Álvaro Antón Sancho

Triality and fixed points of Spin*-bundles.*

This work is concerned with the variety of fixed points of the moduli space of $\mathrm{Spin}(8,\mathbb{C})$-bundles by the triality automorphism.

This is motivated by recent research of O. García-Prada and S. Ramanan, who studied the action of the group $\mathrm{Out}(G)$ of a complex semisimple Lie group G on the moduli space of G-Higgs bundles on a compact Riemann surface. In particular, they identified the fixed points under involutions in $\mathrm{Out}(G)$, showing that they are in one-to-one correspondence with the representations of the fundamental group of the surface in real forms of G.

Here we start a similar analysis for the fixed points under the action of elements of order three in $\mathrm{Out}(G)$. As a first step in this direction, in this work we consider the fixed points of the moduli space $\mathcal{M}(G)$ of G-bundles, for the group $G = \mathrm{Spin}(8,\mathbb{C})$, by the triality automorphism (an outer automorphism of order 3). We fix our attention in $G = \mathrm{Spin}(8,\mathbb{C})$ because the only simple Lie algebra which admits automorphisms of order three is $\mathfrak{so}(8,\mathbb{C})$ and $\mathrm{Spin}(8,\mathbb{C})$ is the simply connected Lie group with this Lie algebra. Our main result describes the variety of stable and simple fixed points of $\mathcal{M}(\mathrm{Spin}(8,\mathbb{C}))$ under the triality automorphism.

This lecture is devoted to the following topics:

1. General facts about Lie groups and Lie algebras. The case of $\mathfrak{so}(8,\mathbb{C})$ and $\mathrm{Spin}(8,\mathbb{C})$.
2. The moduli space of principal G-bundles, $\mathcal{M}(G)$.
3. The action of $\mathrm{Out}(G)$ on $\mathcal{M}(G)$. The fixed points of the moduli space of Spin-bundles.

Iván Blanco Chacón (UCM)

Rational points on elliptic curves

Let K be a perfect field and L an algebraic closure. An elliptic curve is a pair (E,O) where E is a smooth curve of genus 1 and O a distinguished point of E. Due to Riemann-Roch's theorem, such a curve admits a Weierstrass equation, particularly simple in characteristic different form 2 and 3, and what's more, a group structure, being possible to define it, both from the Weierstrass equation and from the Picard group of the curve, intrinsically.

In diophantine geometry, the interest of such a curve are its K-rational points, those being characterized by the trivial action of the Galois group of L/K. Such points inherit the group structure, forming the Mordell-Weil group E(K) of E/K. Unfortunately, from degree 3 onwards, there is no "Hasse-Minkowsky's principle", and henceforth, the study of the relations between p-adic solutions of the elliptic and the rational solutions (if any) becomes a very important question, arising objects as the Tate-Schafarevich and Selmer group. Attached to an elliptic curve, exist a L-function, over which, BSD announces a relation between the order of vanishing at s=1 and the rank of Mordel-Weil's group.

I will briefly discuss these questions, concretely, the meaning of BSD conjecture, and Wiles' thesis, as well as Gross-Zagier work.

Energy Balance Climate Models and control results for discrete problems

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The aim of this communication is to present some new results related to a unidimensional discrete climate model obtained from a difference scheme discretization. We consider the following model (introduced by Budyko and Sellers in 1969)

$$(P)\quad \begin{cases} y_t - (k(1-x^2)y_x)_x = R_a(x,y,v) - R_e(x,y,u), \quad (t,x) \in (0,\infty)\times(-1,1)\,, \\ y(0,x) = y_0(x), \quad x \in (-1,1)\,. \end{cases}$$

where k represents the diffusion coefficient, $R_a(y,v)$ determines the amount of energy absorbed by the Earth surface (which depends on the solar constant Q) and $R_e(y,u)$ is the energy emmitted by Earth (which will be considered strictly increasing) defined as $R_e(y,u) = u\,|y|^3\,y$ (Stefan-Boltzmann law). The parameter u repesents the emissivity of our model and will be considered as a control variable (indicating the regulations on the emission of greenhouse gases). We also have another control, v, that influences on the amount of solar energy absorbed by Earth. These controls take into account our activities over the climate system. We will assume the pressence of this controls in some latitude interval $(l_1,l_2) \subset (-1,1)$, representing the region of the Earth surface where the controls are considered.

It iswell known (véase [**?**] and [**?**]) that the set of stationary solutions of (P) is large enough and that its bifurcation diagram (with respect to the solar constant Q) has a principal branch that is S-shaped with at least one turning point to the right and another one to the left.

Our aim is to analyse and solve the problem of transferring the system (P) from a steady-state to another one for a sufficiently large time $T > 0$ with a suitable choosing of the control variables u y v. For example, consider the problem of transferring our system from an unstable steady-state $y_0(x)$ to a stable one $y_f(x)$. This kind of problems was first considered by J. von Neumann in a general context (see [**?**] y [**?**]).

Our exposition will be divided into three steps. In the frst one, we will introduce some climate modelling concepts and energy balance climate models. Lately, we will present new results for the discrete problem concerning the bifurcation diagram of the stationary model. And finally, we will introduce some controlability technics for nonlinear differential equations to analyse the control problem mentioned above.

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Carolyn Gordon (U. Dartmouth)

You can't hear the shape of a drum

Inverse spectral geometry is the study of the extent to which spectral data associated with an object. E.g, viewing a plane domain as a drumhead, the associated spectral data are the characteristic frequencies of vibration. Mark Kac popularized this question as "Can one hear the shape of a drum?" We will describe an elegant technique of T. Sunada for constructing objects with the same spectral data. Using this technique, we will explicitly construct a pair of "sound-alike" polygonal shaped drumheads.

Juan Manuel Hernández Rubio (UCM)

Right and strong topologies in Banach spaces

Right and Strong* topologies are defined within the Banach spaces framework. Their most important properties and their relationship with other natural topologies defined in Banach spaces are studied. Theorems that characterize the continuity of linear operators with respect to these topologies are given in terms of well known properties in operators theory. This shows a narrow link among locally convex topologies and operator ideals setting a general framework for the study of such topologies.

Paltin Ionescu (U. Genova)

Some applications of adjoint systems

Adjoint systems reflect the interaction between the intrinsic and the extrinsic geometry of an embedded projective manifold. After recalling the main result on the structure of the classical adjunction map, I will present three, quite different, applications. The first one is the finiteness of the Gauss map (following Zak and Ein). Next I will present a recent joint work with F. Repetto, proving a generalization of a theorem by Van de Ven on characterizing linear spaces by a splitting property of the normal bundle sequence.

The main application concerns a classification of non-degenerate manifolds in P^N, of degree at most N. Modulo the Hartshorne Conjecture, this classification is complete. As a corollary, such manifolds turn out to be simply connected, a result for which no topological proof is known yet.

Álvaro Köhn Luque (UCM)

Models of vascular networks formation

In this talk it will be commented some models proposed to describe the process of vascular networks formation, and it will be proposed some of the possible lines of work in this field.

Miguel Ángel López Muñoz (UCM)

Man in the Middle and quantum protocols

The Man in the Middle attack has been useful breaking cryptographic protocols such as Diffie-Hellman key exchange. This method supposes that Eve (the enemy) has complete control over the channel that Alice and Bob uses to communicate among themselves, not only knowing all the information they send to each other but also stopping communication and masquerade as one of them to the other. In quantum cryptography the Man in the Middle strategy has also been successfully used to break some protocols, especially those based in the sharing of EPR pairs. The EPR pairs are pairs of qubits —the quantum analogue of the classical bits— entangled, that is, certain measures done in one of them somewhat affects the other, even if they are not together.

We propose a protocol that allows to sometimes unmask the presence of Man in the Middle in a quantum channel used by Alice and Bob. It is mainly based in two important quantum properties: the inability of distinguish general quantum states and the inability of knowing if a single qubit is or not part of an EPR pair.

This protocol could be used to improve other protocols, something useful because the Man in the Middle attack in general cannot be avoided. The reason is that very often the channel is public or property of a third party.

Nicolas Puignau (U. Lyon-UCM)

A Quick Overview in Enumerative geometry

Enumerative geometry has known an important breakthrough this last decade under impulsion of Gromov-Witten theory and developments of Tropical Geometry.

In this talk, we will take a look on this methods, approaching some classical complex enumerative problems recently solved and their real counterpart (by name: Welschinger's invariants).We will enter upon news questions and expected results of this very active area of mathematics.

Flavia Repetto (U. Milan)

A Barth-Lefschetz result for submanifolds of a product of projective spaces

It is well known (as a consequence of a topological result of Lefschetz type, due to Barth and Larsen) that if X is a submanifold of the complex projective space P^N of positive dimension d, then the restriction map between the Picard groups:

$$\mathrm{Pic}(P^N) \to \mathrm{Pic}(X)$$

is an isomorphism if $N<2d-1$, and injective with torsion free cokernel if $N=2d-1$.

In my talk I will present a result (joint work with Lucian Badescu) concerning the Picard group of a small codimensional submanifold X of the product of two projective spaces, improving some bounds due to Sommese. I will also give two examples showing that the bounds obtained are optimal.

Constanza Riera Burger (U. Bergen)

Graph states and unitary transforms

A new state, the two-graph state, is presented, being a generalisation of the graph state. It describes precisely the coefficients of the pure quantum state vector resulting from the action of a member of the local Clifford group on a graph state. This description facilitates a computationally efficient spectral analysis of the graph state with respect to operations from the local Clifford group on the state, as all operations can be realised graphically. By focusing on the so-called local transform group, which is a size 3 cyclic subgroup of the local Clifford group over one qubit, and over n qubits is of size 3^n, we can efficiently compute spectral properties of the graph state up to about $n = 25$ qubits.

Jonathan Sánchez Hernández (UCM)

Contraction of Jordan's algebras in dimension 2

We determine the isomorphism classes of Jordan algebras in dimension two over the field of real numbers. Using techniques of non-standard analysis we study the properties of the variety of Jordan algebras, and also the contractions among these algebras.

Partial Differential Equations and Perturbations

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Working with Sobolev spaces we can use standard extension operators to extend functions defined in Ω. Usually these operators require strong regularity conditions on Ω. These conditions can be improved in an optimal way by constructing a more complicated extension operator, which is due to Stein. After comparing these two kind of operators, we will see their influence on some known results about Sobolev embeddings and trace operators.

These tools will help us to study the behaviour of eigenvalues of the Laplace operator with the following boundary conditions

$$\frac{\partial u}{\partial n} + \beta u = 0 \qquad \partial\Omega_\epsilon$$

where $\beta = a(x)\epsilon^\gamma$, $a(x) > 0$, when the domain is Hölder perturbed.

Ignacio Santa-María Megía (UCM)

Connections in Principal Fibre Bundles and applications to Field Equations

In this short communication we present the definitions of fibre bundles, principal fibre bundles and the notion of connection. These objects are finally applied to the study of Electromagnetism and other equations from physics, specially through their Hamiltonian formulation.

Damiano Testa (U. La Sapienza)

Rationally connected varieties

The main goal of the talk is to define rationally connected varieties. I will present evidence supporting the claim that they are a large class of simple varieties with interesting properties.

The talk is entirely self-contained, with as little background required as possible.