On strong $\mathfrak{c}$-algebrability of some families of functions

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47th Summer Symposium in Real Analysis

$f : \mathbb{R} \to \mathbb{R}$

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L. Bukovský, *The structure of the Real line*, Monografie Matematyczne (New Series), vol. 71. Birkhäuser, Basel (2011).

- $\Sigma^0_0 = \Pi^0_0 = \Delta^0_0$ the set of all clopen subsets of $(\mathbb{R}, \tau_e)$
- Σ^0_1 the set of all open subsets of $(\mathbb{R}, \tau_e)$
- Π^0_1 the set of all closed subsets of $(\mathbb{R}, \tau_e)$
- $\Sigma^0_\alpha = \left\{ \bigcup_n A_n : \ A_n \in \bigcup_{\beta<\alpha} \Pi^0_\beta, \ n \in \omega \right\}$
- $\Pi^0_\alpha = \{\mathbb{R} \setminus A : A \in \Sigma^0_\alpha\} = \left\{ \bigcap_n A_n : \ A_n \in \bigcup_{\beta<\alpha} \Sigma^0_\beta, \ n \in \omega \right\}$
- $\Delta^0_\alpha = \Sigma^0_\alpha \cap \Pi^0_\alpha$

for $1 < \alpha < \omega_1$.

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for $1 < \alpha < \omega_1$.

Σ^0_2 - the family of F_σ sets
Π^0_2 - the family of G_δ sets.

$$
\begin{tikzcd}
 & \Sigma^0_1 \arrow[dr, dash, "\subseteq"] & & \Sigma^0_2 \arrow[dr, dash, "\subseteq"] & & & \Sigma^0_{\alpha+1} \arrow[dr, dash, "\subseteq"] & & \\
\Delta^0_0 \arrow[ur, dash, "\subseteq"] \arrow[dr, dash, "\subseteq"'] & & \Delta^0_1 \arrow[ur, dash, "\subseteq"] \arrow[dr, dash, "\subseteq"'] & & \cdots \arrow[r, dash, "\subseteq"] & \Delta^0_\alpha \arrow[ur, dash, "\subseteq"] \arrow[dr, dash, "\subseteq"'] & & \Delta^0_{\alpha+1} \arrow[r, dash, "\subseteq"] & \cdots \\
 & \Pi^0_1 \arrow[ur, dash, "\subseteq"] & & \Pi^0_2 \arrow[ur, dash, "\subseteq"] & & & \Pi^0_{\alpha+1} \arrow[ur, dash, "\subseteq"] & &
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- $\mathcal{B}_0 = \mathcal{C}$;
- $\mathcal{B}_\alpha$ - the family of all functions f such that there exists a sequence $(f_n)_{n\in\mathbb{N}}$ from $\bigcup\limits_{\beta<\alpha} \mathcal{B}_\beta$ satisfying $\lim\limits_{n\to\infty} f_n = f$.
- $\mathcal{B}_\alpha^0 = \mathcal{B}_\alpha \setminus \bigcup\limits_{\beta<\alpha} \mathcal{B}_\beta$

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- $\mathcal{B} = \bigcup\limits_{\alpha<\omega_1} \mathcal{B}_\alpha$;
- $f \in \mathcal{B}_\alpha$ iff for each $V \in \tau_e$ holds $f^{-1}(V) \in \Sigma^0_{\alpha+1}$

S. Kempisty, *Sur les fonctions quasicontinues*, Fund. Math. **19** (1932), 184–197.

Definition (S. Kempisty, 1932)

A function f is quasi-continuous at a point x if for every neighbourhood U of x and for every neighbourhood V of $f(x)$ there exists a non-empty open set $G \subset U$ such that $f(G) \subset V$.

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$\mathcal{Q}$ - the family of all quasi-continuous functions

S. Marcus, *Sur les fonctions quasicontinues au sens de S. Kempisty*, Coll. Math. 8 (1961), 47-53.

$$\boxed{\text{for each } \alpha,\ 0 < \alpha < \omega_1,\ \text{holds } \mathcal{Q} \cap \mathcal{B}^0_\alpha \neq \emptyset}$$

H. P. Thielman, *Types of functions*, Amer. Math. Monthly **60** (1953), 156–161.

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A function f is cliquish at a point x if for every neighbourhood U of x and for each $\epsilon > 0$ there exists a non-empty open set $G \subset U$ such that $\mid f(y) - f(z) \mid < \epsilon$ for each $y, z \in G$.

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$\mathcal{C}_q$ - the family of all cliquish functions

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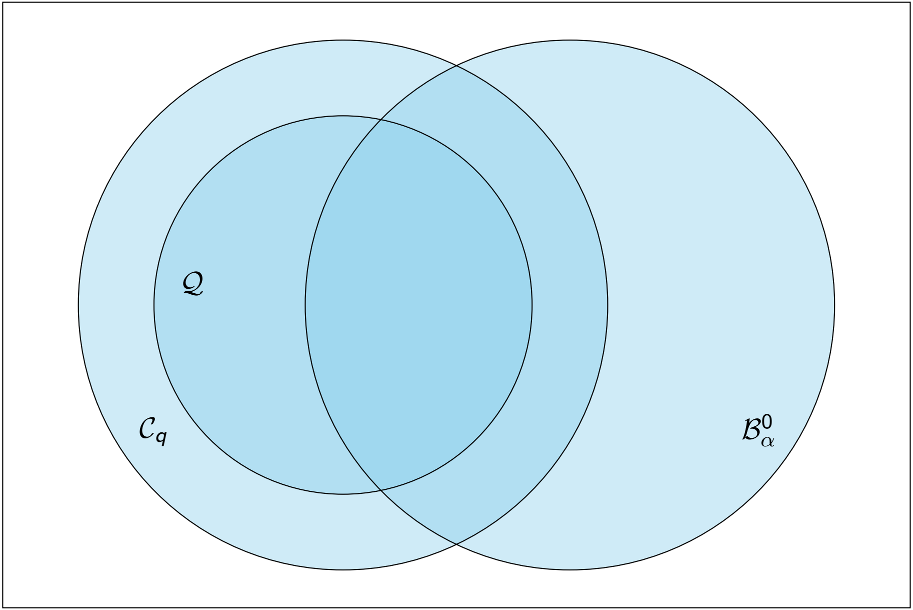
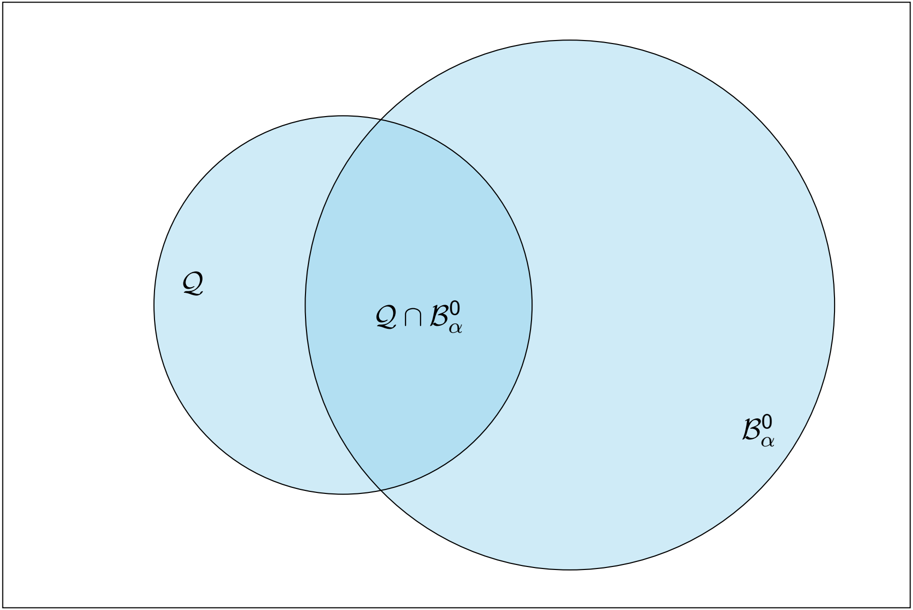
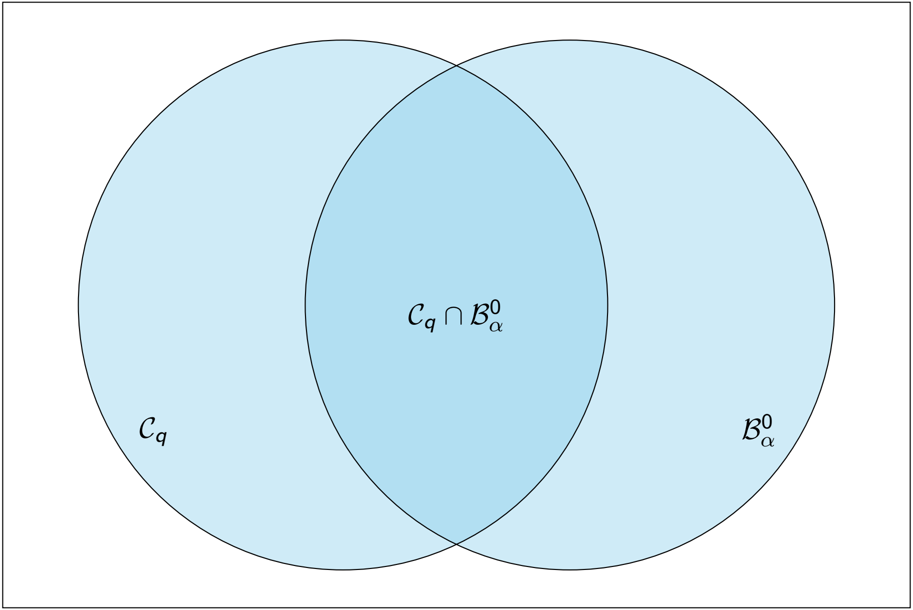
for each α, $0 < \alpha < \omega_1$, holds $\mathcal{Q} \cap \mathcal{B}^0_\alpha \neq \emptyset$

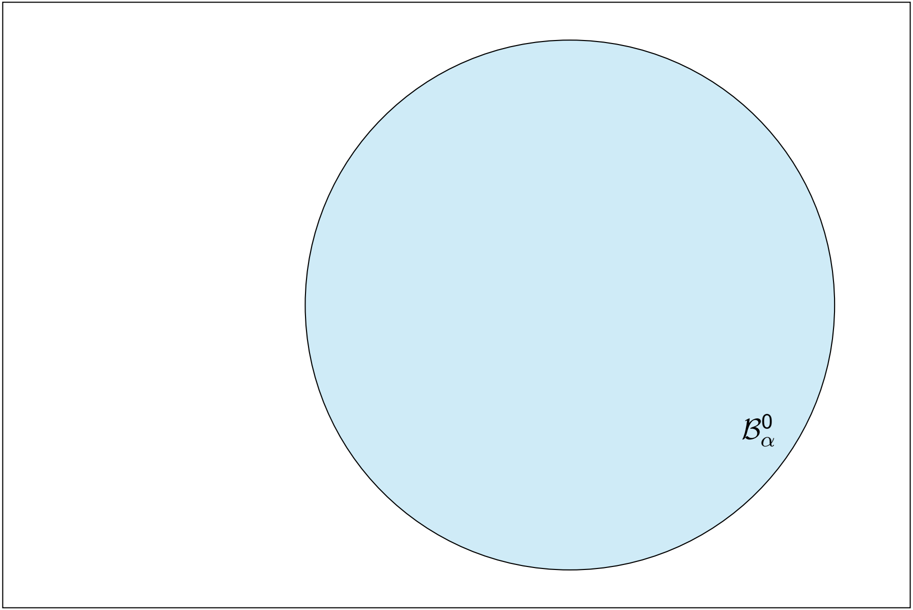
$$\mathcal{Q} \subset \mathcal{C}_q$$

for each α, $0 < \alpha < \omega_1$, holds $\mathcal{C}_q \cap \mathcal{B}^0_\alpha \neq \emptyset$


$\mathcal{Q}$
$\mathcal{C}_q$
$\mathcal{B}_\alpha^0$


$\mathcal{Q}$
$\mathcal{Q} \cap \mathcal{B}_\alpha^0$
$\mathcal{B}_\alpha^0$


$\mathcal{C}_q \cap \mathcal{B}^0_\alpha$
$\mathcal{C}_q$
$\mathcal{B}^0_\alpha$


$\mathcal{B}^0_\alpha$

$$\boxed{\mathcal{Q} \cap \mathcal{B}^0_\alpha \subset \mathcal{C}_q \cap \mathcal{B}^0_\alpha \subset \mathcal{B}^0_\alpha, \text{ for } \alpha < \omega_1}$$

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$$f \in \mathcal{C}_q \Leftrightarrow D(f) \text{ is meager}$$

$$\mathcal{B}_1 \subsetneq \mathcal{C}_q$$

$$\boxed{\mathcal{Q} \cap \mathcal{B}^0_\alpha \subset \mathcal{C}_q \cap \mathcal{B}^0_\alpha \subset \mathcal{B}^0_\alpha, \text{ for } \alpha < \omega_1}$$

$$f \in \mathcal{C}_q \Leftrightarrow D(f) \text{ is meager}$$

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1. $\mathcal{C} = \mathcal{Q} \cap \mathcal{C} \subset \mathcal{C}_q \cap \mathcal{C} \subset \mathcal{C}$, for $\alpha = 0$
2. $\mathcal{Q} \cap \mathcal{B}_1 \subset \mathcal{C}_q \cap \mathcal{B}_1 = \mathcal{B}_1 \subset \mathcal{C}_q$, for $\alpha = 1$

L. Zajiček, *On σ-porous sets in abstract spaces*, Abstr. Appl. Anal. **5** (2005), 509–534.

(X, ρ) - metric space

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$M \subset X$, $x \in X$, $r > 0$

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$$\gamma\left(x, r, M\right) = \sup\{t \geq 0 : \exists_{z \in X} B\left(z, t\right) \subset B\left(x, r\right) \setminus M\}$$

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The set $M \subset X$ is

1. porous if $p(M, x) > 0$ for each $x \in M$
2. strongly porous if $p(M, x) = 1$ for each $x \in M$

$f, g \in \mathcal{B}$

$$\boxed{\rho(f, g) = min\{1, sup\{| f(t) - g(t) |: t \in \mathbb{R}\}\}}$$

J.Hejduk, G.I., *On the porosity of Baire class functions* (to submitted)

for $1 < \alpha < \omega_1$:

$$\mathcal{Q} \cap \mathcal{B}_\alpha \subset \mathcal{C}_q \cap \mathcal{B}_\alpha \subset \mathcal{B}_\alpha$$

strongly porous strongly porous

$$\underbrace{\mathcal{C}_q \cap \mathcal{B}_\alpha \subset \mathcal{C}_q \cap \mathcal{B}_{\alpha+1}}_{\text{strongly porous}}$$

$$\underbrace{\mathcal{B}_\alpha \subset \mathcal{B}_{\alpha+1}}_{\text{strongly porous}}$$

for $1 \leq \alpha < \omega_1$:

$$\underbrace{\mathcal{Q} \cap \mathcal{B}_\alpha \subset \mathcal{Q} \cap \mathcal{B}_{\alpha+1}}_{\text{strongly porous}}$$

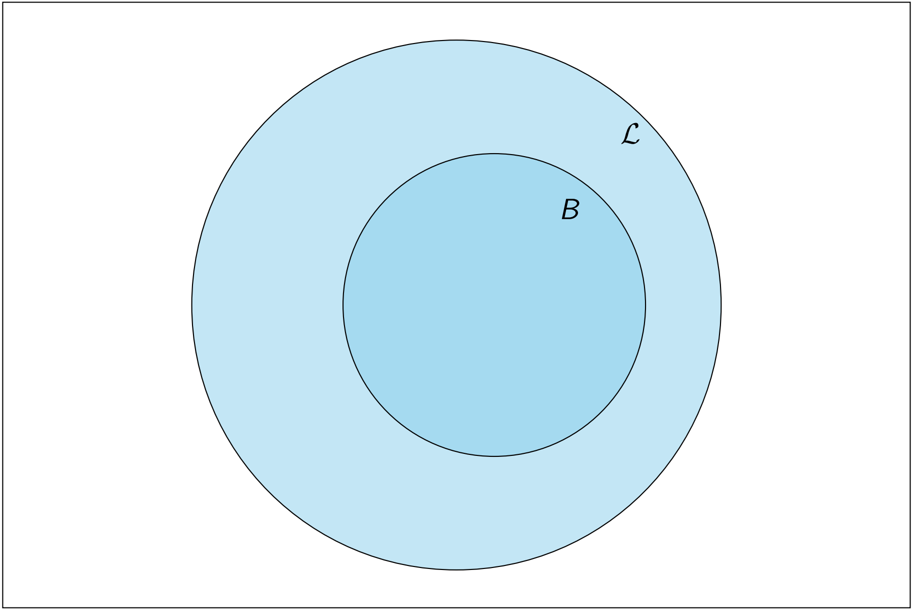
M. Balcerzak, A. Bartoszewicz, M. Filipczak, *Nonseparable spaceability and strong algebrability of sets of continuous singular functions*, J. Math. Anal. Appl. **407** (2013), 263–269.

Definition

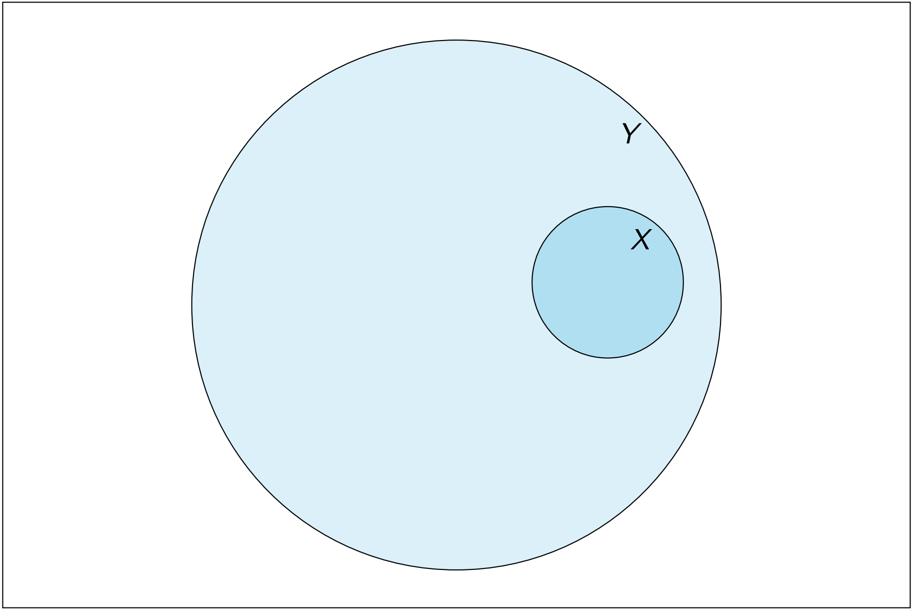
Let $\mathcal{L}$ be a linear commutative algebra. We say that $A \subset \mathcal{L}$ is strongly $\mathfrak{c}$-algebrable if $A \cup \{\Theta\}$ contains a $\mathfrak{c}$-generated algebra B that is isomorphic with a free algebra.

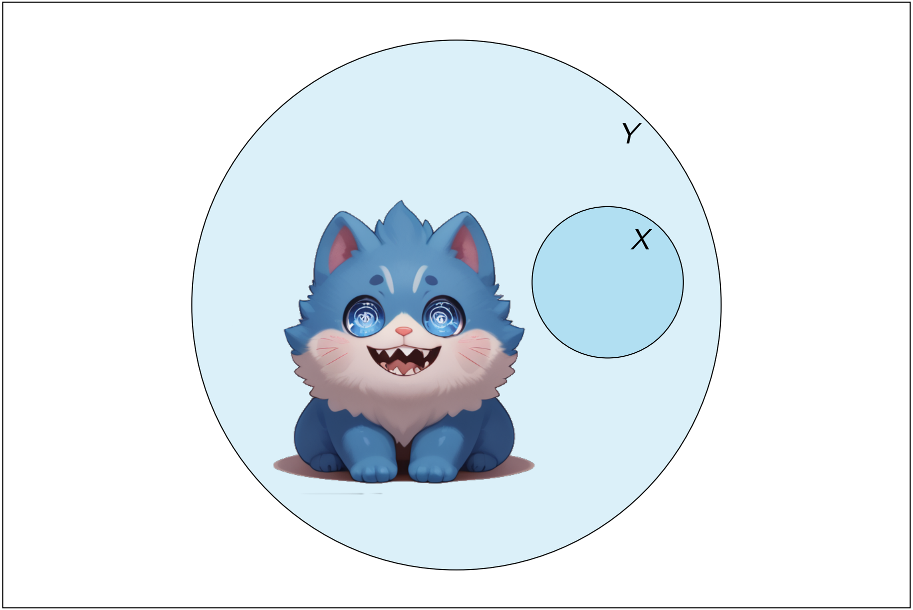
$X = \{x_\alpha : \alpha < \mathfrak{c}\}$ - the set of generators of this free algebra.

$X = \{x_\alpha : \alpha < \mathfrak{c}\}$ is the set of generators of some free algebra contained in $A \cup \{\Theta\}$ iff the set $\widetilde{X}$ of elements of the form $x_{\alpha_1}^{k_1} x_{\alpha_2}^{k_2} \dots x_{\alpha_n}^{k_n}$ is linearly independent and all linear combinations of elements from $\widetilde{X}$ are in $A \cup \{\Theta\}$.


$\mathcal{L}$
B


$\mathcal{L}$


Y
X

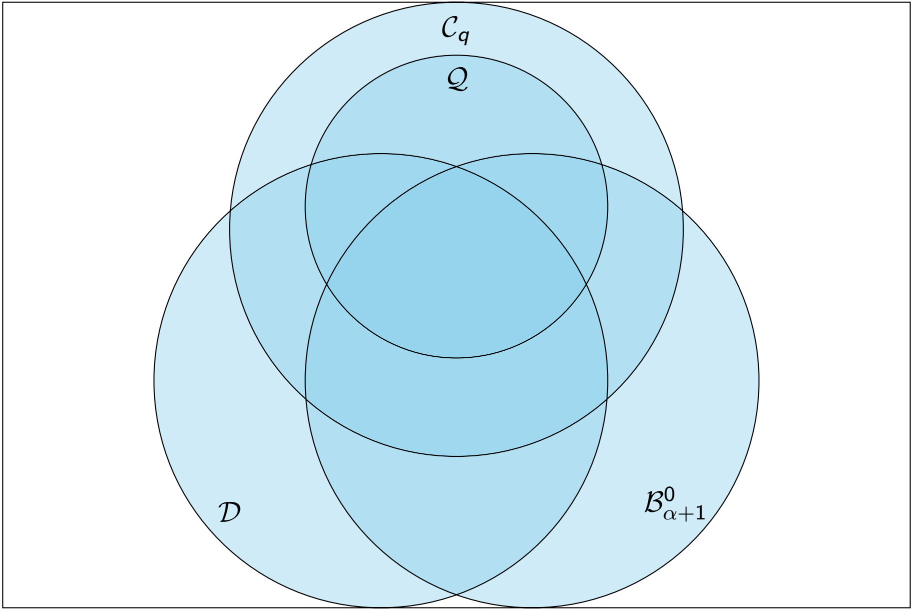

Y
X

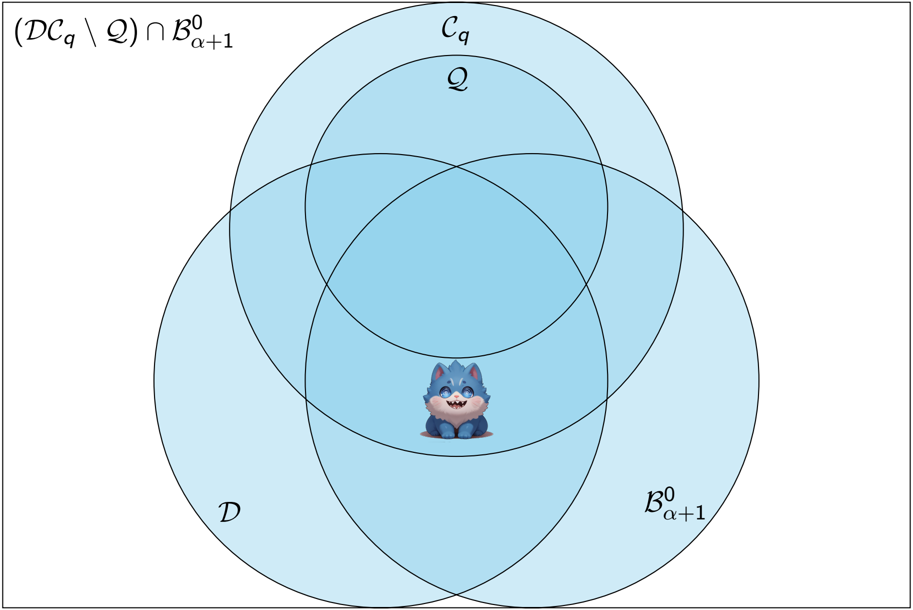
$\mathcal{Q}, \mathcal{C}_{\boldsymbol{q}}, \mathcal{B}^0_\alpha,$

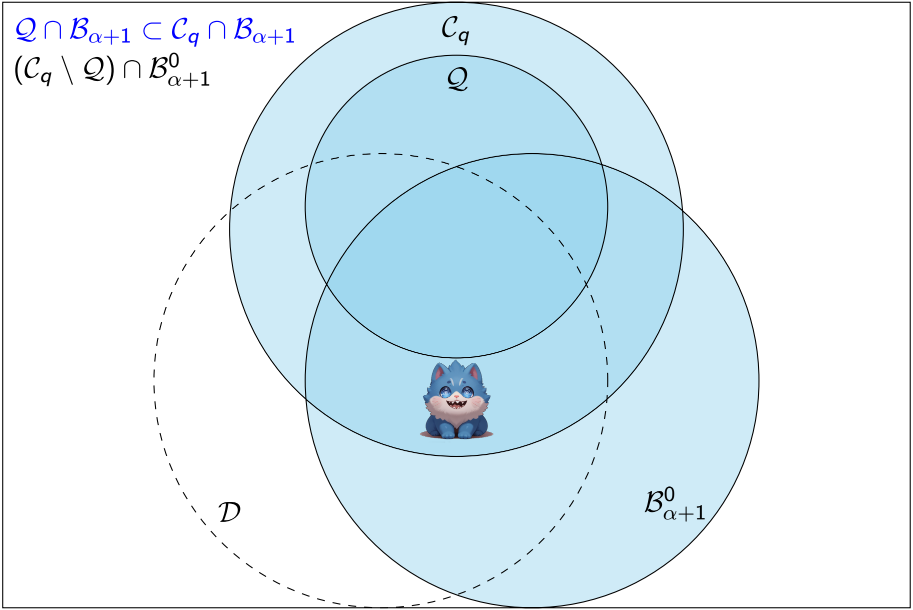
$\mathcal{Q}, \mathcal{C}_{\boldsymbol{q}}, \mathcal{B}^0_\alpha, \mathcal{D}$

Theorem

For each ordinal number α, $1 \leq \alpha < \omega_1$, the set $(\mathcal{D}\mathcal{C}_{\boldsymbol{q}} \setminus \mathcal{Q}) \cap \mathcal{B}^0_{\alpha+1}$ is strongly $\mathfrak{c}$-algebrable.


$\mathcal{C}_q$
$\mathcal{Q}$
$\mathcal{D}$
$\mathcal{B}^0_{\alpha+1}$


$(\mathcal{DC}_q \setminus \mathcal{Q}) \cap \mathcal{B}^0_{\alpha+1}$
$\mathcal{C}_q$
$\mathcal{Q}$
$\mathcal{D}$
$\mathcal{B}^0_{\alpha+1}$


$\mathcal{Q} \cap \mathcal{B}_{\alpha+1} \subset \mathcal{C}_q \cap \mathcal{B}_{\alpha+1}$
$(\mathcal{C}_q \setminus \mathcal{Q}) \cap \mathcal{B}^0_{\alpha+1}$
$\mathcal{C}_q$
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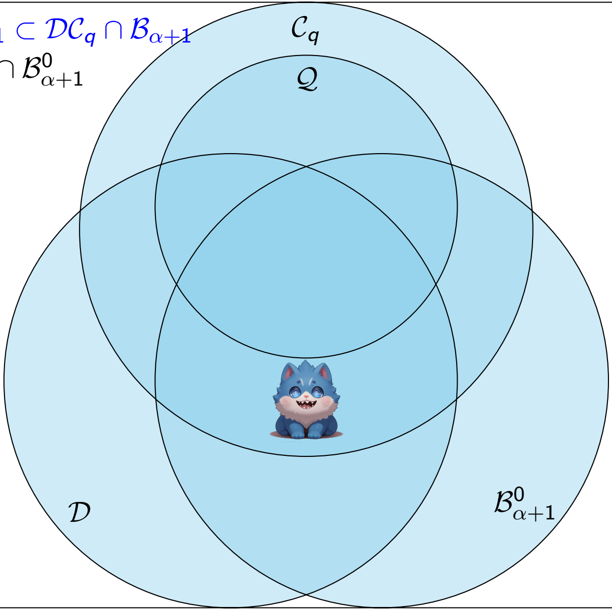
$\mathcal{D}$
$\mathcal{B}^0_{\alpha+1}$

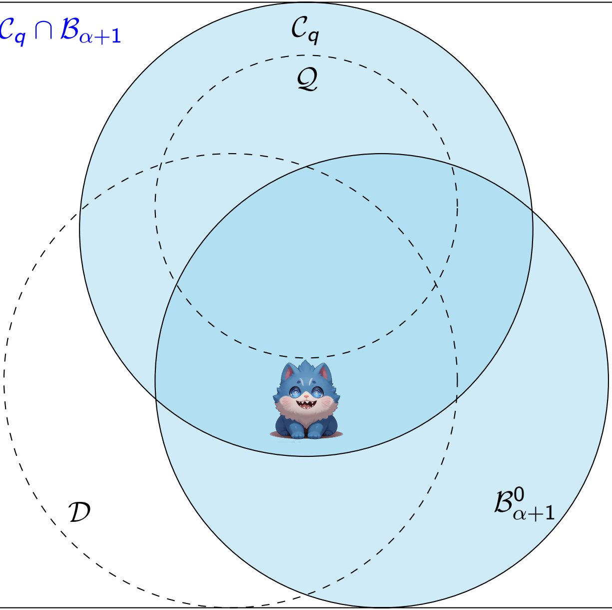
$\mathcal{DQ} \cap \mathcal{B}_{\alpha+1} \subset \mathcal{DC}_q \cap \mathcal{B}_{\alpha+1}$

$(\mathcal{DC}_q \setminus \mathcal{Q}) \cap \mathcal{B}^0_{\alpha+1}$



$\mathcal{C}_q \cap \mathcal{B}_\alpha \subset \mathcal{C}_q \cap \mathcal{B}_{\alpha+1}$

$\mathcal{C}_q \cap \mathcal{B}^0_{\alpha+1}$



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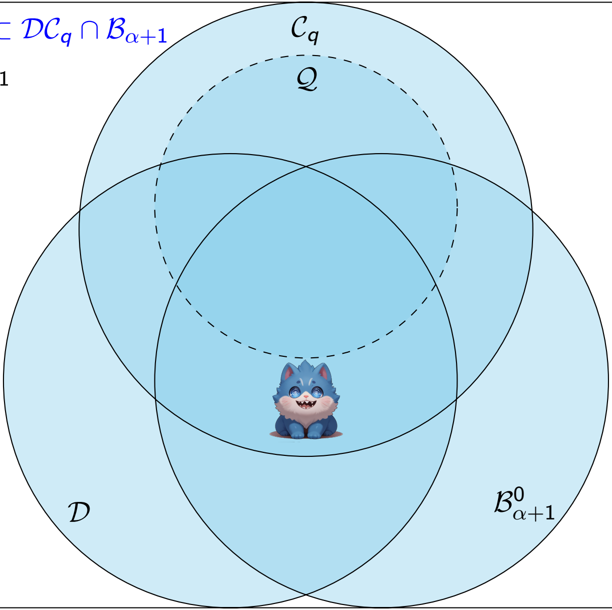
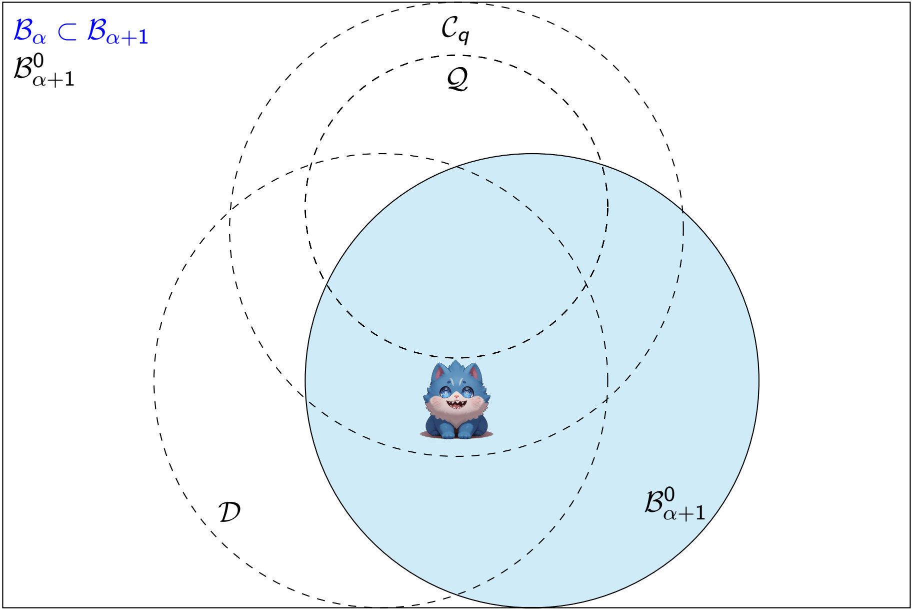
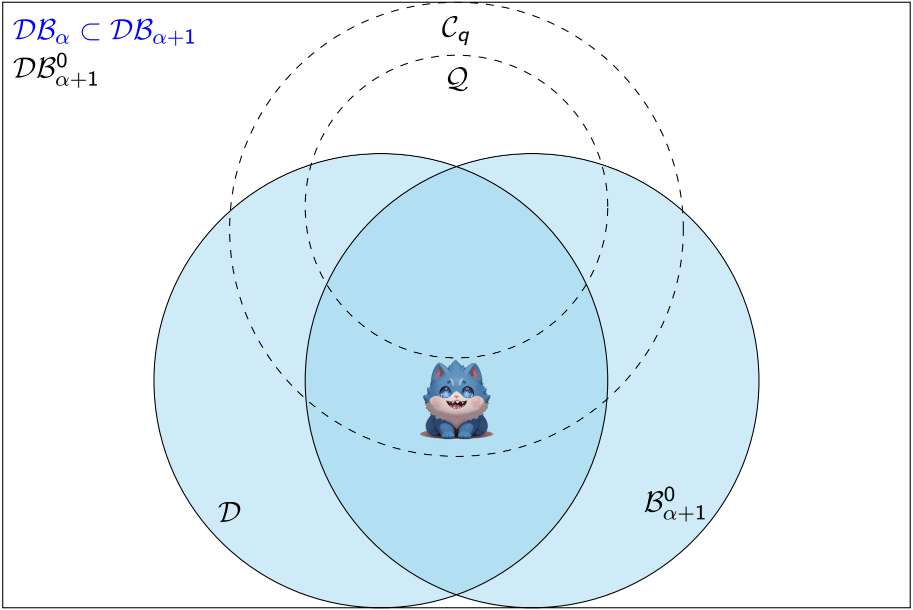



$\mathcal{B}_\alpha \subset \mathcal{B}_{\alpha+1}$
$\mathcal{B}^0_{\alpha+1}$
$\mathcal{C}_q$
$\mathcal{Q}$
$\mathcal{D}$
$\mathcal{B}^0_{\alpha+1}$


$\mathcal{DB}_\alpha \subset \mathcal{DB}_{\alpha+1}$
$\mathcal{DB}^0_{\alpha+1}$
$\mathcal{C}_q$
$\mathcal{Q}$
$\mathcal{D}$
$\mathcal{B}^0_{\alpha+1}$

Theorem

For each ordinal number α, $1 \leq \alpha < \omega_1$, the set $(\mathcal{DC}_q \setminus \mathcal{Q}) \cap \mathcal{B}^0_{\alpha+1}$ is strongly $\mathfrak{c}$-algebrable.

Corollary

Let α be an ordinal number α such that $1 \leq \alpha < \omega_1$. Then

1. *the set $\mathcal{C}_q \cap \mathcal{B}^0_{\alpha+1}$ is strongly $\mathfrak{c}$-algebrable;*
2. *the set $\mathcal{DC}_q \cap \mathcal{B}^0_{\alpha+1}$ is strongly $\mathfrak{c}$-algebrable;*
3. *the set $(\mathcal{C}_q \setminus \mathcal{Q}) \cap \mathcal{B}^0_{\alpha+1}$ is strongly $\mathfrak{c}$-algebrable;*
4. *the set $\mathcal{B}^0_{\alpha+1}$ is strongly $\mathfrak{c}$-algebrable;*
5. *the set $\mathcal{DB}^0_{\alpha+1}$ is strongly $\mathfrak{c}$-algebrable.*

Corollary

For each ordinal number α, $1 \leq \alpha < \omega_1$, *there exists a cliquish function from the family* $\mathcal{B}^0_{\alpha+1}$ $(\mathcal{DB}^0_{\alpha+1})$, *which is not quasi-continuous.*

Corollary

For each ordinal number α, $1 \leq \alpha < \omega_1$, *there exists a function* f *from the family* $\mathcal{B}^0_{\alpha+1}$ $(\mathcal{DB}^0_{\alpha+1})$, *such that the set of points of discontinuity of* f *is meager.*

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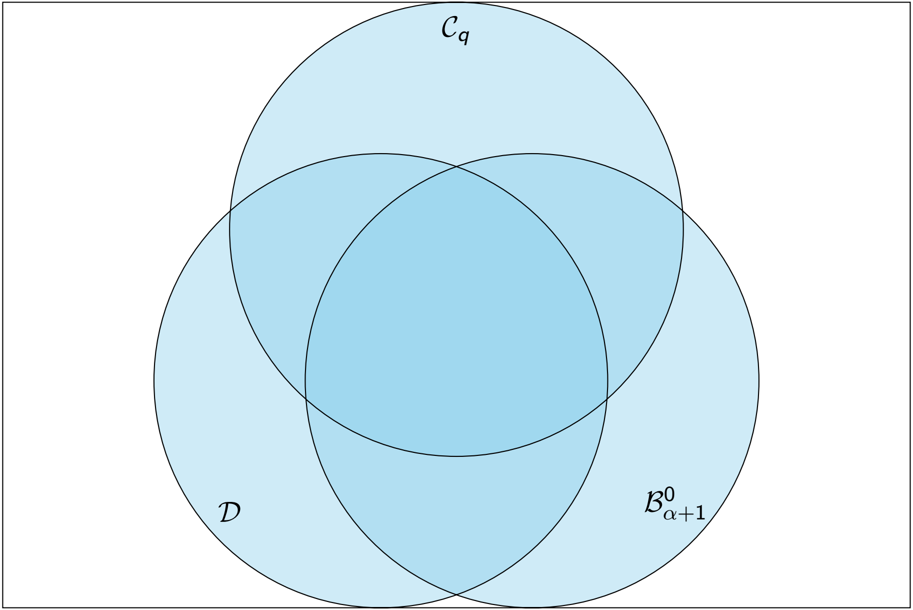
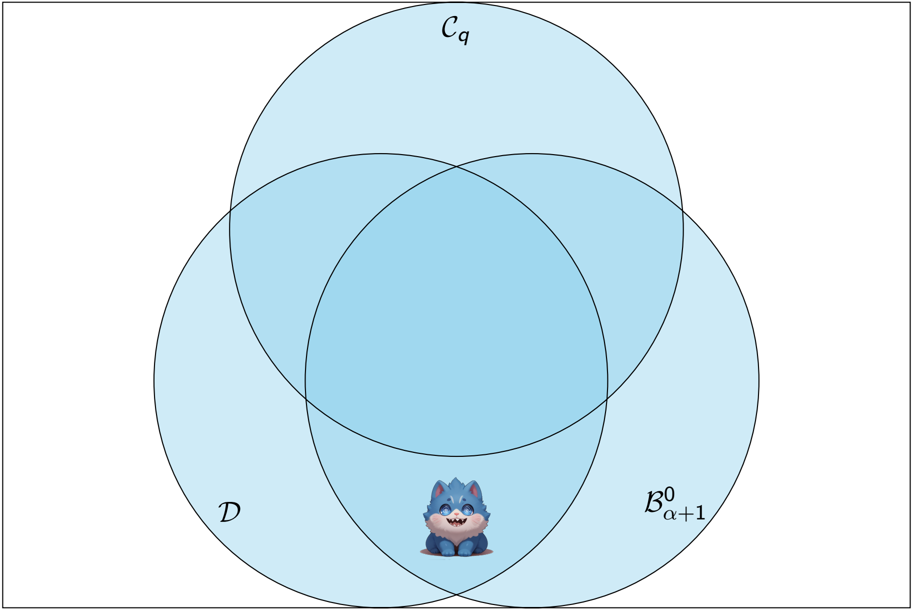
Theorem

For each ordinal number α, $1 \le \alpha < \omega_1$, *the set* $\mathcal{DB}^0_{\alpha+1} \setminus \mathcal{C}_q$ *is strongly* $\mathfrak{c}$*-algebrable.*


$\mathcal{C}_q$
$\mathcal{D}$
$\mathcal{B}^0_{\alpha+1}$


$\mathcal{C}_q$
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$\mathcal{B}_\alpha \setminus \mathcal{C}_q \subset \mathcal{B}_{\alpha+1} \setminus \mathcal{C}_q$

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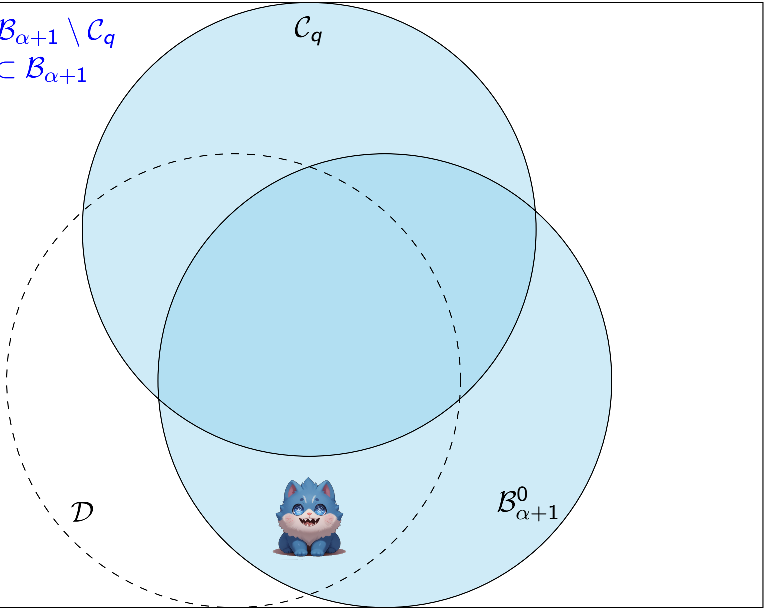
$\mathcal{B}^0_{\alpha+1} \setminus \mathcal{C}_q$



$\mathcal{DB}_\alpha \setminus \mathcal{C}_q \subset \mathcal{DB}_{\alpha+1} \setminus \mathcal{C}_q$

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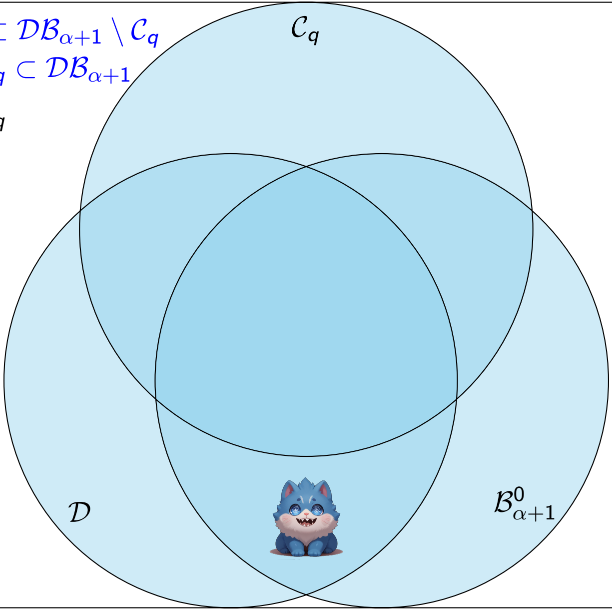
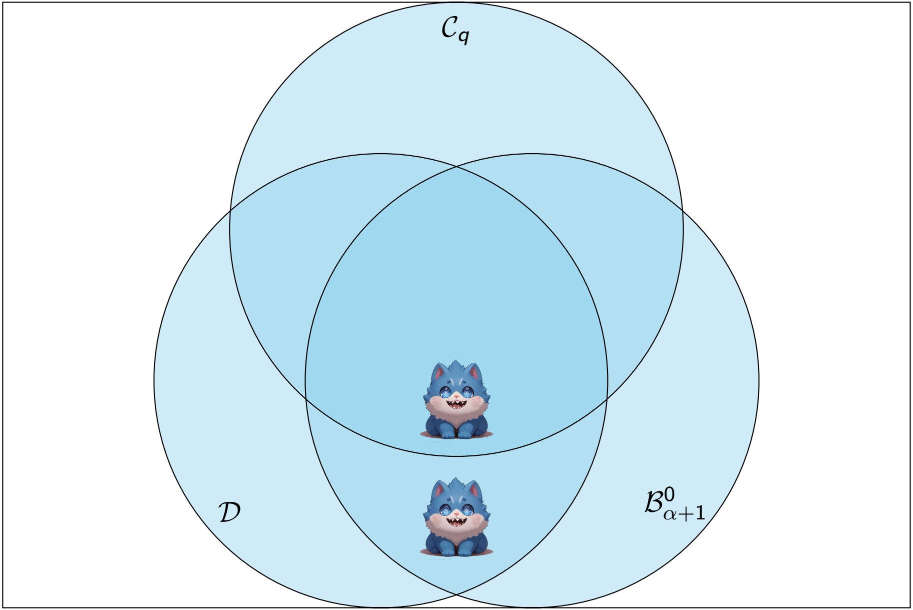
$\mathcal{DB}^0_{\alpha+1} \setminus \mathcal{C}_q$




$\mathcal{C}_q$
$\mathcal{D}$
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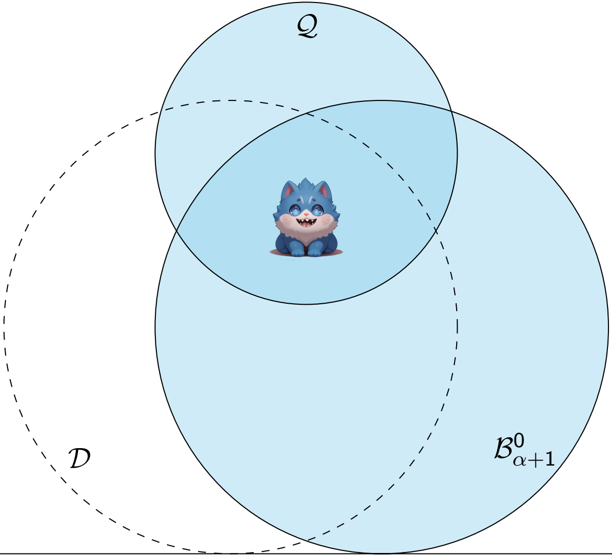
Theorem

For each ordinal number α, $1 \leq \alpha < \omega_1$, the set $\mathcal{B}^0_{\alpha+1} \cap \mathcal{Q}$ ($\mathcal{DB}^0_{\alpha+1} \cap \mathcal{Q}$) is strongly $\mathfrak{c}$-algebrable.

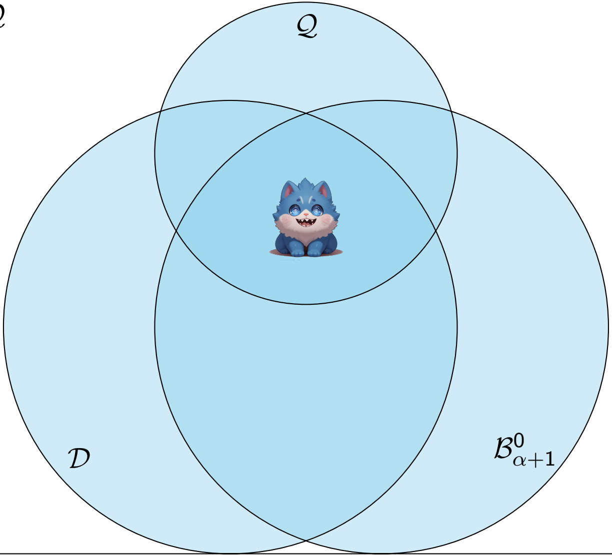
$\mathcal{Q} \cap \mathcal{B}_\alpha \subset \mathcal{Q} \cap \mathcal{B}_{\alpha+1}$

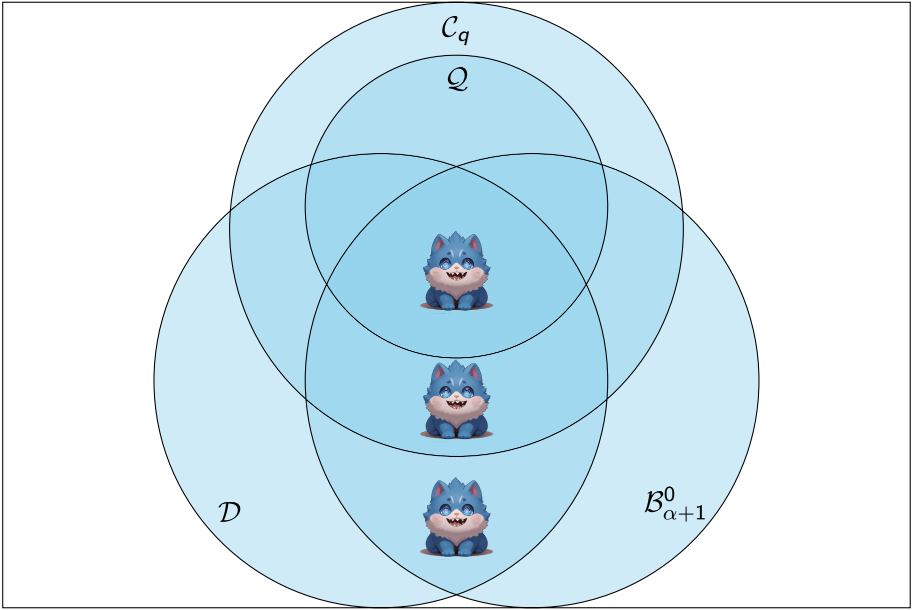
$\mathcal{B}^0_{\alpha+1} \cap \mathcal{Q}$



$\mathcal{DQ} \cap \mathcal{B}_\alpha \subset \mathcal{DQ} \cap \mathcal{B}_{\alpha+1}$

$\mathcal{DB}^0_{\alpha+1} \cap \mathcal{Q}$




$\mathcal{C}_q$
$\mathcal{Q}$
$\mathcal{D}$
$\mathcal{B}^0_{\alpha+1}$

J.Hejduk, G.I., *On the porosity of Baire class functions* (to submitted)

for $1 < \alpha < \omega_1$:

$$\mathcal{Q} \underbrace{\cap\, \mathcal{B}_\alpha \subset \mathcal{C}_q}_{\text{strongly porous}} \underbrace{\cap\, \mathcal{B}_\alpha \subset \mathcal{B}_\alpha}_{\text{strongly porous}}$$

$$\mathcal{C}_q \underbrace{\cap\, \mathcal{B}_\alpha \subset \mathcal{C}_q \cap}_{\text{strongly porous}} \mathcal{B}_{\alpha+1}$$

$$\mathcal{B}_\alpha \underbrace{\subset}_{\text{strongly porous}} \mathcal{B}_{\alpha+1}$$

for $1 \leq \alpha < \omega_1$:

$$\mathcal{Q} \underbrace{\cap\, \mathcal{B}_\alpha \subset \mathcal{Q} \cap}_{\text{strongly porous}} \mathcal{B}_{\alpha+1}$$