POLARIZATION CONSTANTS IN NORMED SPACES

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Abstract. According to a fairly well-known result from linear algebra, for each n-homogeneous polynomial P on a normed space E there is a unique n-linear form L on E^n (called the polar of P) such that $P(x) = L(x, \ldots, x)$ for all $x \in E$. Morevoer, if P is continuous, then L is continuous too and

$$\|P\| \leq \|L\| \leq \frac{n^n}{n!}\|P\|.$$

Interestingly, the constant $\frac{n^n}{n!}$ appearing in the previous inequality cannot be replaced, in general, by a smaller value. As a matter of fact, there is a normed space (namely ℓ_1^n) and an n-homogeneous polynomial, $(P(x_1, \ldots, x_n) = x_1 \cdots x_n)$ such that $\|L\| = \frac{n^n}{n!}\|P\|$. However, for a specific normed space E, the best constant $C > 0$ in the inequality $\|L\| \leq C\|P\|$, where P is any n homogeneous polynomial on E and L is its polar, might be smaller that $\frac{n^n}{n!}$. Let $\mathbb{K}$ be either $\mathbb{C}$ or $\mathbb{R}$. We define the n-th polarization constant of a normed E space over $\mathbb{K}$ as

$$\mathbb{K}(E, n) := \inf\{C > 0 : \|L\| \leq C\|P\|\},$$

where the infimum is taken over all the n-homogeneous polynomials P on E (observe that here, L is always the polar of P). Obviously,

$$1 \leq \mathbb{K}(E, n) \leq \frac{n^n}{n!},$$

for all E and every $n \in \mathbb{N}$.

In this work we gather a selection of the most significant results appearing in the literature related to the calculation of the constants $\mathbb{K}(E, n)$ for several classical normed spaces E. Also, a number of general properties of the constants $\mathbb{K}(E, n)$ will also be proved.

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