Book of Abstracts

47th Summer Symposium in Real Analysis

"The Meninas Symposium"

Facultad de Ciencias Matemáticas
Universidad Complutense de Madrid

June 16-20, 2025

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Plenary talks

Per Enflo

Kent State University, USA

Construction of Invariant Subspaces for Operators on Hilbert Spaces

I will present a method to construct invariant subspaces - non-cyclic vectors - for operators on Hilbert space. It represents a new direction of a method of "extremal vectors", first presented in Ansari-Enflo [1]. One looks for an analytic function $l(T)$ of T, of minimal norm, which moves a vector y near to a given vector x. The construction produces a non-cyclic vector, by gradual approximation by almost non-cyclic vectors. But for certain weighted shifts, almost non-cyclic vectors will not always converge to a non-cyclic vector. The construction recognizes this, and when the construction does not work, it will show, that T has some shift-like properties. The method also leads to problems and conjectures in analysis, which may be of interest in themselves.

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Audrey Fovelle

IMAG-Universidad de Granada, Spain

Asymptotic smoothness and concentration properties in Banach spaces

In 2008, in order to show that L_p is not uniformly homeomorphic to $\ell_p \oplus \ell_2$ for $1 < p \neq 2 < \infty$, Kalton and Randrianarivony introduced a new technique based on a certain class of graphs and asymptotic smoothness ideas. More specifically, they proved that reflexive asymptotically uniformly smooth Banach spaces satisfy some concentration property for Lipschitz maps defined on the Hamming graphs. After introducing all the objects at stake and explaining their interest, we will see how one can construct the first example of a Banach space that has such concentration property without being asymptotically smooth.

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Krystal Taylor

Ohio State University, USA

Fractals and the Buffon Circle Problem

From the delicate geometry found in a snowflake to the intricate patterns of a coastal shoreline, nature holds infinite patterns and scales. The world is not easily described us- ing mere lines and cones, and classic Euclidean geometry falls short. The notion of fractals gives us a language and a set of tools to understand more complex phenomena. The modern application of fractals spans both pure and applied mathematics – from the study of lung vasculature to surprising constructions of counter examples.

There are many classical results relating the geometry, dimension, and measure of a fractal set to the structure of its orthogonal projections. Among them, the Favard length problem, also known as Buffon's needle problem (after Count Buffon), concerns the average length of the linear projections of a subset of the plane. This fascinating and difficult problem lies in the intersection of harmonic analysis, combinatorics, and number theory. In more detail, let $K = \bigcup_{n=1}^{\infty} K_n$ be a self-similar set in the plane, constructed as a limit of Cantor iterations K_n. Assuming that K has finite length and is purely unrectifiable (so that its intersection with any Lipschitz graph has zero length), a classic theorem of Besicovitch implies that the Favard length of K vanishes. It is an open problem to determine the exact rate of decay. Substantial progress has only been achieved in recent years. In this talk, we survey these developments with emphasis on main ideas and present new developments in a nonlinear setting.

Luis Bernal González

University of Sevilla, Spain

Modes of convergence of sequences of real or complex functions: a linear point of view

The aim of this talk is to study the implications among the diverse types of convergence of sequences of functions, with focus on the space L_0 of real measurable functions on a measure space, and on the space $H(\Omega)$ of holomorphic functions on a planar domain. After reducing the problem to convergence to the zero function, the comparison between two given families $\mathcal{A}$, $\mathcal{B}$ of null sequences satisfying $\mathcal{A} \not\subset \mathcal{B}$ will be faced from a linear point of view, in the context of the modern theory of lineability, initiated by

V.I. Gurariy at the beginning of the millennium. If X is a vector space, then a subset A of X is said to be *lineable* if it contains, except for zero, an infinite dimensional vector space M. If, in addition, X is a topological vector space, then one can demand for M to be dense, or closed. This problem of finding large algebraic or algebraic-topological structures inside a set A becomes more interesting when A is far from being linear.

Several degrees of lineability, including existence of large linear algebras, will be exhibited in the present talk for the diverse difference sets $\mathcal{A} \setminus \mathcal{B}$ induced by the most relevant modes of convergence on L_0 and $H(\Omega)$.

Short talks

Izabella Abraham

Transilvania University of Braşov

The Invariant Measure for an at most Countable Topological Generalized Iterated Function System

We prove the existence and uniqueness of the invariant measure for generalized iterated function systems consisting of an at most countable number of maps, in the context of Hausdorff topological spaces. Our results give a positive answer to an open problem posed in [F. Strobin, On the existence of the Hutchinson measure for generalized iterated function systems, Qual. Theory Dyn. Syst. **19**(3), 85 (2020)]. As a corollary, we derive the existence and uniqueness of the invariant measure for classical countably infinite topological iterated function systems.

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Nacib G. Albuquerque

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A summability principle and applications

In this talk, we explore recent advances in summability principles for multilinear summing operators. Our main result introduces a new inclusion theorem for a class of summing operators:

$$\Pi^{\mathcal{B}_{\mathcal{I}}}_{(r;\mathbf{p})}\left(E_1, \ldots, E_m; F\right) \subset \Pi^{\mathcal{B}_{\mathcal{I}}}_{(\mathbf{s};\mathbf{q})}\left(E_1, \ldots, E_m; F\right),$$

where $E_1, \ldots, E_m, F$ are Banach spaces, $r, s \geq 1$, and $\mathbf{p} = (p_1, \ldots, p_m)$, $\mathbf{q} = (q_1, \ldots, q_m)$ $\in [1, \infty)^m$ are parameters subject to specific technical conditions. The class $\Pi^{\mathcal{B}_{\mathcal{I}}}$ refers to summing operators associated with blocks $\mathcal{B}_{\mathcal{I}}$. This framework not only recovers several classical results, such as inclusion theorems for absolutely and multiple summing operators, but also leads to new applications, including improved estimates for Hardy–Littlewood inequalities in multilinear settings and a Grothendieck-type coincidence result in anisotropic contexts. The results presented are part of a joint work with Gustavo Araújo, Lisiane Rezende, and Joedson Santos.

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Aliasghar Alikhani-Koopaei

Pennsylvania State University, Berks College, Reading, PA 19610

On dynamics generated by a convergent sequence of maps

Let (X, d) be a compact metric space. Denote by $C(X)$ the set of continuous self-maps of X, and by $B_1(X)$ the set of Baire class one self-maps of X. Let $E(X)$ be a class of self-maps of X that contains $C(X)$. When studying the dynamical behavior of a function $f \in E(X)$, or of a sequence of functions $\{f_n\} \subset E(X)$ converging pointwise to a limit $f \in E(x)$, various sets naturally arise whose structure, size, and properties are of interest. In this paper, we examine some aspects of the dynamics of continuous and Baire one functions, in order to highlight the difficulties involved in extending similar results to functions in $E(X)$, particularly when X is the unit interval.

In studying the dynamical systems of non-continuous members of $E(X)$, we encounter difficulties due to the lack of the following properties:

(i) $E(X)$ being compositely closed, that is for f and g in $E(X)$, $f \circ g \in E(X)$;

(ii) $E(X)$ being closed under pointwise or uniform limits;

(iii) if a sequence $\{f_k\} \subset E(X)$ converges to f uniformly (resp. point-wise), then f_k^n converges to f^n uniformly (resp. point-wise).

Here we discuss the conditions needed to be imposed on $E(X)$ and relate the dynamical behaviour of $(X, \mathbb{F})$ with the dymical behaviour of the limit function f, where $\mathbb{F} = \{f_n\} \subset E(X)$; especially when $E(X) = B_1(X)$.

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Bogdan-Cristian Anghelina

Transilvania University of Brașov

The canonical projection associated with a mixed possibly infinite iterated function system

In this presentation we provide an alternative description for the fixed points of the fractal operator associated with a mixed possibly infinite iterated function system (mIIFS). Such a system is a possibly infinite iterated function system (i.e. a possibly infinite family of Banach contractions on a complete metric space, satisfying some extra conditions) enriched with an orbital possibly infinite iterated function system (i.e. a possible infinite family of nonexpansive functions which need not be Banach contractions on the entirely previously mentioned complete metric space, but just on the orbits of space's elements).

To this end, we develop a canonical projection type theory for mIIFSs. Finally, some visual aspects concerning our results are presented.

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Daria Bugajewska

Adam Mickiewicz University, Poznań, Poland

Superposition operators in spaces of functions of generalized bounded variation

It is well known that superposition operators play an essential role in many areas of nonlinear analysis. A very important in the theory of autonomous superposition operators in the space of functions of bounded variation in the sense of Jordan is the Josephy theorem, which states that the autonomous superposition operator maps the space of functions of bounded variation in the sense of Jordan into itself if and only if its generator satisfies the local Lipschitz condition. This result has many generalizations in various spaces of functions of generalized bounded variation. During the talk I would like to focus on some such generalized spaces like spaces of functions of bounded variation in the sense of Young, in the sense of Waterman and in the sense of Wiener (both for $1 < p$ and $0 < p < 1$) and I would like to present, in particular, some new results connected with Josephy's theorem. Other properties of autonomous superposition operators I am going to discuss as well.

Ibrahim Dahi

Moulay Ismail University of Meknes

On the Well-Posedness of the global attractor for a triply nonlinear thermistor problem

In this work, we examine a broader variant of the nonlocal thermistor problem, which characterizes the temperature diffusion that occurs when an electric current flows through a substance. We analyze the doubly nonlinear problem in which the equation describing the temperature evolution has a nonlocal term on the right-hand side whereas the first nonlinearity is present in the diffusion term. Therefore, we investigate the existence and uniqueness of bounded weak solutions for a triply nonlinear thermistor problem formulated within Sobolev spaces. Additionally, we establish the existence of an absorbing set, which leads to the existence of a global (universal) attractor, thereby characterizing the long-term behavior of the system.

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Emma D'Aniello

Università degli Studi della Campania "Luigi Vanvitelli"

Adding machines, composition operators, and chaos induced by measures

A systematic study of dynamical properties of composition operators on L^p spaces has been carried out in the last decades. In this talk, we investigate some types of chaotic behaviour for composition operators induced by measures on adding machines (odometers), illustrating some recent results and open problems.

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Oswaldo R. B. de Oliveira

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Do the Change of Variable Formula Integrals have equal value?

Assuming that the two integrals in the *Change of Variable Formula* for the unidimensional Riemann integral are finite, one can ask if they have equal value. We give an answer to this question. The proof is very easy to follow and to keep in mind. An example is given.

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Thierry De Pauw

Institute for Theoretical Sciences, Westlake University

Radon-Nikodýmification of integral geometric measures

I will review necessary and sufficient conditions on a measure space for the canonical embedding $L_\infty \to L_1^*$ to be (A) injective and (B) surjective. I will describe the situation for Hausdorff measure and explain how this depends on the σ-algebra considered. Next, I will present a result in collaboration with Ph. Bouafia: the existence, for any measure, of a Radon-Nikodýmification akin the completion of a measure space or the compactification of a separated topological space. Finally, I will explicitly describe this construction in the case of integral geometric measures.

Layan El Hajj

Carnegie Mellon University in Qatar

A free boundary problem for systems (The symmetric regime)

We study a system of PDEs with free boundaries inside the unit ball. In particular, we prove that solutions to our problem exist and, furthermore, that any solution must be symmetric.The core difficulty arises from the non-variational nature of the system, coupled with a highly singular right-hand side term. These characteristics preclude the application of standard techniques. However, the inherent symmetry of the problem, derived from the geometry, allows us to circumvent these challenges. Although we treat the Laplacian case in more detail, the approach is easily generalized to more general operators, m-component case, and p-Laplacian operator as stated in the paper.

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Bruce Hanson

St. Olaf College

Sets where Lip f is infinite and lip f vanishes

Assume $f : \mathbb{R} \to \mathbb{R}$ is continuous. Then the Big Lip and little lip functions are defined by

$$\operatorname{Lip} f(x) = \limsup_{r\to 0^+} \sup_{|x-y|<r} \frac{|f(x)-f(y)|}{r} \qquad \operatorname{lip} f(x) = \liminf_{r\to 0^+} \sup_{|x-y|<r} \frac{|f(x)-f(y)|}{r}.$$

In [1] the authors construct a function $f : \mathbb{R} \to \mathbb{R}$ and a set $E \subset \mathbb{R}$ such that $|\mathbb{R} \backslash E| = 0$, $\operatorname{Lip} f = \infty$ on E and $\operatorname{lip} f = 0$ on E. We consider the problem of characterizing the sets $E \subset \mathbb{R}$ such that there exists $f : \mathbb{R} \to \mathbb{R}$ with $\operatorname{Lip} f = \infty$ and $\operatorname{lip} f = 0$ on E.

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Jacek Hejduk

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Overview of the generalizations of the classical density and corresponding topologies

In the recent years the notion of the classical Lebesgue density were genralized by weakening the assumptions on the sequences of intervals and consequently several notions like $\langle s \rangle$-density by M. Filipczak, J. Hejduk [3], $\mathcal{J}$-density by J. Hejduk and R. Wiertelak [11], $\mathcal{S}$-density point by F. Strobin and R. Wiertelak [13] were obtained.

The presentation contains the essential ideas of the above generalizations and some properties of related density topologies. The results related $\mathcal{S}$-density investigated by M. Widzibor in his PhD thesis [15] are also included.

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Grażyna Horbaczewska

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On multiadditive set-valued maps "modulo K"

Let $p \in \mathbb{N}$, Y be a real vector metric space and K be a closed convex cone in Y satisfying $K \cap (-K) = \{0\}$.

A set valued map $F\colon X^p \to n(Y)$ is called *K-p-additive* (or *K-multiadditive*, or *p-additive "modulo K"*), if it is K-additive with respect to each variable, i.e.

$$F(x_1, \dots, x_i + y_i, \dots, x_p) =_K F(x_1, \dots, x_i, \dots, x_p) + F(x_1, \dots, y_i, \dots, x_p) \tag{3.1}$$

for every $x_1, \dots, x_p, y_i \in X$ and $i \in \{1, \dots, p\}$.

The relation $=_K$ is defined as follows:

$$\forall\, A, B \in n(Y)\ \ \big[A =_K B \iff (A \subset B + K \ \ \wedge\ \ B \subset A + K)\big].$$

We are especially interested in looking for sufficient conditions making such set-valued map continuous in our setting. We prove that a K-p-multiadditive s.v. map $F\colon \mathbb{R}^p \to n(Y)$ which is K-continuous with respect to each coordinate, or K-measurable in the sense of Lebesgue/Baire, is K-continuous on the whole domain, and we give an explicit formula of such s.v. maps. These results, obtained with E. Jabłońska, W. Jabłoński and M. Terepeta, generalize well-known results for multiadditive real functions from [5].

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Gertruda Ivanova

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On strong $\mathfrak{c}$-algebrability of some families of functions

The idea of quasi-continuity emerged from a modification of the definition of continuity, naturally leading to the question of whether such functions belong to specific Baire classes. In response to this question, Marcus demonstrated ([4]) that within every Baire class, one can find quasi-continuous functions that do not belong to any preceding Baire class.

The introduction of cliquish functions broadened the scope of quasi-continuous functions considerably, prompting the question of the relative size of this expanded family within a given Baire class. The paper [3] showed that, concerning topology and porosity, the set of quasi-continuous functions is significantly smaller than the set of cliquish functions within certain Baire classes, and both are negligible compared to the Baire class itself.

Furthermore, it was established that, in terms of porosity, moving from a Baire class $\mathcal{B}_\alpha$ to the next, $\mathcal{B}_{\alpha+1}$, substantially increases the size of the families of quasi-continuous and cliquish functions, as well as the Baire class itself. This naturally led to comparisons of these families using different frameworks, such as algebraic properties.

Algebrability, a concept attracting widespread mathematical interest across various fields, includes the construction of large algebraic structures within function spaces ([1], [2], [5]). We used these methods to demonstrate that there are large algebraic structures between function spaces mentioned above. This means that, concerning algebrability, the set of quasi-continuous functions is also significantly smaller than the set of cliquish functions within certain Baire classes, and both are also negligible compared to the Baire class itself.

We also strengthen these results by focusing on more restricted function classes: namely, the class of quasi-continuous functions possessing the Darboux property and the class of Darboux cliquish functions. It turns out that the transition from $\mathcal{B}_\alpha$-class to $\mathcal{B}_{\alpha+1}$ also significantly expands the family of Darboux quasi-continuous functions and the family of Darboux functions which are not cliquish.

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Eliza Jabłońska

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On set-valued maps aditive "modulo K"

Let X, Y be real vector spaces, $K \subset Y$ be a convex cone and $n(Y) := 2^Y \setminus \{\emptyset\}$. A set-valued map $F\colon X \to n(Y)$ is called K-additive, if

$$F(x+y) =_K F(x) + F(y) \quad \text{for} \;\; x, y \in X,$$

where $=_K$ is the equivalence relation defined in the following way:

$$\forall\, A, B \in n(Y) \quad (A =_K B \quad \iff \quad A \subset B + K \;\; \wedge \;\; B \subset A + K).$$

If Y is additionally topological, we can consider K-continuity of set-valued maps introduced in [3]. More precisely, a set-valued map $F\colon X \to n(Y)$ is called K-continuous at $x_0 \in X$, if for every neighborhood $W \subset Y$ of 0 there is a neighborhood U of x_0 such that

$$F(x) \subset F(x_0) + W + K \quad \wedge \quad F(x_0) \subset F(x) + W + K$$

for all $x \in U$.

During the talk we would like to present selected properties of K-additive set-valued maps. We focus on K-continues K-additive set-valued maps and, among others, we give an explicit formula of K-continuous K-additive set-valued maps defined on the line.

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Mar Jiménez-Sevilla

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Approximate Morse-Sard type results for non-separable Banach spaces

In this talk we will review some results on ranges of derivatives and present several results on approximation by smooth functions with no critical points. For E and F Banach spaces, if $g : E \to F$ is a C^1 smooth function, a point $x \in E$ is called critical if $Dg(x)$, the derivative of g at x, is not a surjective operator. The set of critical points is denoted by $\mathcal{C}_g$. The classical Morse-Sard theorem states that if $g : \mathbb{R}^n \to \mathbb{R}^m$ is C^k-smooth with $k > \max\{n-m, 0\}$, then the set of critical values $g(\mathcal{C}_g)$ is of Lebesgue measure zero in $\mathbb{R}^m$. In many cases Morse-Sard theorem fails when $dim(E) = \infty$. In this talk we present sufficient conditions to ensure an approximate strong version of the Morse-Sard theorem for mappings from an infinite-dimensional Banach space E to a Banach space F. Namely, under some appropriate conditions on an infinite-dimensional (not necessarily separable) Banach space E, for every (non-zero) quotient F of E (or a certain class of quotients F of E), every continuous function $f : E \to F$, and for every continuous function $\varepsilon : E \to (0, \infty)$, there exists a C^k smooth function $g : E \to F$ with no critical points such that $\|f(x) - g(x)\| \leq \varepsilon(x)$ for all $x \in E$. Here k depends on the smoothness of E. Joint work with D. Azagra and M. García-Bravo [1].

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Pedro L. Kaufmann

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Nonabolute integration on compact lines

We present a nonabsolute integration process on compact lines (linearly ordered compact topological spaces) which extends the notion of Kurzweil-Stieltjes nabla integration on time scales. We establish some basic properties of this integral and provide illustrative examples. Moreover, we propose a derivation process on compact lines motivated by the nabla derivative and which, together with the introduced integral, allows for a formulation of a Fundamental Theorem of Calculus in this setting. Contrasts with the time scale theory are highlighted, since for general compact lines we cannot rely on metrizability to prove, for instance, Cousin's Lemma or Vitali's Covering Theorem.

Joint work with Leandro Candido.

Jun Kawabe

Shinshu University

Nonadditive measure representation theorems for nonlinear functionals

To deal with various problems in game theory and expected utility theory, it is necessary to consider set functions that do not satisfy additivity due to interactions among sets of players or states of nature. A nonadditive measure is a monotone set function vanishing at the empty set. Among the integral concepts based on nonadditive measures, the Choquet [1], Shilkret [4], and Sugeno [5] integrals are particularly popular, which are in turn defined by

$$\mathrm{Ch}(\mu, f) = \int_0^\infty \mu(\{f > t\})dt, \quad \mathrm{Sh}(\mu, f) = \sup_{t>0} t\mu(\{f > t\}),$$

and

$$\mathrm{Su}(\mu, f) = \sup_{t>0} t \wedge \mu(\{f > t\})$$

for a function $f\colon X \to [0, \infty]$ on a nonempty set X and a nonadditive measure $\mu\colon 2^X \to [0, \infty]$. The Choquet integral is not linear in general, but it coincides with the Lebesgue integral when μ is σ-finite. In contrast, the Shilkret and Sugeno integrals are not linear even if μ is additive. Each of these integrals can be viewed as a monotone nonlinear functional on a collection of functions and has unique characteristics such as comonotonic additivity, comonotonic maxitivity, and constant minitivity. This talk gives necessary and sufficient conditions under which a given monotone functional on a collection of functions can be described by each of the above nonlinear integrals for some nonadditive measure. Furthermore, the uniqueness and continuity of a representing measure will be discussed [2, 3].

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Stanisław Kowalczyk

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On $\mathfrak{c}$-lineability

We would like to present some results about $\mathfrak{c}$-lineability and $\mathfrak{c}$-spaceability of some families $\mathcal{F}$ of real functions defined on an interval I. The main goal is to formulate general conditions under which any non-empty family $\mathcal{F} \subset \mathbb{R}^I$ of functions is $\mathfrak{c}$-spaceable or $\mathfrak{c}$-lineable. Generally, we consider the families of function of the form $\mathcal{F} = \mathcal{F}_1 \setminus \mathcal{F}_2$. In most cases, families of functions for which lineability and spaceability are studied have such a form. Most often, family $\mathcal{F}_2$ is seemingly "very close" to $\mathcal{F}_1$ or consists of "almost all" functions. The results obtained by us are a generalization of previous ideas, [1].

The main idea of our constructions is to "reproduce" one function to obtain $\mathfrak{c}$-dimensional (closed) linear space. For this "reproduction" we use the Fichtenholz-Kantorovich Theorem, applied to a countable family of pairwise disjoint intervals contained in the domain of functions from the considered class. The initial function is "squashed" and "pasted" into disjoint intervals included in the domain of constructed function.

We present some natural conditions under which families constructed in this way belongs to considered class (it always forms $\mathfrak{c}$ dimensional linear space). Finally, we present numerous examples of applications of the presented theory.

Presented results was obtained jointly with Małgorzata Turowska.

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Center of distances and Bernstein sets

The notion of center of distances has been introduced in [5] as follows. If (X, d) is a metric space, then the set

$$S(X) := \{a \in [0, \infty) \colon \forall_{x \in X} \exists_{y \in X} d(x, y) = a\}$$

is called the *center of distances* of X. In [3], the question to characterize sets $S(X)$ for subsets of the real line has been raised. At the conferences 'Inspirations in Real Analysis II' (2024) and '46th Summer Symposium in Real Analysis' (2024), M. Filipczak asked whether, for any given subset $A \subset [0, \infty)$ with $0 \in A$, there exists $X \subset \mathbb{R}$ such that $S(X) = A$. We give a positive answer to Filipczak's question and improve this result showing that in addition X can be chosen to be a Bernstein set, see [6].

It is worth noting that the answer to Filipczak's question was already known for compact A and in that case X can be chosen to be closed, see [3, Corollary 4.11]. The notion of the center of distances is quite general, but most applications concern subsets of real numbers $\mathbb{R}$. For example, this notion has been successfully used in proofs that some sets are not achievement sets of any series, see [5] or [4]. Exact calculation of the center of distances is interesting on its own, compare [2] and [1].

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Ondřej Kurka

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The ergodicity of Orlicz sequence spaces

According to Ferenczi and Rosendal, a separable Banach space X is called ergodic if the equivalence relation E_0 is Borel reducible to the isomorphism relation of subspaces of X. In particular, an ergodic space has continuum many pairwise non-isomorphic subspaces. It is conjectured that any space which is not Hilbertian (i.e., isomorphic to a Hilbert space) is ergodic.

The study of ergodicity of Banach spaces has yielded several remarkable results. In the talk, we will show that every Orlicz sequence space which is not Hilbertian is ergodic. The proof is based on a construction of an asymptotically Hilbertian subspace. As a consequence, we prove that the twisted Hilbert spaces $\ell_2(\phi)$ constructed by Kalton and Peck are either Hilbertian, or ergodic.

The talk is based on a work in collaboration with Noé de Rancourt.

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Bastien Lecluse

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On the almost everywhere convergence of two-parameter ergodic averages along directional rectangles

In this talk, we study the almost everywhere convergence of sequences of two-parameter ergodic averages over rectangles in the plane. On the one hand, we see that if the rectangles we consider have their sides with slopes in a finitely lacunary set, then the averages converge almost everywhere in all L^p spaces, $1 < p \leq \infty$. On the other hand, given some non-lacunary sets of directions, we construct sequences of rectangles oriented along these directions for which the associated ergodic averages fail to converge almost everywhere in any L^p space, $1 < p < \infty$.

Martina Maiuriello

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Infinite-dimensional dynamics: (frequently) hypercyclic phenomena for composition operators

Inspired by recent developments in the study of lineability and spaceability within the framework of linear dynamics, we explore the existence of infinite-dimensional closed subspaces consisting of (frequently) hypercyclic vectors for (frequently) hypercyclic composition operators on $L^p(X), 1 \leq p < \infty$. Given the central role played by (frequent) hypercyclicity in the chaotic theory of linear operators, we further investigate its intricate connections with other key dynamical properties: we analyze, in particular, the relationship between (frequent) hypercyclicity and (frequent) recurrence in the setting of dissipative composition operators. This analysis leads to interesting applications for the widely studied class of weighted shift operators as well.

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Relaxation result for differential inclusions on finite and infinite intervals

Applying a recent Filippov Lemma for measure differential inclusions, we first present on a compact interval a Filippov-Ważewski-type theorem for the very general setting of differential inclusions involving the Stieltjes derivative with respect to a non-decreasing, left-continuous map g. The solutions $y : [0,1] \to \mathbb{R}^d$ of the relaxed problem

$$y'_g(t) \in \begin{cases} \overline{co}F(t, y(t)), & t \notin D_g, \qquad y(0) = \xi_0 \\ F(t, y(t)), & t \in D_g \end{cases} \tag{3.2}$$

can be approximated by solutions $z : [0,1] \to \mathbb{R}^d$ of

$$z'_g(t) \in F(t, z(t)), \qquad z(0) = \xi_0, \tag{3.3}$$

where D_g is the set of discontinuity points of g.

Using the relaxation result on a compact interval we get in the second part a relaxation theorem on an infinite domain. In this case the approximation can be achieved once we allow to the initial value to differ (but remaining close to) from the initial value of the considered solution of the relaxed inclusion. New relaxation results can be deduced for generalized differential problems, for impulsive differential inclusions with multivalued impulsive maps and possibly countable impulsive moments and also for dynamic inclusions on time scales.

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Radu Miculescu

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Relational generalized iterated function systems

In this talk, we introduce a wider class of generalized iterated function systems, called relational generalized iterated function systems. More precisely, the classical contraction condition for functions defined on product spaces is weakened by means of an equivalence relation. In particular, if we consider the total equivalence relation, we recover the classical generalized iterated function systems. Our main result states that each compact subset of the underlying metric space generates, via a sequence of iterates, a fixed point of the associated fractal operator, called an attractor of the system. We also establish a structure result for the attractors and a theorem concerning the continuous dependence of the attractor on the associated compact set. Ultimately, we provide some examples which illustrate our main results.

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Janusz Morawiec
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On an operator induced by invariant measures

We investigate a linear operator associated with a functional equation that arises by studying some class of invariant measures under multidimensional transformations. By examining its iterates, we derive an explicit solution formula for the functional equation and establish a result on the existence of an invariant measure under a multidimensional transformation that can be viewed as a generalization of classical p-adic maps to higher dimensions.

Piotr Nowakowski
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Cantor sets and their algebraic differences

Denote by $C(a) \subset [0,1]$ the central Cantor set generated by a sequence $a = (a_n) \in (0,1)^{\mathbb{N}}$. It is known that the difference set $C(a) - C(a)$ has one of three possible forms: a finite union of closed intervals, a Cantor set, or a Cantorval ([1]). Our main results describe two different conditions for (a_n) which guarantee that $C(a) - C(a)$ is a Cantorval. Examples of application of both results will be also presented. We will also consider a class of special Cantor sets (or S-Cantor sets) and a full classification of their difference (with one common parameter). The presentation is based on the papers: [2], [3] and [4].

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Maximal operators and differentiation with respect to collections of shifted convex bodies

Let Ω be a collection of shifted balls in $\mathbb{R}^n$. The term "shifted ball" here means that the ball does not contain the origin. Maximal operators and differentiation of integrals with respect to the collection Ω, i.e. with respect to the basis $\mathbf{B}_\Omega$ for which $\mathbf{B}_\Omega(x) = \{B + x : B \in \Omega\}$ $(x \in \mathbb{R}^n)$, have been studied by Nagel and Stein, Stein, Aversa and Preiss, Csörnyei, and Hagelstein and Parissis. One of the motivations for the research was the intimate connection with the boundary behaviour of Poisson integrals along regions more general than cones. Moonens and Rosenblatt examined the case of collections of shifted two-dimensional intervals, while Laba and Pramanik studied the topic for dilation invariant collections of sparse one-dimensional sets.

We will discuss some new results related with maximal operators and differentiation of integrals with respect to collections of shifted convex bodies.

The talk is based on a joint research with Emma D'Aniello and Laurent Moonens.

Cheng-Han Pan

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A Mazurkiewicz Set as the Union of Two Sierpinski-Zygmund Functions

A Mazurkiewicz set is a plane subset that intersect every straight line at exactly two points, and a Sierpiński-Zygmund function is a function from real from $\mathbb{R}$ into $\mathbb{R}$ that has as little of the standard continuity as possible. In this talk, we will construct a Mazurkiewicz set that contains no graph of Sierpiński-Zygmund function and another one that can be expressed as a union of two Sierpiński-Zygmund functions.

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Weak Amenability Constant in Banach Algebras

Recall that a locally compact group G is *amenable* if there is an invariant mean μ on $L^\infty(G)$, i.e. a linear functional such that $\mu(1_G) = 1$ and $\mu(L_g f) = \mu(f)$ for all $g \in G$ and $f \in L^\infty(G)$ (here L_g is a left-translation operator). Soon after a connection to Banach–Tarski Paradox had been found. And in 1972 B.E. Johnson showed in [1] that a locally compact group G is amenable if and only if the convolution algebra $L^1(G)$ satisfies a specific condition: namely all derivations into dual bimodules are inner. This condition then served as a definition of an amenable Banach algebra and launched a vast development of this theory. In this talk I will start with a historical note on the topic after which I will focus on the quantitative approach to amenability of Banach algebras. This will then be applied to characterization of some matrix algebras as well as vector-valued sequence algebras. This is a joint work with my Ph.D. student Krzysztof Koczorowski.

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Franciszek Prus-Wiśniowski

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Four open problems in achievement set theory

We present four of the open problems in achievement set theory. In some cases, we will mention some limited recent progress in solving them. In particular, we will observe that the set of uniqueness of an achievement set is always a G_δ set. In addition, we will demonstrate a solution to the problem of the Lebesgue measure of the boundary of a multigeometric achievement set, a solution which unfortunately does not work in the general case.

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Emilia Przemska

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A topological aspect for rough sets approximations inspired by supra-topology structures

In this presentation, we propose extending the rough set model inspired by supra topology $\mathcal{U}_k$, introduced by Al-Shami [1]. We introduce the approximation operators determined by the family of the complements of a supra topology's members $\mathcal{U}_k^c$. Any supra-topology and the family of its complements generate a pair of symmetric Alexandrov topologies. Those topologies characterize the approximations determined by the base families. What is more, the new characterizations have granular-based representations. As a result, we obtain an assessment of the accuracy of the investigated approximations. Using these topologies gives more accurate approximations than those generated by the basic families.

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Blanca Radillo-Murguia

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Directional Maximal Operators and Kakeya-Type Sets

We will discuss a recent result that provides a condition on a set of directions $\Omega \subseteq \mathbb{S}^1$ sufficient to show the admissibility of Kakeya-type sets, extending prior work of Bateman and Katz. This condition guarantees that the associated directional maximal operator M_Ω is unbounded on $L^p(\mathbb{R}^2)$ for every $1 \leq p < \infty$. This work is joint with Paul Hagelstein and Alex Stokolos.

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Martin Rmoutil

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Little Lipschitz constant of functions on the real line

The Little Lipschitz constant of a function (also known as the lower scaled oscillation or just "little lip") has now been studied by many authors from various viewpoints. Some basic open questions are still left without complete answers, however. One such question concerns the fact that, given any continuous real function f of one real variable, the set of points at which $lip(f)$ equals infinity is of the type $F_{\sigma\delta}$; the natural question of the validity of the converse, i.e. that for any $F_{\sigma\delta}$ set there exists an "appropriate" function, immediately springs to mind of a typical real-analyst. However, only partial answers have so far been published; namely, for F_σ sets, as well as for G_δ sets, this is the content of a 2019 joint paper with Zoltán Buczolich, Bruce Hanson, and Thomas Zürcher. Much more recently, with Thomas Zürcher, we published a construction of an "appropriate function" for any Lebesgue null $F_{\sigma\delta}$ set. I want to give a brief account of this (fairly involved) result as well as of our ongoing work on the most general case (arbitrary $F_{\sigma\delta}$ set) which is now nearing completion. Both proofs are based on ideas of David Preiss.

Jakub Rondoš

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Trees of continuous functions

We present several new results concerning the interaction of the topology of a compact Hausdorff space with its Banach space of continuous real-valued functions. The proofs of these results are mainly based on a new descripiton of spaces of continuous functions over metrizable compact spaces in terms of certain trees.

Anna Rita Sambucini

University of Perugia, Department of Mathematics and Computer Sciences

A survey on the Riemann-Lebesgue integrability in non-additive setting

We present some results regarding Riemann-Lebesgue integral of a vector (real resp.) function relative to an arbitrary non-additive set function. Then these results are generalized to the case of Riemann–Lebesgue integrable interval-valued multifunctions.

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Alina Shalukhina

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The Hardy–Littlewood maximal function on spaces of homogeneous type

We explore the boundedness properties of the Hardy–Littlewood maximal operator M on quasi-Banach function lattices $X(\Omega, d, \mu)$ over a space of homogeneous type (Ω, d, μ). Spaces of homogeneous type are quasi-metric spaces with a doubling measure; they are equipped with adjacent systems of dyadic "cubes" designed by Hytönen and Kairema [1] in logic of the Euclidean dyadic decomposition. With the dyadic tool in hand, we prove the self-improving boundedness property of M, which is the main result in [2]:

$$\text{if } M \text{ is bounded on a lattice } X(\Omega, d, \mu) \text{ with the Fatou property,}$$
$$\text{then } M \text{ is bounded on its lower convexification } X^{(\frac{1}{s})}(\Omega, d, \mu) \text{ for some } s > 1.$$

An important application of this result is the special case when $X(\Omega, d, \mu)$ is a variable Lebesgue space $L^{p(\cdot)}(\Omega, d, \mu)$. Then the class of exponents $p(\cdot)$ such that M is bounded on $L^{p(\cdot)}(\Omega, d, \mu)$ is left-open, meaning that every $p(\cdot)$ is contained there together with a smaller exponent $\frac{p(\cdot)}{s}$, $s > 1$. In addition, we obtain a nice interpolation result for this class of exponents using the left-openness.

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Antonín Slavík

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A general form of Gronwall inequality with Stieltjes integrals

The Gronwall inequality is a fundamental tool in the theory of differential and integral equations. In its classical version, if $u, K, L\colon [t_0, t_0+T] \to [0, \infty)$ are continuous functions such that

$$u(t) \leq K(t) + \int_{t_0}^{t} L(s)u(s)\,\mathrm{d}s, \quad t \in [t_0, t_0 + T],$$

then the inequality provides an a priori bound for u in terms of K and L, namely

$$u(t) \leq K(t) + \int_{t_0}^{t} K(s)L(s)\exp\left(\int_{s}^{t} L(\tau)\,\mathrm{d}\tau\right)\mathrm{d}s, \quad t \in [t_0, t_0 + T].$$

In the present talk, we are interested in Gronwall-type results with ordinary integrals replaced by Kurzweil-Stieltjes (or Lebesgue-Stieltjes) integrals with respect to a nondecreasing function. Several results of this type are available in the literature, but their assumptions are not completely satisfactory. In short, it is usually assumed that K a constant function, or that the integrator is a left-continuous or right-continuous function. We will show that these assumptions are not necessary, and provide a new general version of Gronwall's lemma formulated in terms of the generalized exponential function. The proof is based on a new quotient rule for Stieltjes integrals.

References

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Filip Strobin

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Topological prevalence of variable speed of convergence in the deterministic chaos game

The chaos game algorithm gives one of the most popular methods of generating images of attractors of iterated function systems. Its validity can be explained by the fact that with probability 1, randomly chosen sequences of a finite alphabet are disjunctive, meaning that they contain all finite words as subwords. This observation leads to deterministic version of the chaos game - given a (initially defined) disjunctive sequence, we are sure that the generated orbit approximates the attractor.

In the main part of my talk I will show that a typical (in the sense of Baire's category) disjunctive sequence do not provide any control over the speed of convergence of the orbit toward the attractor. This will need a certain tool for measuring the speed of convergence - *the set* $\lambda(\mathbf{i})$ *of exponents of rate of convergence* of a given driver $\mathbf{i}$. I will show that a typical driver, the set $\lambda(\mathbf{i})$ is the biggest possible interval: $[\underline{D}(A), \infty]$, where $[\underline{D}(A)$ is the lower box dimension of the attractor A. Finally, I will show that this result cannot be extended to certain σ-porosity.

In the last part of will also recall a recent result of Bárány, Jurga and Kolossváry which shows that things changes when we focus to drivers chosen with probability one, and I will presenta some additional properties of disjunctive sequences that can give some control of the speed of convergence.

The presented results (except from the mentioned one of Bárány, Jurga and Kolossváry) were obtained together with Krzysztof Leśniak and Nina Snigireva.

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Piotr Sworowski

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A partitioning property of δ-fine intervals and the HK_r-integral

Let δ be a positive-valued function on the real line. We say that an interval-point pair $([c,d],\xi)$ is *δ-fine* if $[c,d] \subset (\xi - \delta(\xi), \xi + \delta(\xi))$. We say that an interval $[c,d]$ is *doubly δ-fine* if both $([c,d],c)$ and $([c,d],d)$ are δ-fine.

The notion of doubly δ-fine has proven useful in an ε-δ characterization of Baire one functions [1]. In this talk, we will discuss its application in the monotonicity theorem for the HK_r-integral [2].

Definition 1 ([2]). *A function* $f: [a,b] \to \mathbb{R}$ *is* L^r-Henstock–Kurzweil integrable *(*HK_r-integrable*) on* $[a,b]$ *if there exists a function* $F \in L^r$ *such that for any* $\varepsilon > 0$ *there exists a gauge* δ *so that for any* δ*-fine partition* $\{([c_i,d_i],x_i)\}_{i=1}^n$ *of* $[a,b]$ *we have*

$$\sum_{i=1}^{n} \left(\frac{1}{d_i - c_i} \int_{c_i}^{d_i} |F(y) - F(x_i) - f(x_i)(y - x_i)|^r \, dy \right)^{1/r} < \varepsilon.$$

The talk is based on a jont work co-authored by Paul Musial, Valentin Skvorstov, and Francesco Tulone.

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Małgorzata Turowska

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On $[r,s]$-superporosity

We characterize families of $[r,s]$-(upper) superporous subsets of $\mathbb{R}$. $[r,s]$-superporosity generalize notions of superporosity and strong superporosity of subsets of $\mathbb{R}$, [3, 4]. Definitions and properties of $[r,s]$-superporosity are similar to definitions and properties of superporosity and strong superporosity. Superporous sets preserve positive porosity and strongly superporous sets preserve strong porosity, i.e. if E is superporous (respectively, E is strongly superporous) then for every $x \in E$ and for every F such that porosity of F at x is greater than 0 (respectively, is equal to 1), porosity of $E \cup F$ at x is greater than 0 (respectively, is equal to 1). Taking arbitrary positive numbers, instead of 0 or 1, we obtain the following definition of $[r,s]$-superporosity for $0 < r \leq s < 1$, $[r,s]$-superporous sets transfer s-porous sets to r-porous sets, i.e. the set E is $[r,s]$-superporus iff for every $x \in E$ and for every F such that porosity of F at x is not less than s, porosity of $E \cup F$ at x is not less than r.

Even though the definition and properties of $[r,s]$-superporosity, superporosity and strong superporosity are similar and all of them consist of very small sets, the families of these sets are essentially different. We focus on relationships between $[r,s]$-superporous sets for different indices $[r,s]$. Furthermore, we compare $[r,s]$-superporosity to superporosity and strong superporosity. Finally we apply the notion of $[r,s]$-superporosity to find multipliers and adders of porouscontinuous functions, introduced by J. Borsík and J. Holos in [1].

This is jointly work with Stanisław Kowalczyk.

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Renata Wiertelak

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On functions continuous with $\mathcal{I}(\mathcal{Y})$–density topology

Let Y be a bounded set of the second category, and let $\{a_n\}_{n\in\mathbb{N}}$ and $\{b_n\}_{n\in\mathbb{N}}$ be sequences of real numbers converging to zero, such that $a_n \neq 0$ for every $n \in \mathbb{N}$. Define $Y_n = a_nY + b_n$ for every $n \in \mathbb{N}$, and let $\mathcal{Y} = \{Y_n\}_{n\in\mathbb{N}}$.

We say that x_0 is an $\mathcal{I}(\mathcal{Y})$–**density point** of a set A having the Baire property if

$$\chi_{(A-x_0-b_n)\frac{1}{a_n}\cap Y}(x) \xrightarrow[n\to\infty]{\mathcal{I}} \chi_Y(x).$$

The concept of an $\mathcal{I}(\mathcal{Y})$–density is a generalization of the so-called $\mathcal{I}(\mathcal{J})$–density ([2]), $\mathcal{I}\langle s\rangle$–density ([1]), and finally $\mathcal{I}$–density ([3]).

For every Baire set A, let us define

$$\Phi_{\mathcal{I}(\mathcal{Y})}(A) = \{x \in \mathbb{R} : x \text{ is an } \mathcal{I}(\mathcal{Y})\text{–density point of } A\}.$$

Then $\Phi_{\mathcal{I}(\mathcal{Y})}$ is a lower density operator, and the family

$$\mathcal{T}_{\mathcal{I}(\mathcal{Y})} = \left\{A \in \mathcal{B}a : A \subset \Phi_{\mathcal{I}(\mathcal{Y})}(A)\right\}$$

is a topology containing the natural topology.

For a sequence $\mathcal{Y}$ of intervals tending to zero, we consider four families of continuous functions defined as follows:

$$\begin{aligned}
\mathcal{C}_{nat,nat} &= \{f\colon (\mathbb{R}, \mathcal{T}_{nat}) \to (\mathbb{R}, \mathcal{T}_{nat})\},\\
\mathcal{C}_{nat,\mathcal{I}(\mathcal{Y})} &= \{f\colon (\mathbb{R}, \mathcal{T}_{nat}) \to (\mathbb{R}, \mathcal{T}_{\mathcal{I}(\mathcal{Y})})\},\\
\mathcal{C}_{\mathcal{I}(\mathcal{Y}),nat} &= \{f\colon (\mathbb{R}, \mathcal{T}_{\mathcal{I}(\mathcal{Y})}) \to (\mathbb{R}, \mathcal{T}_{nat})\},\\
\mathcal{C}_{\mathcal{I}(\mathcal{Y}),\mathcal{I}(\mathcal{Y})} &= \{f\colon (\mathbb{R}, \mathcal{T}_{\mathcal{I}(\mathcal{Y})}) \to (\mathbb{R}, \mathcal{T}_{\mathcal{I}(\mathcal{Y})})\}.
\end{aligned}$$

The aim of this talk is to discuss the properties of continuous functions equipped with the $\mathcal{I}(\mathcal{Y})$–density topology or natural topology in the domain or the range.

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Maxim Zinchenko

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Bounds for L^∞ extremal polynomials

In this talk I will give an overview of some of the recent results on norm estimates for L^∞ extremal polynomials. In particular, I will discuss sharp lower and upper bounds and an analog of the Szegő theorem for the L^∞ extremal polynomials.

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Ondřej Zindulka

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Combinatorics of null-additive sets

A set of reals $X \subseteq \mathbb{R}$ is termed *null-additive* if $X + N$ is Lebesgue null for any Lebesgue null set $N \subseteq \mathbb{R}$. Replacin Lebesgue measure with Haar measure it easily extends to other locally compact Polish groups.

This notion is around over 30 years. It is related to the classical notion of strong measure zero and therefore of interest in set theory. The basic result of Shelah [1] provides a combinatorial characterization of null-additive sets in the Cantor set 2^ω.

We discuss characterizations of null-additive sets in terms of

- fractal measures,
- selection principles and
- games

in the Cantor set, the line and Euclidean spaces and have a look at the general case of a locally compact Polish group.

References

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