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Around Tjurina number: A new Hope

Patricio Almirón Cuadros
UCM

Abstract

In this talk I will present the state of art about one of the main research lines of my PhD Thesis: Tjurina number. The principal motivation is a question posed by A. Dimca and G.M. Greuel in 2017 which gives a surprising relation between two of the main invariants of plane curves singularities: Milnor number, of topological nature, and Tjurina number, of analytical nature. I will present a partial answer to this question (joint work with G. Blanco) which constitute a new hope to achieve a positive answer to Dimca and Greuel's question in the general case.

Enumeration in geometry

Ciro Ciliberto
Roma Tor Vergata

Abstract

Enumeration of geometric objects verifying some specific properties is an old and venerable subject. In this talk I will start by briefly reviewing some of its history and problems. In the last decades, enumerative geometry saw the flourishing of new problems and underwent a tremendous change of perspective and a spectacular progress, with the introduction of extremely refined new mathematical ideas and tools which launched unexpected bridges between different parts of mathematics. This has been due also, sometimes mainly, to the input of questions coming from physics. Being impossible to present all this material in a one hour talk, I will limit myself to give general information on some aspects of these topics, the ones which are closer to my own research and (limited) knowledge.

Coupled systems of PDEs related to the fifth state of matter

Eduardo Colorado Heras
ICMAT, UC3M

Abstract

Along this talk, will be shown some results for systems of coupled nonlinear Schrödinger equations of the following type,

$$\begin{cases} iu_t + \Delta u + g_u(u,v) = 0 \\ iv_t + \Delta v + g_v(u,v) = 0 \end{cases}$$

where i denotes the imaginary unit, u, v depend on $(x,t) \in \mathbb{R}^n \times \mathbb{R}_+$, $n = 1, 2, 3$ and g is a nonlinear function on its arguments.

The previous kind of system has applications mainly in nonlinear optics and Bose-Einstein condensates (BEC); corresponding the latter to the fifth state of matter achieved at ultra-cold temperatures close to the absolute zero.

Hölder-Zygmund spaces associated to Schrödinger operators and some applications.

Marta de León-Contreras
UAM

Abstract

In [1], E. Stein introduced the classical Hölder-Zygmund spaces defined through the derivatives of the Poisson semigroup $e^{-t\sqrt{-\Delta}}$. He proved that these spaces coincide with the classical Hölder spaces $C^{k,\alpha}$, $0 < \alpha < 1$, $k \in \mathbb{N}$.

In this talk we are going to extend the Hölder-Zygmund spaces introduced by E. Stein for a weaker class of functions and also we will introduce some Hölder-Zygmund spaces associated to Schrödinger operators, $\mathcal{L} = -\Delta + V$, where V is a nonnegative potential satisfying the reverse Hölder inequality. We shall characterize these spaces by means of the heat and the Poisson semigroups as well as pointwise estimates. Finally, we will show some applications.

The content of this talk is based on a joint work with José L. Torrea.

References

[1] E. M. Stein. *Singular integrals and differentiability properties of functions*. Princeton Mathematical Series, No. 30. Princeton University Press, Princeton, N.J., 1970.

How to use a quantum sledgehammer to crack a topological nut (and to get something in return)

Ángel González Prieto
UCM,UPM

Abstract

Back in the 80's, Witten showed that a knot invariant, the Jones polynomial, can be computed from a physical theory known as Chern-Simmons theory. Aware of the importance of this discovery, Atiyah generalized Witten's constructions, putting them into a purely categorical language and giving them the name Topological Quantum Field Theories (TQFTs for short).

From a mathematical point of view, TQFTs are powerful algebraic tools that provide deep insight into the behaviour of topological invariants. In this talk, we will review the fundamentals of TQFTs and we will show how to build them using a physically-inspired construction that only needs two ingredients: a field theory and a quantisation in the spirit of Grothendieck' six operations. We will finish the talk with a collection of examples that show that the use of a sledgehammer (the TQFT) to crack a nut (the topological invariant) may be very productive as it gives in return an effective new method of computation of very subtle topological invariants.

Joint work with M. Logares and V. Muñoz.

A class of companion pencils

Carla Hernando Fuster
UC3M

Abstract

In this talk, we are mainly interested in sparse companion linearizations (pencils). Our main goal is to extend the results in [3] to such kind of constructions.

First, we introduce a general class of potential companion pencils for arbitrary square matrix polynomials over an arbitrary field, which extends the class introduced in [3] for monic scalar polynomials. We provide a canonical form, up to permutation, for companion pencils in this class. We also relate these companion pencils with other relevant families of companion linearizations known so far. Finally, we determine the number of different sparse companion pencils in the class, up to permutation.

References

[1] F. De Terán, F. M. Dopico, D. S. Mackey. Spectral Equivalence of matrix polynomials and the Index Sum Theorem. Linear Algebra Appl. 459 (2014) 264-333.

[2] F. M. Dopico, P. Lawrence, J. Pérez, P. Van Dooren. Block Kronecker linearizations of matrix polynomials and their backward errors. Submitted. Available as MIMS Eprint 2016.34. The University of Manchester. UK.

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Post–quantum cryptography with polynomials.

Ignacio Luengo Velasco
UCM

Abstract

Post–quantum cryptography is the public-key cryptography resistant to future quantum computers. In this talk we will talk about a post–quantum cryptosystem called DME (Double Matrix Exponentiation) based on multivariate polynomial applications that we have developed (using ideas of Algebraic Geometry), patented and present it to the NIST contest to choose the future post–quantum cryptography standard (`https://csrc.nist.gov/Projects/Post\OT1\textendashQuantum-Cryptography/Round-1-Submissions`).

We will also present the questions and problems of Conmutative Algebra related with the algebraic cryptoanalysis of the scheme DME.

The preprint can be found in the web above and in `http://www.mat.ucm.es/~iluengo/DME`

The conjecture of Birch and Swinnerton-Dyer (and some equivariant refinements)

Daniel Macías Castillo
ICMAT,UAM

Abstract

Let E be an elliptic curve defined over $\mathbb{Q}$. The rank part of the Conjecture of Birch and Swinnerton-Dyer (or BSD) for $E/\mathbb{Q}$ is one of the celebrated 'Millennium Problems'. It predicts that the rank of the abelian group $E(\mathbb{Q})$ of points of E that are defined over $\mathbb{Q}$ (known to be finitely generated by the Theorem of Mordell-Weil) coincides with the order of vanishing at $s = 1$ of the L-function associated to E. This L-function is constructed through local methods and, consequently, the latter invariant is relatively easy to calculate in practice.

The stronger Conjecture of BSD further relates the leading term at $s = 1$ of this L-function to a number of additional invariants associated to $E/\mathbb{Q}$. Furthermore, given a finite Galois extension $F/\mathbb{Q}$ with Galois group G, the (again finitely gener- ated) abelian group $E(F)$ has additional structure through the natural action of G. Certain equivariant refinements of the original BSD Conjecture aim to encapsulate this finer structure on $E(F)$.

In this talk we will give a brief and informal introduction to these problems.

Sierpinski-Zygmund functions: Historical overview and recent findings

María Elena Martínez Gómez
UCM

Abstract

H. Blumberg showed (1922) that for every function $f \in \mathbb{R}^{\mathbb{R}}$, R, there exists a subset D dense in $\mathbb{R}$, such that $f_{|D}$ is continuous. Then, one year later, W.Sierpinski and A. Zygmund showed (1923) that there exists a function $f : \mathbb{R} \to \mathbb{R}$ such that, for any set $Z \subset \mathbb{R}$ of cardinality $\mathfrak{c}$, the restriction $f_{|Z}$ is not continuous. Functions enjoying this property are called, in the literature, Sierpinski-Zygmund function.

Some of the very interesting properties enjoyed by these functions are related to other (more modern) classes of functions (such as " *everywhere surjective*" functions, Darboux functions, almost continuous functions, among others).

The existence of large structures within the class of Sierpinski–Zygmund functions depends, strongly, on the set theoretical setting we are working on, reaching some undecidability results unless some additional hypothesis are considered. In this short talk we shall provide an overview on what it is known about the class of Sierpinski–Zygmund functions and its topological and algebraic genericity within different set theoretical settings and also we will provide a new proof of the Blumberg's result that is simpler and much shorter than the original one.

This is a joint work with Profs. Krzysztof C. Ciesielski, Gustavo A. Muñoz, and Juan B. Seoane.

References

[1] H. Blumberg. *Properties of all real functions*, Trans. Amer. Math. Soc. 24 (1922), no. 2, 113–128.

[2] K.C. Ciesielski, M.E. Martínez-Gómez, G.A. Muñoz-Fernández, and J.B. Seoane-Sepúlveda. *On the class of Sierpinski-Zygmund functions. An expository set theoretical approach*, Preprint (2018). 113–128.

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Unitary operators in Hilbert spaces of analytic functions

Alejandro Mas Mas
UAM

Abstract

In many Banach spaces of analytic functions all the isometries or, at least, the surjective isometries (unitary operators) are known. In the Hilbert spaces there are much more isometries, so a natural question is trying to describe all unitary operators which are, for instance, weighted composition operators.

In this talk we will answer this question in a large family of Hilbert spaces of analytic functions in the unit disk with reproducing kernels. These spaces are called weighted Hardy spaces.

This is a joint work with María J. Martín and Dragan Vukotić, part of the author's thesis project.

Degraded mixing solutions for the Muskat problem

Francisco Mengual Bretón
UAM

Abstract

The dynamic of two incompressible fluids moving through a 2D porous medium and under the action of gravity can be modelled by the IPM system. When initially one of the fluids lies above the other, the Muskat problem describes the evolution of IPM under the assumption that the fluids remain in contact at a moveable interface Graph(f), which turns out a Cauchy problem for f. If the heaviest fluid stays down (stable regime) such problem is well-posed in Sobolev spaces, otherwise (unstable regime) it is ill-posed. In spite of this, Á. Castro, D. Córdoba and D. Faraco proved recently the existence of weak solutions of IPM in the unstable regime by replacing the free boundary assumption with the opening of a mixing zone. In this talk I will sketch a new construction in which we show that such mixing solutions display a linearly degraded macroscopic behaviour.

This is joint work with Á. Castro and D. Faraco.

Power-regularity and spectral properties of Bishop-type operators.

Miguel Monsalve López
ICMAT, UCM

Abstract

While the invariant subspace problem remains open in the Hilbert space setting, several families of operators have arisen as candidates of counterexamples along the years: one of the simplest and longest-standing example is the Bishop's operators.

In this talk, we will present some ideas which relate some spectral properties with a particular asymptotic behaviour of Bishop's operators.

Joint work with Eva A. Gallardo Gutiérrez.

Mathematical Models in Material Science.

Carlos Mora Corral
UAM

Abstract

We will explain how mathematics help to model and understand the behaviour of elastic bodies, brittle bodies, materials with shape memory and liquid crystals.

A Whitney Extension Theorem for convex functions

Carlos Mudarra Díaz-Malaguilla
ICMAT, UCM

Abstract

Given a closed subset E of $\mathbb{R}^n$ and two functions $f : E \to \mathbb{R}$, $G : E \to \mathbb{R}^n$, the famous Whitney Extension Theorem provides necessary and sufficient conditions on f, G for the existence of a function F of class $C^1(\mathbb{R}^n)$ such that $F = f$ and $\nabla F = G$ on E. It is natural to ask for further necessary and sufficient conditions on f, G which ensure that the extension F can be taken to be *convex* as well. In this talk we present the solution to this problem. Also, given a Hilbert space X and a modulus of continuity ω, we solve a similar problem for convex functions of class $C^{1,\omega}(X)$. If time allows, we will present some applications of these results.

Periodic solutions in a non spatial periodic Lotka-Volterra predetor-prey model.

Eduardo Muñoz Hernández
UCM

Abstract

In this talk, I will explain the work done under the supervision of Julián López Gómez during my Master's Thesis about a non spatial periodic Lotka-Volterra predator-prey model. This work is developed from [2], a paper where López-Gómez, J., Ortega, R. and Tineo, A. showed the multiplicity of periodic coexistence states in a non spatial periodic Lotka-Volterra predator-prey model. In my Master's Thesis, we have proved, in the first place, the uniqueness of the multiple periodic coexistence states. Secondly, inspired by a paper by Ding, T. and Zanolin, F. [1], we have also demonstrated the existence of infinite periodic solutions for a reduced model of [2], but out of the hypothesis of the main theorem of [1]. These results are in strong contrast with the uniqueness result about coexistence states obtained in [3].

References

[1] Ding, T., Zanolin, F. (1996). Periodic solutions and subharmonic solutions for a class of planar systems of Lotka-Volterra type, *Proc. First World Congress Nonlin. Anal.,* pp. 395-406.

[2] López-Gómez, J., Ortega, R. and Tineo, A. (1996). The periodic predator-prey Lotka–Volterra model,*Adv. Diff. Eqns.* **1**, pp. 403-423.

[3] López-Gómez and Pardo, R. M. (1993). The existence and the uniqueness for the predator-prey model with diffusion, *Diff. Int. Eqs.* **6**, pp. 1025-1031.

The Brezis-Nirenberg problem for the fractional Laplacian with mixed Dirichlet-Neumann boundary conditions

Alejandro Ortega García
UC3M

Abstract

In this work we study the existence of solutions to the critical Brezis-Nirenberg problem when one deals with the spectral fractional Laplace operator and mixed Dirichlet-Neumann boundary conditions. We obtain results in the spirit of Brezis-Nirenberg as well as we study the effect that the mixed Dirichlet-Neumann boundary condition has on the existence issues. In particular, if the size of the Dirichlet part is 'small enough' we will be able to obtain existence results for a certain critical problem.

Quantum entanglement in nonlocal games

Carlos Palazuelos Cabezón
ICMAT, UCM

Abstract

In this talk we will explain how quantum entangled states can be used to improve the probability of success in nonlocal games. We will see how nonlocal games can be understood from the point of view of Banach spaces, and how some fundamental results in Functional Analysis can be used to deduce interesting properties about such games.

Duality and normality on topological groups

Víctor Pérez Valdés
UCM

Abstract

This talk can be divided essentially in two parts. In the first one, we introduce basic concepts and classical results of the class of topological groups. Concretely, we deal with the Pontryagin Duality Theorem for locally compact abelian groups.

In the second one, we present a result of normality using techniques of duality theory of topological groups. A group topology ν in a topological group (G, τ) is said to be a compatible topology with the duality $(G, G^\wedge)$ if the dual groups of G induced by τ and ν coincide, i.e. $(G, \nu)^\wedge = (G, \tau)^\wedge$.

The result we show states that for the group $G := \mathbb{Z}^{\mathbb{R}}$, there do not exist normal topologies compatible with the duality $(G, G^\wedge)$ that belongs to a particular family; namely, the family of the locally quasi-convex topologies.

This result has been recently published in a joint article with the professor E. Martín-Peinador.

References

[1] M. J. Chasco, E. Martín-Peinador, V. Tarieladze. *On Mackey topology for groups.* Studia Mathematica 132 (3) (1999) 257-284.

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Mathematical properties on the hyperbolicity of interval graphs

Rosalío Reyes Guillermo
UC3M

Abstract

Gromov hyperbolicity is an interesting geometric property, and so it is natural to study it in the context of geometric graphs. In particular, we are interested in interval and indifference graphs, which are important classes of intersection and Euclidean graphs, respectively.

It is well-known that interval graphs (with a very weak hypothesis) and indifference graphs are hyperbolic. In this paper we give a sharp bound for their hyperbolicity constants. The main result in this paper is the study of the hyperbolicity constant of every interval graph with edges of length 1. Moreover, we obtain sharp bounds for the hyperbolicity constant of the complement of any interval graph with edges of length 1.

Polarization constants in normed spaces

Daniel L. Rodríguez Vidanes
UCM

Abstract

A fairly well-known result from linear algebra states that for every n-homogeneous polynomial P on a normed space E there is a unique n-linear form L on E^n such that $P(x) = L(x, \dots, x)$ for all $x \in E$. Moreover, R. S. Martin in his Ph. D. Dissertation (1932) proved that if P is continuous, then L is continuous too and

$$\|P\| \leq \|L\| \leq \frac{n^n}{n!}\|P\|.$$

The constant appearing in the previous inequality cannot be replaced, in general, by a smaller value, that is, there is a normed space E and an n-homogeneous polynomial P such that $\|P\| = \frac{n^n}{n!}\|L\|$ where $P(x) = L(x, \dots, x)$ for all $x \in E$. Nonetheless, for a specific normed space the best constant in the inequality might be smaller than $\frac{n^n}{n!}$, this is what is known as the n-polarization constant.

The main goal of this talk is to show some of the known values of the n-polarization constants, some approximations of these constants and other results involving the value of these constants.

Espacios L_p y $L_{p(.)}$ de exponente variable

Mauro Sanchiz Alonso
UCM

Abstract

En la rama del Análisis funcional los espacios más sencillos y que primero se estudian, ya en el grado de matemáticas, son los espacios l_p de sucesiones y L_p de funciones. Una generalización de los espacios L_p son los espacios $L_{p(.)}$ de exponente variable, que son espacios que guardan mucha relación con los espacios L_p pero que son mucho más ricos en estructura. En esta presentación veremos algunas de esas propiedades. La exposición realizada saldrá de mi trabajo final de Master, "Estructura de espacios L_p y de exponente variable. Operadores estrictamente singulares".

Cones of non-negative Polynomials

Christoph Schulze
Universität Konstanz

Abstract

Let $n, d \in \mathbb{N}$ and $\mathbb{R}[\underline{X}]_{2d} := \mathbb{R}[X_0, X_1, \ldots, X_n]_{2d}$ be the vector space of homogeneous polynomials of degree $2d$. We denote by $\mathcal{P} \subseteq \mathbb{R}[\underline{X}]_{2d}$ the subset of polynomials which are non-negative on the real projective space $\mathbb{P}^n(\mathbb{R})$ (or more generally on some subset $S \subseteq \mathbb{P}^n(\mathbb{R})$). Then $\mathcal{P}$ is convex and closed under multiplication with $\mathbb{R}^+$ - therefore $\mathcal{P}$ is a convex cone.
The aim of this talk is to give an overview about results concerning the convex structure of $\mathcal{P}$. This includes a complete characterization of the facial structure for very small n and d, results about extreme rays of $\mathcal{P}$ (which may be seen as the generating points of the cone), connections between faces of $\mathcal{P}$ and their zero sets, relations to interpolation problems and information about faces of low codimension.

Higher order recurrences and row sequences of Hermite-Padé approximation

Yanely Zaldívar
UC3M

Abstract

We obtain extensions of the Poincaré and Perrón theorems for higher order recurrence relations and apply them to obtain an inverse type theorem for row sequences of (type II) Hermite-Padé approximation of a vector of formal power series.

List of Speakers

Almirón Cuadros, Patricio (UCM): *Around Tjurina number: A new Hope*

Ciliberto, Ciro (Roma Tor Vergata): *Enumeration in geometry*

Colorado Heras, Eduardo (ICMAT, UC3M): *Coupled systems of PDEs related to the fifth state of matter*

De León-Contreras, Marta (UAM): *Hölder-Zygmund spaces associated to Schrödinger operators and some applications.*

González Prieto, Ángel (UCM, UPM): *How to use a quantum sledgehammer to crack a topological nut (and to get something in return)*

Hernando Fuster, Carla (UC3M): *A class of companion pencils*

Luengo Velasco, Ignacio (UCM): *Post–quantum cryptography with polynomials.*

Macías Castillo, Daniel (ICMAT,UAM): *The conjecture of Birch and Swinnerton-Dyer (and some equivariant refinements).*

Martínez Gómez, María Elena (UCM): *Sierpinski-Zygmund functions: Historical overview and recent findings.*

Mas Mas, Alejandro (UAM): *Unitary operators in Hilbert spaces of analytic functions.*

Mengual Bretón, Francisco (UAM): *Degraded mixing solutions for the Muskat problem.*

Monsalve López, Miguel (ICMAT, UCM): *Power-regularity and spectral properties of Bishop-type operators.*

Mora Corral, Carlos (UAM): *Mathematical Models in Material Science.*

Mudarra Díaz-Malaguilla, Carlos (ICMAT, UCM): *A Whitney Extension Theorem for convex functions.*

Muñoz Hernández, Eduardo (UCM): *Periodic solutions in a non spatial periodic Lotka-Volterra predetor-prey model.*

Ortega García, Alejandro (UC3M): *The Brezis-Nirenberg problem for the fractional Laplacian with mixed Dirichlet-Neumann boundary conditions.*

Palazuelos Cabezón, Carlos (ICMAT, UCM): *Quantum entanglement in nonlocal games.*

Pérez Valdés, Víctor (UCM): *Duality and normality on topological groups.*

Reyes Guillermo, Rosalío (UC3M): *Mathematical properties on the hyperbolicity of interval graphs*

Rodríguez Vidanes, Daniel L. (UCM): *Polarization constants in normed spaces.*

Sanchiz Alonso, Mauro (UCM): *Espacios L_p y $L_{p(.)}$ de exponente variable.*

Schulze, Christoph (Universität Konstanz): *Cones of non-negative Polynomials.*

Zaldívar, Yanely (UC3M): *Higher order recurrences and row sequences of Hermite-Padé approximation.*