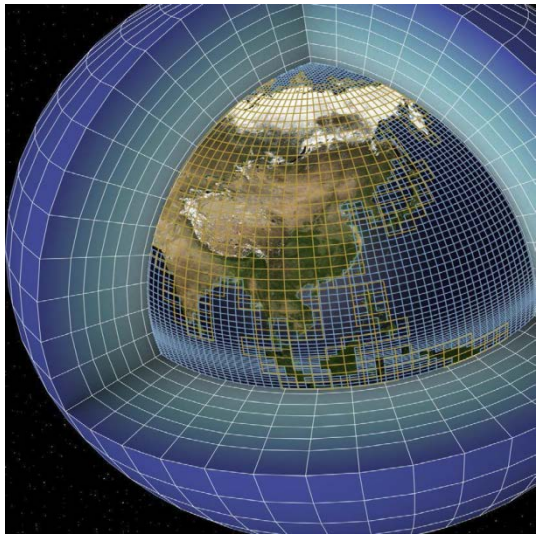
Díaz, J.I., **Lions, J.-L.**, eds., ***Environment, Economics and Their Mathematical Models***, Research Notes in Applied Mathematics 35, Masson, Paris, 1994.

Diffusive energy balance models versus “horrendogram” of Earth System Science



Earth System Science Overview, *NASA Advisory Council*, 1986

One of the last paper on the subject:



PROCEEDINGS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

Multiple solutions and numerical analysis to the dynamic and stationary models coupling a delayed energy balance model involving latent heat and discontinuous albedo with a deep ocean

J. I. Díaz, A. Hidalgo and L. Tello

Proc. R. Soc. A 2014 470, 20140376, published 27 August 2014

$$\frac{\partial \gamma(U)}{\partial t} - \operatorname{div}(\nabla U) + w\frac{\partial U}{\partial z} \ni 0 \quad \text{in } (0,T)\times\Omega,$$

$$\frac{\partial u}{\partial t} - \operatorname{div}(|\nabla_{\mathcal{M}} u|^{p-2}\nabla_{\mathcal{M}} u) + \frac{\partial U}{\partial n} + F(x,\nabla_{\mathcal{M}} u) + G(t,x,u,u(t-\tau))$$

$$\in \frac{1}{\rho c} QS(t,x)\beta(u) + f(t,x) \quad \text{in } (0,T)\times\mathcal{M},$$

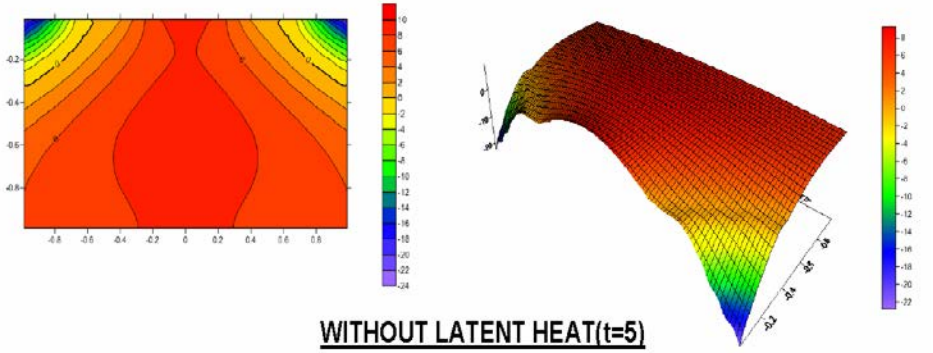
$$U|_{[0,T]\times\mathcal{M}} = u,$$

$$\hat{F}(x,\nabla_N U) + \frac{\partial U}{\partial z} = 0 \quad \text{in } (0,T)\times N,$$

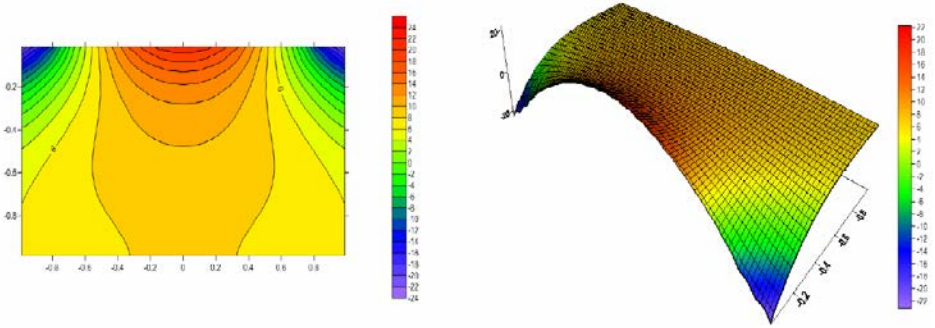
$$U(0,x,z) = U_0(x,z) \quad \text{in } \Omega,$$

$$u(s,\cdot) = u_0(s,\cdot) \quad \text{on } [-\tau,0]\times\mathcal{M}.$$

WITH LATENT HEAT (t=5)

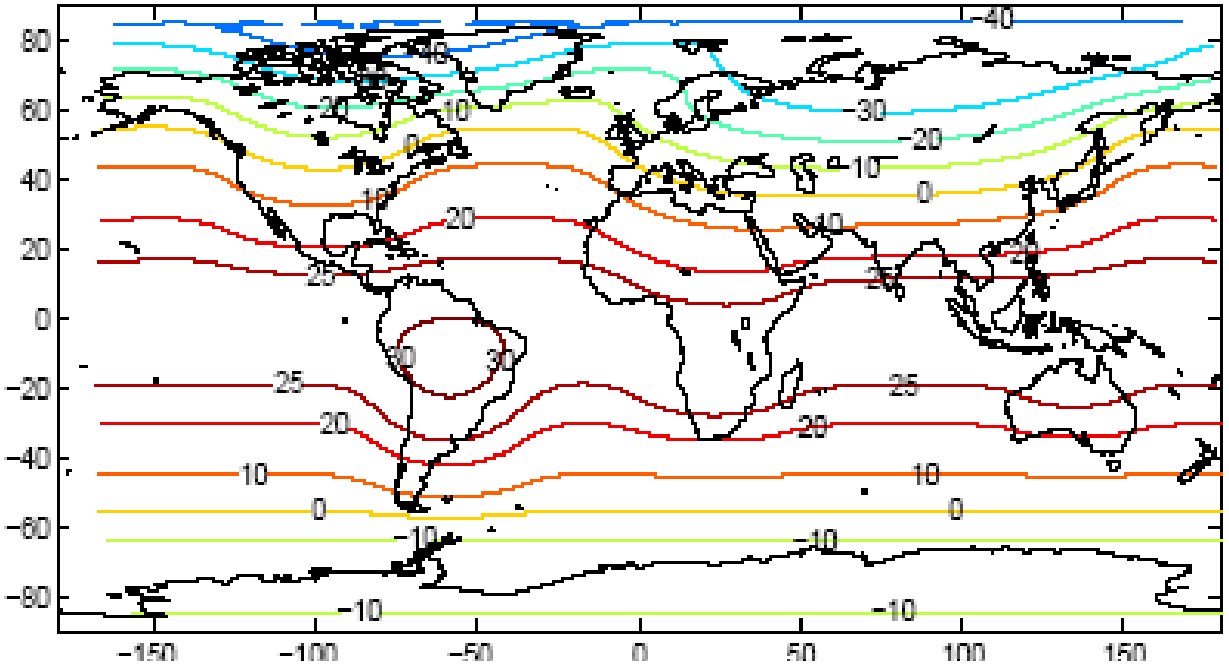


WITHOUT LATENT HEAT(t=5)



R. Bermejo, J. Carpio, J.I. Díaz, P. Galán

A **finite element** algorithm of a nonlinear diffusive climate energy balance model, *Pure and Applied Geophysics*, **165**, 2008, 1025-1048.



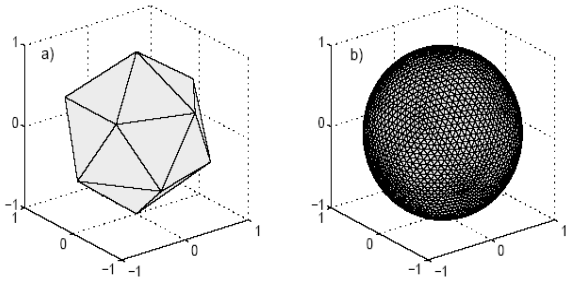


Figure 1: Initial Icosahedron and mesh after 4 refinements.

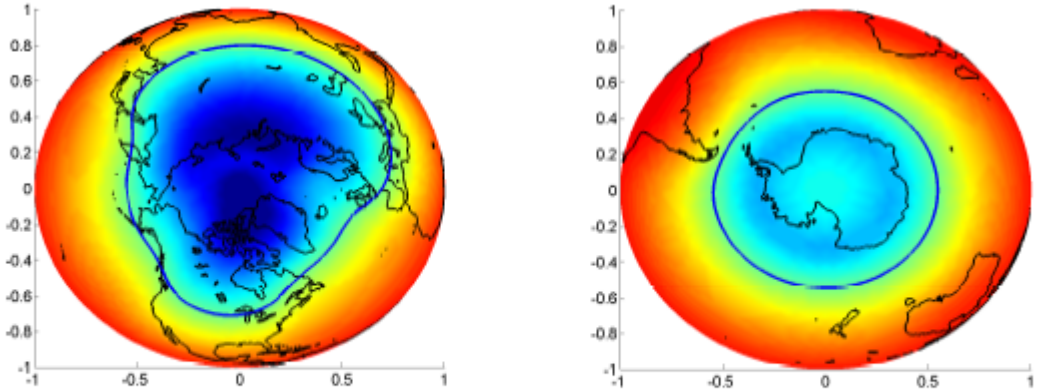


Figure 4: -2^oC January snow line. Left: northern hemisphere; right: southern hemisphere .

Open problem: modeling volcano actions by stochastic EBMs

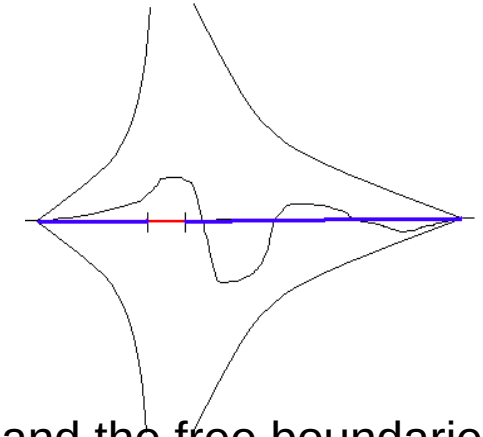
5. Control of nonlinear PDEs

C. R. Acad. Sci. Paris, t. 312, Série I, p. 519-522, 1991

Contrôle optimal/*Optimal Control*

Sur la contrôlabilité approchée des inéquations variationnelles et d'autres problèmes paraboliques non linéaires

Jesus Ildefonso Diaz



"Obstruction phenomenon", controllability of solutions and the free boundaries, von Neumman conjecture on climate modification, …

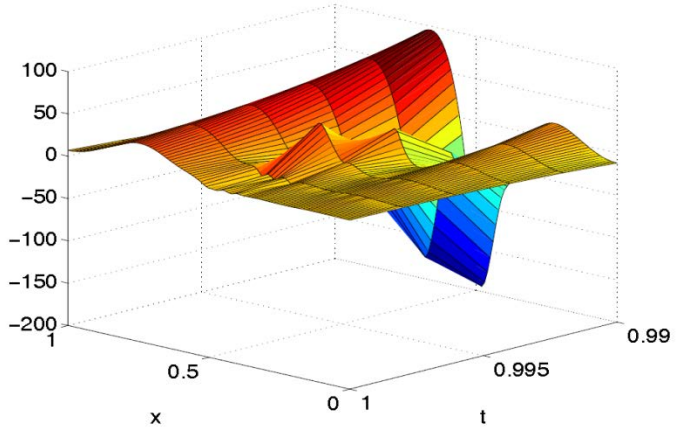
2 Ph.D. Thesis, 21 articles, plenary lecturer 16th IFIP "System Modelling and Optimization" July 5-9, 1993; Compiegne (France), …

J. I. Díaz and **J.L. Lions**. **Sur la contrôlabilté approchée de problèmes paraboliques avec phénomènes d'explosion**, Comptes Rendus Acad. Sci. Paris, t. 327, Série I, 173-177, 1998.

J. I. Díaz and **J. L. Lions**. **On the approximate controllability for some explosive parabolic problems**, *International Series of Numerical Mathematics*, Vol. 133, Birkhäuser Verlag, Basel, pp. 115-132, 1999.

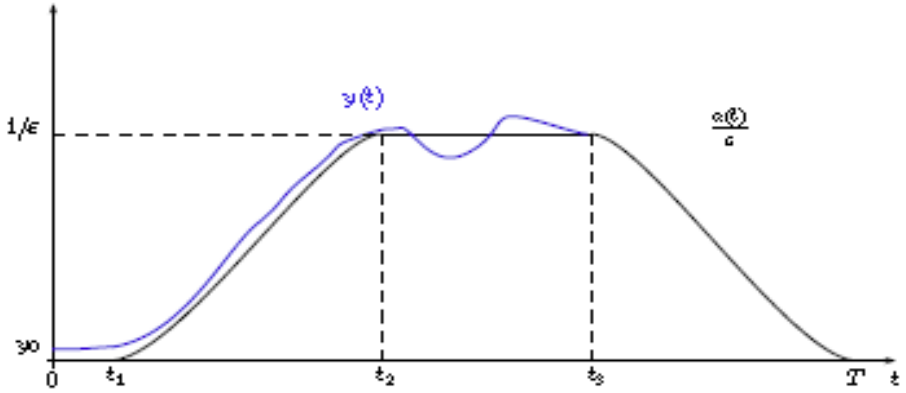
J. I. Díaz and **J. L. Lions**. **On the Approximate Controllability of Stackelberg-Nash Strategies**. In, Ocean Circulation and Pollution Control. A Mathematical and Numerical Inquiry, (J. I. Díaz ed.). Lecture Notes, EMS Volume, Proceedings of the Diderot Videoconference Amsterdam-Madrid-Venice, Lecture-Notes, Springer Verlag 2003, 17-28.

Numerical experiences on the control for non-Lipschitz semilinear problems



J.-M. Coron, J. I. Díaz, A. Drici and T. Mingazzini, "Global Null Controllability of the 1-Dimensional Nonlinear Slow Diffusion Equation", Chinese Annals of Mathematics, 34B(3), 2013, 333–344 (special issue devoted to J.L. Lions)

Non differentiable operators, beyond linearization: return method,…

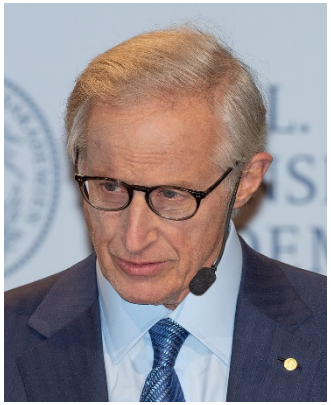


Open: case of d>1

Componente económica: Control en la economia del Cambio Climático

William D.**Nordhaus**
(1941-)

Premio Nobel de Economia de 2018.
Premio Fundación BBVA Fronteras del Conocimiento, 2017 en Cambio Climático.



Tasas CO2 (GHGs)

Modelo “Dynamic Integrated climate economy” DICE

Nordhaus W. D. (nov. 1992). «An Optimal Transition Path for Controlling Greenhouse Gases»; *Science* 258, 1315-1319.

Open: Filtro de Kalman para EDPs no lineales

5. Homogeneization to the critical scale

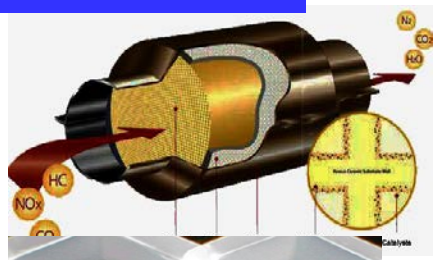
“Piensa globalmente y actúa localmente”

Control como suma de pequeñas acciones: Homogeneización

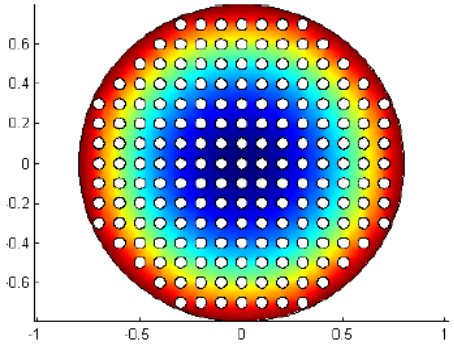
¿Es posible ofrecer un resultado matemático mostrando la eficiencia de una infinitud de acciones locales de pequeña magnitud?

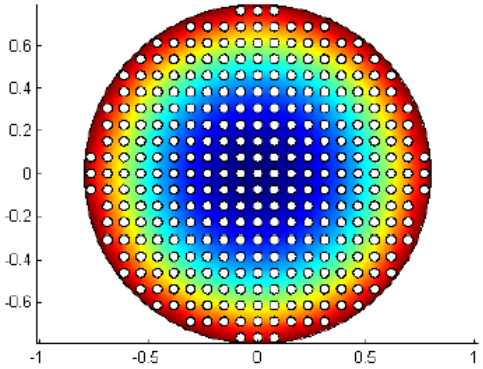
Control del comportamiento a escala macroscópica mediante la actuación de controles (forma de partículas) a escala

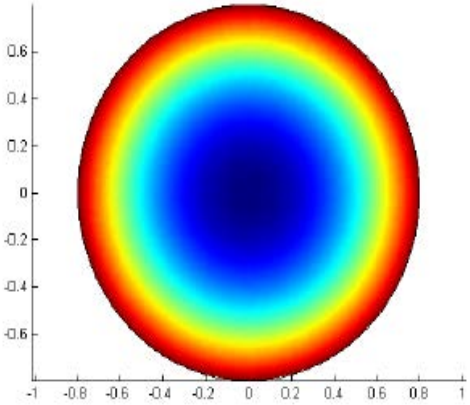
¡¡¡ Si !!!

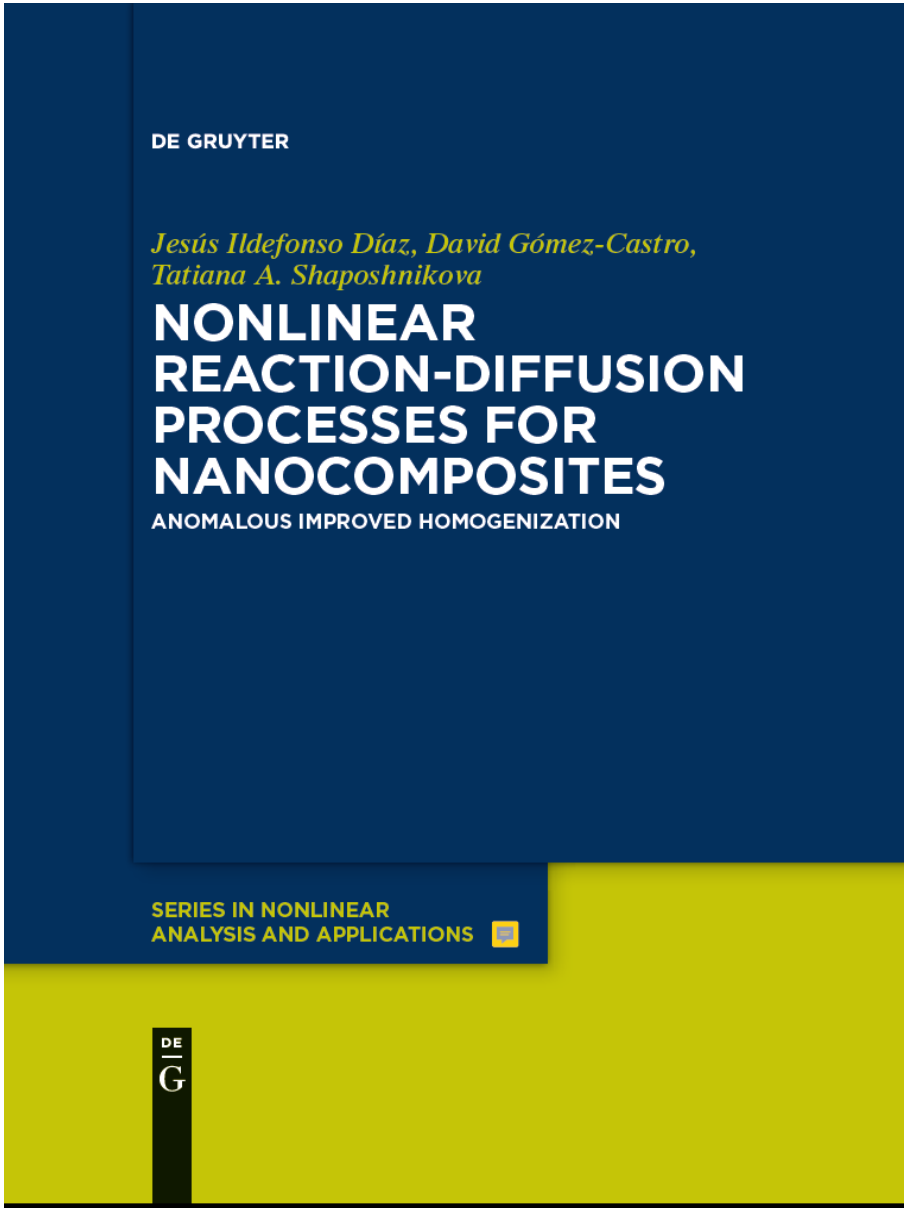


De hecho, esa estrategia permite evitar el fenómeno de “obstrucción” debido a la radiación Stefan-Boltzmann que aparece si los controles son globales









Publicación en papel: 6 de junio 2021

Numerosos *Open problems* en el texto

6. Many other subjects

- Steiner symmetrization (P.L. Lions, ...)
- Singular nonlinear elliptic equations; bifurcation,…
- Schrödinger equations (linear and nonlinear), relativistic (fractional non-local operators,..)
- Nonlinear population dynamics
- Monge-Ampere and other nonlinear models in Differential Geometry,
- Stochastic nonlinear parabolic equations,…
- Quasicrystals (Yves Meyer,…)

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