MR3840089 53B05 53B20 53B30 53C50

Diallo, Abdoul Salam (SNG-UBAMB-ATI);
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★**Walker Osserman metric of signature $(3, 3)$. (English summary)**

Mathematical structures and applications, 199–210, *STEAM-H: Sci. Technol. Eng. Agric. Math. Health*, *Springer, Cham*, 2018.

Let M be an n-dimensional manifold and R the curvature of an affine torsion free connection ∇. The Jacobi operator at a point p, $R_Z\colon T_pM \to T_pM$, is defined as $R_Z(X) = R(X,Z)Z$. A manifold (M,∇) is an affine Osserman manifold if at every point $p \in M$ the characteristic polynomial of R_Z is independent of the direction Z. If ∇ is the Levi-Civita connection of some pseudo-Riemannian metric g on M, then (M,g) is called an Osserman manifold [see E. García-Río et al., Differential Geom. Appl. **11** (1999), no. 2, 145–153; MR1712127].

In the paper under review, the authors show a non-flat example of an affine Osserman manifold $(M = \mathbb{R}^3, \nabla)$. Then, the Riemann extension g^∇ of (M,∇) to the cotangent bundle T^*M provides an example of an Ossermann manifold $(\mathbb{R}^6, g)$ of signature $(3, 3)$ which admits a field of parallel null 3-planes. This is a Walker Ossermann manifold.

{For the collection containing this paper see MR3887578}

Javier Lafuente-López

MR3807128 53A30 53B25 53B30

Xie, Zhenxiao (PRC-CUMT2);
Wang, Changping [**Wang, Chang Ping**[1]] (PRC-FUJN-SMC);
Wang, Xiaozhen [**Wang, Xiao Zhen**] (PRC-FUJN-SMC)

Conformally flat Lorentzian hypersurfaces in $\mathbb{R}^4_1$ with a pair of complex conjugate principal curvatures. (English summary)

J. Geom. Phys. **130** (2018), 249–259.

For a three-dimensional Lorentzian hypersurface $x\colon M^3_1 \to \mathbb{R}^4_1$ immersed in the affine Lorentzian space $\mathbb{R}^4_1$, the shape operator endomorphism $\mathcal{A}_p$ defined in the tangent space $T_pM^3_1$ (isometric to $\mathbb{R}^3_1$) is symmetric. For three-dimensional Lorentzian vector space, it is well known that a symmetric linear transformation such as $\mathcal{A}_p$ may fail to be diagonalizable and that it can be classified into four types: type I, when $\mathcal{A}_p$ is diagonalizable; type II, when $\mathcal{A}_p$ has a pair of complex conjugate eigenvalues; type III, when $\mathcal{A}_p$ is not diagonalizable and has two distinct real eigenvalues; and type IV, when $\mathcal{A}_p$ is not diagonalizable and has just one real eigenvalue.

Three-dimensional conformally flat Lorentzian hypersurfaces of type I and III (that is, $\mathcal{A}_p$ is such a type for all $p \in M^3_1$) were studied by the authors in previous papers [Sci. China Math. **61** (2018), no. 5, 897–916; MR3788967; Internat. J. Math. **28** (2017), no. 13, 1750092; MR3737070]. In the paper under review, they investigate the second type, with the same goal and similar tools.

Using the projective light-cone model of the conformal geometry of $\mathbb{R}^4_1$, the authors study the integrability conditions of conformally flat Lorentzian hypersurfaces of type II in $\mathbb{R}^4_1$, by constructing three conformal fundamental forms, a scalar conformal invariant (they call it conformal curvature) and a canonical moving frame. Then, by using these invariants, they get a congruence theorem and integrability equations. It turns out that these hypersurfaces can also be determined up to a conformal transformation in $\mathbb{R}^4_1$, by solutions of a third-order partial differential equation. On the other hand, all possible examples are given for which the conformal curvature is constant along the curvature lines corresponding to the real principal curvature. *Javier Lafuente-López*

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Gauss curvature in pseudo-Riemannian three-dimensional space forms, J. Math. Phys. 48 (2007) 042301. MR2319885

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR3750251 53A05 53B25
Lee, Jae Won [**Lee, Jae Won**[3]] (KR-GYSG-MD); **Kim, Dong-Soo** (KR-CHON); **Kim, Young Ho** [**Kim, Young Ho**[1]] (KR-KNG); **Yoon, Dae Won** (KR-GYSG-MD)
Generalized null 2-type immersions in Euclidean space. (English summary)
Adv. Geom. **18** (2018), *no. 1,* 27–36.

Generalized null 2-type submanifolds are submanifolds M of the Euclidean space $\mathbb{E}^m$ satisfying the condition $\Delta H = fH + gC$ for some smooth functions f, g on M and a constant vector C in $\mathbb{E}^m$, where Δ and H denote the Laplace operator and the mean curvature vector of M, respectively. This is a generalization of null 2-type submanifolds defined by B.-Y. Chen [see *Total mean curvature and submanifolds of finite type*, Ser. Pure Math., 1, World Sci. Publishing, Singapore, 1984; MR0749575], since these submanifolds satisfy the condition $\Delta H = \lambda H$ for some constant λ.

In this article, the authors focus on developable surfaces in the Euclidean space $\mathbb{E}^3$. They show that a tangent developable generalized null 2-type surface is an open part of a plane and classify conical generalized null 2-type surfaces. Finally, they show that all cylindrical hypersurfaces in Euclidean space $\mathbb{E}^m$ ($m \geq 3$) are generalized null 2-type submanifolds.

Javier Lafuente-López

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR3692383 53A30 53C30 53C40
Li, Tongzhu (PRC-BIT)
Möbius homogeneous hypersurfaces with three distinct principal curvatures in $\mathbb{S}^{n+1}$. (English summary)
Chin. Ann. Math. Ser. B **38** (2017), *no. 5,* 1131–1144.

Let $x\colon M^n \to \mathbb{S}^{n+1}$ be an immersed hypersurface in the $(n+1)$-dimensional sphere $\mathbb{S}^{n+1}$. If for any points $p, q \in M^n$ there exists a Möbius transformation $\phi\colon \mathbb{S}^{n+1} \to \mathbb{S}^{n+1}$ such that $\phi \circ x(M^n) = x(M^n)$ and $\phi \circ x(p) = x(q)$, then the hypersurface is called a Möbius homogeneous hypersurface.

In [Manuscripta Math. **96** (1998), no. 4, 517–534; MR1639852], C. P. Wang studied the Möbius (conformal) geometry of general submanifolds in the sphere, introducing a Möbius invariant metric g and a Möbius invariant 2-form B called the Möbius second fundamental form. He proved that for a hypersurface in $\mathbb{S}^{n+1}$, $n \geq 3$, the pair (g, B) forms a complete Möbius invariant system which determines the hypersurface $x(M^n)$ up to a Möbius transformation in $\mathbb{S}^{n+1}$. Then, the Möbius scalar invariants on the homogeneous hypersurfaces are constant, hence its Möbius principal curvatures (i.e., the eigenvalues with respect to the Möbius shape operator) are also constant. Umbilic-free hypersurfaces with constant Möbius principal curvatures and null Möbius 1-form C are called isoparametric [see Z. J. Hu and S. Zhai, Pacific J. Math. **249** (2011), no. 2, 343–370; MR2782673]. These hypersurfaces have been systematically studied [see also É. Cartan, Math. Z. **45** (1939), 335–367; MR0000169; T. E. Cecil, *Lie sphere geometry*, Universitext, Springer, New York, 1992; MR1219311].

In the paper under review, the Möbius homogeneous hypersurfaces with three distinct principal curvatures are classified completely up to a Möbius transformation in $\mathbb{S}^{n+1}$. The author first proves that these hypersurfaces have Möbius 1-form $C = 0$, and then he recovers the classification theorem of isoparametric hypersurfaces with three distinct principal curvatures of [Z. J. Hu and S. Zhai, op. cit.] to obtain the main theorem. As a corollary, he concludes that, conversely, the Möbius isoparametric hypersurfaces with three distinct principal curvatures are Möbius homogeneous. *Javier Lafuente-López*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR3626203 53A30 51M15 53A35
Shipman, Barbara A. (1-TXA); **Shipman, Patrick D.** (1-COS);
Shipman, Stephen P. (1-LAS)

Lorentz-conformal transformations in the plane. (English summary)

Expo. Math. **35** (2017), *no. 1,* 54–85.

The (locally) conformal transformations $(x, y) \to (u, v)$ in the Euclidean plane are characterized by the property of preserving the solutions of the Laplace equation $f_{xx} + f_{yy} = 0$. In the same way, conformal transformations in the Lorentzian plane can be seen as those that preserve the solutions of the wave equation, $f_{xx} - f_{yy} = 0$. In the Euclidean case $u = u(x, y)$ and $v = v(x, y)$ must be analytical because they satisfy the Cauchy-Riemann equations for holomorphic functions or the corresponding equations for antiholomorphic functions. In the Lorentzian case these functions u and v satisfy one of the two systems $[u_x = v_y,\ u_y = v_x]$ or $[u_x = -v_y,\ u_y = -v_x]$, whose solutions are written, respectively, as

$$(u, v) = \frac{1}{2}(h(x + y) - k(-x + y), h(x + y) + k(-x + y))$$

or

$$(u, v) = \frac{1}{2}(-h(x + y) + k(-x + y), h(x + y) + k(-x + y)),$$

where h and k are smooth.

In the article under review the authors consider mappings $(x, y) \to (u, v)$ similar to the previous ones, but where h and k are not necessarily differentiable, and they refer to them as Lorentz-conformal maps. Of course these maps include the linear Lorentz-conformal group.

Much of this paper is devoted to discovering what shapes in the xy-plane can be mapped by an invertible Lorentz-conformal transformation to the u-constant or v-constant contours, or also to the standard unit square in the uv-plane. Explicit constructions and many examples are exhibited, including computational contour plots of the corresponding transformations.

On the other hand, classes of Lorentz-conformal maps are characterized in terms of symmetries in the contour plot, according to a natural action of the dihedral group D_4. Unfolding for a Lorentz-conformal mapping is defined, and the authors show how unfoldings of non-invertible mappings into invertible ones are reflected in a change of the symmetry group. *Javier Lafuente-López*

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MR3581668 53A05 53B25 53B30 53C42
Yang, Dan [Yang, Dan[1]] (PRC-LIAO-SM); **Fu, Yu** [Fu, Yu[1]] (PRC-NEFE-SM); **Li, Lan** [Li, Lan[2]] (PRC-SZU-CMC)

Geometry of spacelike generalized constant ratio surfaces in Minkowski 3-space. (English summary)

Front. Math. China **12** (2017), *no. 2,* 459–480.

A surface in the Euclidean 3-space $\mathbb{E}^3$ is called a generalized constant ratio (GCR) surface if the tangential component of the position vector points in a principal direction. This subject was studied in a previous paper [Bull. Braz. Math. Soc. (N.S.) **45** (2014), no. 1, 73–90; MR3194083] where Y. Fu and M.-I. Munteanu gave an explicit description of these surfaces, classifying the flat GCR ones, and also those that have constant mean curvature. The definition of GCR surface is a generalization of the concept of constant slope surface studied in [M.-I. Munteanu, J. Math. Phys. **51** (2010), no. 7, 073507; MR2681099].

In the paper under review the authors generalize in an obvious way the GCR concept for spacelike surfaces in the 3-dimensional Minkowski space $\mathbb{L}^3$ and then they use similar arguments to those used in [Y. Fu and M.-I. Munteanu, op. cit.] to give an explicit and exhaustive description of these surfaces, whether the position vector lies always in the timelike cone or in the spacelike cone. Except in some specific cases, these surfaces are of revolution with respect to an axis which can be spacelike, timelike or null (see [J. Hano and K. Nomizu, Tohoku Math. J. (2) **36** (1984), no. 3, 427–437; MR0756026] for definitions).

Finally the authors, using the preceding description, show that the spacelike GCR surfaces with constant mean curvature are surfaces of revolution and the flat ones are open parts of planes or cylinders. *Javier Lafuente-López*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR3680210 53A30 53A40
Çivi, Gülçin (TR-ISTNTS);
Arsan, Güler Gürpinar [**Arsan, Güler Gürpınar**] (TR-ISTNTS)
On Weyl manifolds with harmonic conformal curvature tensor. (English summary)
An. Ştiinţ. Univ. Al. I. Cuza Iaşi. Mat. (N.S.) **62** (2016), *no. 2, vol. 1,* 327–335.

A differentiable manifold of dimension n having a conformal class $[g]$ of metrics and a torsion-free connection ∇ preserving $[g]$ is called a Weyl manifold, and is denoted by $W_n(g,T)$, where T is a 1-form satisfying the compatibility condition $\nabla g = 2g\otimes T$. It is well known that the pair $W_n(\overline{g}=\lambda^2 g, \overline{T})$ generates the same Weyl manifold, iff $\overline{T} = T + d\ln\lambda$. If a tensor A changes to $\overline{A} = \lambda^p A$ when g changes to $\overline{g} = \lambda^2 g$ (i.e. A is a satellite of weight $\{p\}$), its prolonged covariant derivative is defined by $\dot{\nabla}_k A = \nabla_k A - pT_k A$ in some given coordinates (see [E. Ö. Canfes and A. Özdeğer, J. Geom. **60** (1997), no. 1-2, 7–16; MR1477068] for details). We will denote by C^m_{ijk}, R^m_{ijk}, and $R_{ij} = R^h_{ijh}$ the components of the conformal curvature tensor of $[g]$, the curvature tensor and the Ricci tensor of ∇ respectively. All these tensors have height $\{0\}$.

In this paper the authors consider a Weyl manifold $W_n(g,T)$ $(n>3)$ with harmonic conformal curvature tensor (i.e. $\dot{\nabla}_h C^h_{ijk} = 0$). First they give conditions for such a space $W_n(g,T)$ to be equipped with a harmonic curvature tensor ($\dot{\nabla}_h R^h_{ijk} = 0$), to be conformally recurrent ($\dot{\nabla}_l C^h_{ijk} = \lambda_l C^h_{ijk}$, for some $\lambda_l \neq 0$) or Ricci recurrent ($\dot{\nabla}_l R_{ij} = \lambda_l R_{ij}$). Using this, they prove that if $W_n(g,T)$ is conformally recurrent then it is conformally symmetric ($\dot{\nabla}_l C^h_{ijk} = 0$) or conformally flat ($C^h_{ijk} = 0$). Also they prove that if $W_n(g,T)$ is Einstein-Weil [A. Özdeğer, Acta Math. Sin. (Engl. Ser.) **29** (2013), no. 2, 373–382; MR3016537] then it has harmonic curvature tensor if and only if its scalar curvature tensor $R = g^{ij}R_{ij}$ is prolonged covariant constant.

Javier Lafuente-López

MR3516832 53C50 53A30 53B30
Dzhalilov, Akhtam (UZ-TRNP-MNS); **Musso, Emilio** (I-TRNP);
Nicolodi, Lorenzo (I-PARM-MI)
Conformal geometry of timelike curves in the $(1+2)$-Einstein universe. (English summary)
Nonlinear Anal. **143** (2016), 224–255.

The $(1+2)$-Einstein universe $\mathcal{E}^{1,2}$ is defined as the quadric

$$\mathcal{E}^{1,2} = \left\{[\xi] \in \mathbb{R}\mathrm{P}^4 : \langle \xi, \xi\rangle = 0\right\},$$

where $\langle\,,\rangle$ is the inner product in $\mathbb{R}^{2,3}$, i.e. with negative index 2 in $\mathbb{R}^5$. We may regard $\mathcal{E}^{1,2}$ with its canonical conformal structure as the common conformal compactification of the Minkowski space $\mathbb{R}^{1,2} = \mathbb{M}^{(1,2)}_0$ and the de Sitter and anti-de Sitter spheres $\mathbb{M}^{1,2}_1$ and $\mathbb{M}^{(1,2)}_{-1}$.

In this paper, the authors state the basic theory of local and global conformal geometry for timelike curves in $\mathcal{E}^{1,2}$. The group of conformal transformations considered here is restricted to the action on $\mathcal{E}^{1,2}$ of the connected component of the group of linear isometries of $\mathbb{R}^{2,3}$.

First, by analogy with the Möbius geometry of curves in $\mathbb{S}^3$ [E. Musso, Math. Nachr. **165** (1994), 107–131; MR1261366; C. Schiemangk and R. Sulanke, Math. Nachr. **96** (1980), 165–183; MR0600808; R. Sulanke, Math. Nachr. **100** (1981), 235–247; MR0632630], the authors define the infinitesimal conformal strain (arc length) for a timelike curve, the osculating conformal cycle, and the notion of conformal vertex. Then they prove the existence of a canonical conformal frame field $\mathbf{M}$ along a generic timelike curve (i.e. without vertex) parametrized by the conformal arc-length, and using the Cartan moving frame method, they obtain two conformal curvatures $(k \geq 0, h)$. These curvatures determine the curve up to a restricted conformal transformation, through the Frenet conformal equations $\mathbf{M}' = \mathbf{M}\mathcal{K}(k,h)$. Here the authors identify the curves with $k = 0$, and classify the generic timelike curves with constant conformal curvatures h and $k > 0$, in terms of the stratification of $\mathbb{R}^2_+$ determined by the orbit-type of $\mathcal{K}(k,h)$.

Next, they use the canonical frame to compute the Euler-Lagrange equations of the conformal strain functional and then they show that the conformal equivalence classes of critical curves depend on two real constants and also that there exist countably many distinct classes of closed ones.

Finally, the authors state a connection between the conformal global geometry of generic timelike closed curves and the geometry of transversal knots in the sphere $\mathbb{S}^3$ (with its standard contact structure) via the directrices of the timelike curve. These are two immersed curves in $\mathbb{S}^3$ transverse to the contact distribution, which are built using the symplectic lift of the canonical conformal frame $\mathbf{M}$. If such directrices are simple curves, then their linking and Bennequin numbers [D. Fuchs and S. L. Tabachnikov, Topology **36** (1997), no. 5, 1025–1053; MR1445553] provide three global conformal invariants, which are computed in the special class of closed timelike curves of constant curvature. *Javier Lafuente-López*

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MR3454547 53C50 53A30 53C22
Mounoud, Pierre (F-UBORD-IM); **Suhr, Stefan** (D-HAMB)
On spacelike Zoll surfaces with symmetries. (English summary)
J. Differential Geom. **102** (2016), *no. 2,* 243–284.

A spacelike Zoll surface is a Lorentzian surface all of whose spacelike geodesics are simple closed curves of the same length. This gives a Lorentzian analogue to Zoll surfaces, where the basic example is the sphere or the projective plane with the canonical Riemannian structures. Exotic Zoll structures on spheres were extensively studied by several authors [see, for example, A. L. Besse, *Manifolds all of whose geodesics are closed*, Ergebnisse der Mathematik und ihrer Grenzgebiete, 93, Springer, Berlin, 1978 (Chapter 4); MR0496885], but there are no exotic Riemannian metrics on the projective plane for which all geodesics are closed [see C. Pries, Geom. Funct. Anal. **18** (2009), no. 5, 1774–1785; MR2481742].

The basic example of a spacelike Zoll surface is the de Sitter space and its finite coverings. In fact the authors proved in [Math. Z. **274** (2013), no. 1-2, 225–238; MR3054326] that any spacelike Zoll surface is diffeomorphic to a cylinder or a Möbius strip.

This article tries to initiate a study of the conformal classification of the spacelike Zoll surfaces. With this aim, the authors provide three explicit families of examples, constructed as deformations of a covering of the de Sitter space, preserving a chosen Killing field of parabolic, elliptic or hyperbolic type. This allows them to prove the existence of spacelike Zoll surfaces not smoothly conformal to a cover of the de Sitter space. Also, in contrast with the rigidity displayed in [C. Pries, op. cit.] for the projective plane, the authors exhibit a Lorentzian Möbius strip of nonconstant curvature, all of whose spacelike geodesics are closed. Finally, the conformality problem for spacelike Zoll cylinders with a nontrivial Killing vector field is studied. *Javier Lafuente-López*

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Mathematical Sciences Regional Conference Series in Applied Mathematics, No. 7, Philadelphia: SIAM, 1972, MR 0469146, Zbl 0321.53001. MR0469146

11. C. Pries, *Geodesics closed on the projective plane*, Geom. Funct. Anal. 18 (2009), no. 5, 1774–1785, MR 2481742, Zbl 1160.53022. MR2481742

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MR3433977 53A04

Castro, Ildefonso (E-JAE); **Castro-Infantes, Ildefonso** (E-GRAN-G)

Plane curves with curvature depending on distance to a line. (English summary)

Differential Geom. Appl. **44** (2016), 77–97.

The fundamental theorem of existence and uniqueness for Euclidean plane curves states that a curve is uniquely determined, up to rigid motion, by its curvature given as a function of its arc-length. In [Amer. Math. Monthly **106** (1999), no. 9, 835–841; MR1732664] D. A. Singer considered the problem of determining a curve when its curvature is a function $\kappa = \kappa(x, y)$ of its position. He proved that the problem for $\kappa = \kappa(r)$ with $r = \sqrt{x^2 + y^2}$ is solvable by quadratures if $r\kappa(r)$ is a continuous function.

In this article the authors propose to study this sort of problem for $\kappa = \kappa(y)$, i.e., when the curvature depends on the signed distance y to the x-axis. They show that if $\kappa(y)$ is a non-null continuous function, the problem of determining such a curve is solvable locally by quadratures, and the curve is uniquely determined by the *primitive curvature* K (that is, $K = K(y)$, such that $K' = \kappa$), up to translations in the x-direction.

The authors study six different situations where they are successful with the procedure described in the main theorem. Namely: $\kappa(y)$ equal to $2\lambda y$, λ/y^2, $\lambda \cos y$, $\lambda \cosh y$, $\lambda \exp(-y)$, $\lambda/\cos^2 y$ (with $\lambda > 0$). In this way, they provide new characterizations of some well-known curves. *Javier Lafuente-López*

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MR3416437 53C50 53A30
Frances, C. [Frances, Charles] (F-STRAS-I)
About pseudo-Riemannian Lichnerowicz conjecture. (English summary)
Transform. Groups **20** (2015), *no. 4,* 1015–1022.

Let (M, g) be a connected compact pseudo-Riemannian manifold of dimension $n \geq 3$. Assume that for any metric $\overline{g}$ in the conformal class $[g]$, the conformal group of $(M, [g])$ is not contained in the isometry group of $(M, \overline{g})$. The conjecture raised by Lichnerowicz was that, in the Riemannian case, (M, g) is conformally diffeomorphic to the standard sphere $\mathbb{S}^n$ (a complete solution to this conjecture was given independently by M. Obata [J. Differential Geometry **6** (1971/72), 247–258; MR0303464] and J. Ferrand [Acad. Roy. Belg. Cl. Sci. Mém. Coll. in–8° (2) **39** (1971), no. 5, 44 pp.; MR0322739]). The Lichnerowicz pseudo-Riemannian conjecture (raised by G. D'Ambra and M. Gromov [in *Surveys in differential geometry (Cambridge, MA, 1990)*, 19–111, Lehigh Univ., Bethlehem, PA, 1991; MR1144526]) says that in the pseudo-Riemannian case, $(M, [g])$ must be conformally flat.

The aim of this paper is to provide a negative answer to the Lichnerowicz pseudo-Riemannian conjecture. In fact, the author constructs for every two integers p, q with $q \geq p \geq 2$, a 2-parameter analytical family of counterexamples of pseudo-Riemannian type (p, q) on the product $\mathbb{S}^1 \times \mathbb{S}^{p+q-1}$. Nevertheless, the Lichnerowicz pseudo-Riemannian conjecture remains open in the Lorentzian case. *Javier Lafuente-López*

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MR3320888 53A05 53C50
Honda, Atsufumi (J-MNCT); **Izumiya, Shyuichi** [Izumiya, Shyūichi] (J-HOKK)
The lightlike geometry of marginally trapped surfaces in Minkowski space-time. (English summary)
J. Geom. **106** (2015), *no. 1,* 185–210.

The article under review studies marginally trapped surfaces in Minkowski space-time $\mathbb{R}^4_1$ using the lightlike differential geometry for spacelike submanifolds of codimension two [see S. Izumiya and M. d. C. Romero Fuster, Selecta Math. (N.S.) **13** (2007), no. 1, 23–55; MR2330586]. The basic idea of this geometry (partially developed in this paper) is to use the two lightlike normal directions along the spacelike surface to define lightcone curvatures, in the same way as the normal vector is used in the classical theory of surfaces.

A spacelike surface in $\mathbb{R}^4_1$ is called marginally trapped if its mean curvature vector is isotropic (lightlike or null) at every point. A first result obtained is that a totally umbilical marginally trapped surface is given, up to rigid Lorentzian motions, by a graph $X_f(u_1,u_2)=(f(u_1,u_2),f(u_1,u_2),u_{1,}u_2)$ for a smooth function $f(u_1,u_2)$. Next, the authors consider a system of partial differential equations for marginally trapped surfaces in this general graph form, and obtain as a consequence that the graph $X_f(u_1,u_2)$ is strongly marginally trapped (i.e. with null mean curvature vector) if and only if f is harmonic. In fact, the class of strongly marginally trapped surfaces includes a generalization of the notion of minimal surfaces in $\mathbb{R}^3_0\subset\mathbb{R}^4_1$. However, the authors show that the classical Bernstein theorem for minimal surfaces does not hold even for the strongly marginally trapped case.

Pursuing the analogy with the minimal Euclidean surfaces [see J. A. Aledo Sánchez, J. A. Gálvez and P. Mira, Ann. Global Anal. Geom. **28** (2005), no. 4, 395–415; MR2200000; H. Liu, Math. Phys. Anal. Geom. **16** (2013), no. 2, 171–178; MR3063956; B. Palmer, Calc. Var. Partial Differential Equations **41** (2011), no. 3-4, 387–395; MR2796236], the authors also give a characterization of marginally trapped surfaces by the variational problem of the area functional with respect to the lightlike normal directions.

Finally, the authors display as special cases of marginally trapped surfaces the maximal surfaces in $\mathbb{R}^3_1$, the spacelike surfaces with constant mean curvature ±1 in de Sitter space and hyperbolic 3-space, as well as (using [S. Izumiya, Mosc. Math. J. **9** (2009), no. 2, 325–357, back matter; MR2567992]) the intrinsic flat spacelike surfaces in the Minkowskian lightcone. *Javier Lafuente-López*

MR3293858 53A05 53A04 53Z05
Karakuş, Fatma (TR-SINOPSA-M); **Yayli, Yusuf** **[Yaylı, Yusuf]** (TR-ANKS)
The Fermi derivative in the hypersurfaces. (English summary)
Int. J. Geom. Methods Mod. Phys. **12** (2015), *no. 1,* 1550002, 12 *pp.*

The Fermi-Walker derivative is used in general relativity and it is defined along any unit speed curve of a semi-Riemannian manifold [see, for instance, R. K. Sachs and H. H. Wu, *General relativity for mathematicians*, Springer, New York, 1977 (p. 51); MR0503498].

This paper is an elementary study of the Fermi derivative, Fermi parallelism and non-rotating frames on a hypersurface M^n in the Euclidean space E^{n+1}. For $n = 2, 3$ a further characterization of the Fermi derivative is given according to the Darboux (for $n = 2$) or Frenet (for $n = 3$) frames, and it is used to study the Fermi derivative and parallelism along some special curves. Finally, the authors apply a similar method using the Frenet frame for curves in the Euclidean space E^n and in a general Riemannian manifold, but no significant results are obtained. *Javier Lafuente-López*

MR3250513 53C21 53A30 53C20
Pina, Romildo **[Pina, Romildo da Silva]** (BR-FGS-MS);
Adriano, Levi (BR-FGS-MS); **Pieterzack, Mauricio** (BR-FGS-MS)
Prescribed diagonal Ricci tensor in locally conformally flat manifolds. (English summary)
J. Math. Anal. Appl. **421** (2015), *no. 1,* 893–904.

The authors study the following problem (P): Consider the Euclidean space $(\mathbb{R}^n, g)$, with $n \geq 3$, coordinates $x = (x_1, \ldots, x_n)$ and $g_{ij} = \delta_{ij}$. Consider the diagonal $(0,2)$-tensor $T = \sum_i f_i(x)dx_i^2$, where each $f_i(x)$ is a smooth function such that $\sum_{i=1}^n f_i(x) \neq (n-1)(f_i(x) + f_j(x))$ for all $x \in \mathbb{R}^n$ and all $i \neq j$. Find all metrics $\overline{g}$, conformal to g, such that $Ric(\overline{g}) = T$.

For such tensors, the authors provide necessary and sufficient conditions for the existence of a metric $\overline{g}$ conformal to g that solves the Ricci tensor equation $Ric(\overline{g}) = T$. These conditions state that the functions f_i must satisfy a well-defined (but very complicated) nonlinear second-order system of differential equations. This result can be extended trivially to locally conformally flat manifolds.

In order to provide explicit examples, the authors consider the problem (P) for some particular cases, such as $Ric(\overline{g}) = T = fg$, and they discuss all solutions for f and $\overline{g}$.

(Pina and K. Tenenblat have obtained analogous results in [Differential Geom. Appl. **24** (2006), no. 2, 101–107; MR2198786].)

Also, they discuss the problem (P), taking T with $f_i(x) = f_i(x_k)$ for all i and some k. Then more examples are presented, including singular tensors T and cases where the metric $\overline{g}$ is complete. *Javier Lafuente-López*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR3159333 53B05 53B20
Vanžurová, Alena (CZ-PLCKS-AG)
On metrizability of locally homogeneous affine 2-dimensional manifolds. (English summary)
Arch. Math. (Brno) **49** (2013), *no. 5,* 347–357.

This paper deals with the metrization problem for a two-dimensional manifold M^2 with a symmetric linear connection ∇, that is, the existence of a semi-Riemannian metric g such that $\nabla g = 0$.

In a previous paper [Acta Univ. Palack. Olomuc. Fac. Rerum Natur. Math. **48** (2009), 157–170; MR2641956] the author and P. Žáčková showed that a nowhere flat ∇ is metrizable if and only if its Ricci tensor Ric is symmetric regular and recurrent with $\nabla Ric = df \otimes Ric$ for some smooth function f, and then the solution is $g = \exp(-f + b)Ric$ for $b \in \mathbb{R}$.

In this paper, the author applies the above result to solve (locally) the metrization problem for the affine locally homogeneous 2-manifold (M^2, ∇) (that is for any points p, q in M there are neighborhoods $U \ni p$, $V \ni q$ and an affine transformation $\varphi\colon (U, \nabla|_U) \to (V, \nabla|_V)$ sending p into q).

According to the classification established in [B. Opozda, Differential Geom. Appl. **21** (2004), no. 2, 173–198; MR2073824] and [T. Arias-Marco and O. Kowalski, Monatsh. Math. **153** (2008), no. 1, 1–18; MR2366132] for these manifolds, the author establishes two types A and B.

Type A manifolds are the ones for which there is a coordinate system around any point of M such that the connection has constant Christoffel symbols. For this type she shows that locally flat connections are exactly the only metrizable connections. This result has been obtained also in [T. Arias-Marco and O. Kowalski, op. cit.].

For type B, the author obtains a two-parameter family of nowhere flat (locally) metrizable connections. *Javier Lafuente-López*

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MR3157055 53A30 53C50

Frances, Charles (F-PARIS11); **Melnick, Karin** (1-MD)

Formes normales pour les champs conformes pseudo-riemanniens. (French. English, French summaries) [[Normal forms for pseudo-Riemannian conformal vector fields]]

Bull. Soc. Math. France **141** (2013), *no. 3,* 377–421.

Let X be a conformal vector field on a pseudo-Riemannian manifold (M, g) of signature (p, q), i.e. a vector field whose local flow leaves invariant the conformal class of g.

In this paper the authors investigate the dynamic of such field in a neighborhood of a singularity $x_0 \in M$. In particular, they wonder if X is locally conjugated to a so-called Möbius vector field, i.e. a vector field given by the action of a 1-parameter subgroup of the stabilizer subgroup P in the conformal homogeneous model $PO(p+1, q+1)/P$ (called Einstein universe, which is the projectivization of the lightcone of $\mathbb{R}^{p+1,q+1}$).

Using the canonical Cartan connection for conformal structures (see [R. W. Sharpe, *Differential geometry*, Grad. Texts in Math., 166, Springer, New York, 1997; MR1453120] or [S. Kobayashi, *Transformation groups in differential geometry*, reprint of the 1972 edition, Classics Math., Springer, Berlin, 1995; MR1336823]), the authors associate a Möbius vector field to each zero of X: its holonomy vector field. It is then proven that some properties of X can be read on its holonomy vector field. The main result of the article then follows:

In the analytic Lorentzian case ($\dim M \geq 3$) they get that X is analytically conjugate to its holonomy vector field and then either X is analytically linearizable around its singularity or (M, g) is conformally flat.

In some cases the same results are obtained for arbitrary signature (p, q).

Furthermore, using a similar method, a local version of the above main result for Riemannian C^∞-manifolds was obtained by the first author in [Geom. Dedicata **158** (2012), 35–59; MR2922702] (see also [D. V. Alekseevskiĭ, Mat. Sb. (N.S.) **89(131)** (1972), 280–296, 356; MR0334077]).

Without the hypothesis of analyticity, the methods allow one to obtain information about the local behavior of certain conformal fields (ones where the differential of the flow at the singular point is bounded), such as linearizability, essentiality (the fact that X preserves a metric or not) or completeness around the singularity.

Javier Lafuente-López

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MR3096876 53B25 53A55 53C25 53C40
Călin, Constantin (R-TUGA);
Crasmareanu, Mircea [**Crâşmăreanu, Mircea**] (R-IASIM)
Slant curves and particles in three-dimensional warped products and their Lancret invariants. (English summary)
Bull. Aust. Math. Soc. **88** (2013), *no. 1,* 128–142.

The authors study slant curves of a three-dimensional warped product $M_3 = I \times_f \mathbb{E}_2$ with Euclidean factors, where $f = f(z)$ is a smooth and strictly positive function on the open interval I. A Frenet curve $\gamma = \gamma(s)$ is said to be slant if its *structural angle* θ defined by the angle of γ' with the vertical vector field ∂_z is constant.

The aim of this paper is to obtain a Lancret-type invariant for these curves, similar to the well-known Lancret invariant (the ratio of torsion and curvature) used in the classical Lancret theorem for the Euclidean case ($f = 1$). Legendre curves ($\theta \equiv \pm\pi/2$) and slant-helices are analyzed as a particular case. They also give an example of a proper slant curve in the hyperbolic space ($f = \exp z$).

Slant curves have been studied by these same authors and others in several three-dimensional geometries [M. Barros, Proc. Amer. Math. Soc. **125** (1997), no. 5, 1503–1509; MR1363411; C. Călin, M. Crâşmăreanu and M.-I. Munteanu, J. Math. Anal. Appl. **394** (2012), no. 1, 400–407; MR2926230; C. Călin and M. Crâşmăreanu, Mediterr. J. Math. **10** (2013), no. 2, 1067–1077; MR3045696; J. T. Cho, J. Inoguchi and J.-E. Lee, Bull. Aust. Math. Soc. **74** (2006), no. 3, 359–367; MR2273746; J. T. Cho and J.-E. Lee, Bull. Aust. Math. Soc. **78** (2008), no. 3, 383–396; MR2472274]. *Javier Lafuente-López*

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MR3104787 53A30 53A07 53C40 53C50
Shu, Shichang (PRC-XYNU-SMI); **Su, Bianping** [Su, Bian Ping] (PRC-XUAT)
Conformal isoparametric spacelike hypersurfaces in conformal spaces $\mathbb{Q}_1^4$ and $\mathbb{Q}_1^5$. (English summary)
Ukrainian Math. J. **64** (2012), *no. 4,* 634–652.

The conformal space $\mathbb{Q}_1^{n+1}$ is defined as the quadric

$$\mathbb{Q}_1^{n+1} = \{[\xi] \in \mathbb{RP}^{n+2} : \langle \xi, \xi \rangle_2 = 0\}$$

where $\langle , \rangle_s$ is the Lorentzian inner product in $\mathbb{R}_s^{n+s}$ with negative index s in $\mathbb{R}^{n+s}$. According to [C. X. Nie et al., Sci. China Math. **53** (2010), no. 4, 953–965; MR2640180] we may regard $\mathbb{Q}_1^{n+1}$ as the common conformal compactification of Lorentzian space $\mathbb{R}_1^{n+1}$ and the de Sitter and anti-de Sitter spheres ($\mathbb{S}_1^{n+1}$ and $\mathbb{H}_1^{n+1}$).

In this paper the authors, following [op. cit.], establish the basic theory of conformal geometry for nondegenerate hypersurfaces $x\colon M \to \mathbb{Q}_1^{n+1}$ and define four basic conformal invariants of x: the conformal metric g, the conformal form Φ, the conformal Blaschke tensor $\mathbf{A}$ and the conformal second fundamental form $\mathbf{B}$. These invariants can be viewed as the analogues of the corresponding Möbius invariants for hypersurfaces in the sphere [Z. J. Hu and H. Li, Sci. China Ser. A **47** (2004), no. 3, 417–430; MR2078352].

The hypersurface is said to be isoparametric if g is nondegenerate, Φ vanishes and the eigenvalues of $\mathbf{B}$ are constant.

In the present paper, the authors obtain a complete classification of conformal isoparametric spacelike hypersurfaces in $\mathbb{Q}_1^4$ and $\mathbb{Q}_1^5$ (see [Z. J. Hu and H. Li, Nagoya Math. J. **179** (2005), 147–162; MR2164403; Z. J. Hu, H. Li and C. P. Wang, Monatsh. Math. **151** (2007), no. 3, 201–222; MR2329083] for the analogous results in $\mathbb{S}^4$ and $\mathbb{S}^5$). The key to establishing this classification is to prove that these hypersurfaces have necessarily parallel conformal second fundamental forms, and then copy the results of [C. X. Nie and C. X. Wu, Acta Math. Sinica (Chin. Ser.) **51** (2008), no. 4, 685–692; MR2454005].

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR3027358 53A55 16R50 53B05
Dušek, Zdeněk (CZ-PLCKS-NDM); **Kowalski, Oldřich** (CZ-KARL-MI)
Rational involutive automorphisms related with standard representations of $\mathrm{SL}(2,\mathbb{R})$**. (English summary)**
Bull. Belg. Math. Soc. Simon Stevin **19** (2012), *no. 3,* 523–533.

It is well known that the group $\mathrm{SL}(2,\mathbb{R})$ admits an irreducible representation in any dimension $n+1$. This representation can be constructed using its natural action on the space P_n of the real homogeneous polynomials of degree n in two variables and the identification of $a_0x^n + a_1x^{n-1}y + \cdots + a_ny^n$ with $(a_0, a_1, \ldots, a_n)$.

In this paper it is proved that each such representation induces an involutive rational mapping of an open dense subset of $\mathbb{R}^{n+1}$ onto itself. The construction of this mapping is based on the relationship between two Hilbert bases for the invariants with respect to the actions on P_n given by the subgroups $\left\{\left(\begin{smallmatrix}1&t\\0&1\end{smallmatrix}\right)\right\}$ and $\left\{\left(\begin{smallmatrix}1&0\\t&1\end{smallmatrix}\right)\right\}$ respectively. The key is that these actions are connected by the involutive permutation $p\colon (a_0, a_1, \ldots, a_n) \mapsto (a_n, a_{n-1}, \ldots, a_0)$. In dimensions 3, 4 and 5, corresponding involutive mappings are constructed explicitly. *Javier Lafuente-López*

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MR3025150 53A30 53C50
Frances, Charles (F-PARIS11-M)
Dégénerescence locale des transformations conformes pseudo-riemanniennes. (French. English, French summaries) [[Local degeneracy of pseudo-Riemannian conformal transformations]]
Ann. Inst. Fourier (Grenoble) **62** (2012), *no. 5,* 1627–1669.

This paper studies the closure of the space $Conf(M,N)$ of conformal C^∞-immersions between two connected semi-Riemannian manifolds M and N, considered as a subspace of the continuous maps $\mathcal{C}^0(M,N)$ with the compact convergence topology. A similar study for quasiconformal injections between Riemannian n-manifolds ($n \geq 2$) was developed in [J. Ferrand, J. Anal. Math. **69** (1996), 1–24; MR1428092].

The main results can be summarized as follows. Suppose (M,g) and (N,h) have the same signature (p,q) and $p+q=n\geq 3$; then:

(A) Let f_k be a sequence in $Conf(M,N)$ converging to a map $f\in\mathcal{C}^0(M,N)$. Then f is C^∞-differentiable, the convergence is C^∞ over compacts and exactly one of the following three cases holds:

(1) $f\in Conf(M,N)$.
(2) The function f is constant; in this case (M,g) is locally conformally Ricci-flat (this means that each $x\in M$ has a neighborhood U and a Ricci-flat metric in the conformal class of $[g]|_U$).
(3) The map f is (locally) a submersion on an isotropic nontrivial submanifold of N.

(B) If $Conf(M,N)$ is not closed in $\mathcal{C}^0(M,N)$ then:

(1) If (M,g) is Riemannian, then it is conformally flat.
(2) If (M,g) is Lorentzian, then it is locally conformally Ricci-flat. Moreover, if there exists a constant map in the boundary of $Conf(M,N)$, then (M,g) is conformally flat.

To make this study the author uses the tools of the Cartan geometry [see R. W. Sharpe, *Differential geometry*, Grad. Texts in Math., 166, Springer, New York, 1997; MR1453120] of $(M,[g])$ modeled on the homogeneous space $Ein^{p,q}\simeq \mathrm{O}(p+1,q+1)/P$, where $Ein^{p,q}$ is the projective null cone in $\mathbf{P}(\mathbb{R}^{n+2}_{p+1})$. *Javier Lafuente-López*