Programa y Abstracts

XIII Workshop of Young Researchers in Mathematics

Facultad de Ciencias Matemáticas, UCM
September 23-25, 2019

Programa

Lunes 23:

- 10:00-10:30: Registro
- 10:30-10:45: Presentación
- 10:45-11:45: J. Ildefonso Díaz Díaz: Beyond the unique continuation: flat solutions for reactive slow diffusions and the infinite walls and confinement singular potentials for the Schrödinger equation
- 11:45-12:15: Café
- 12:15-12:45: Jesús Llorente Jorge: Takagi-Van der Waerden functions
- 12:45-13:15: Luis Alberto Rodríguez Ramírez: El análisis de datos funcionales. Una introducción al PCA funcional
- 15:00-16:00: Javier Aramayona Delgado: Estructuras geométricas en superficies
- 16:00-16:45: Giovanni Bazzoni: Un zoo de variedades complejas, simplécticas y no Kähler
- 16:45-17:15: Café
- 17:15-17:45: Guillermo Gallego Sánchez: Higgs bundles twisted by a vector bundle
- 17:45-18:15: Juan Ángel Rojo Carulli: Pencils of symplectic cubics
- 18:15-18:45: Álvaro Rodríguez Abella: Reducción en Teoría Geométrica de Campos. Ecuaciones de Yang-Mills

Martes 24:

- 10:00-11:00: Simone Marchesi: Terao's conjecture holds for RUA arrangements
- 11:00-11:45: Fernando Abellán García: Combinatorial models for homotopy types
- 11:45-12:15: Café
- 12:15-12:45: John Stewart Fabila-Carrasco: Construction of cospectral graphs
- 12:45-13:15: Ángel Aitor Balmaseda Martín: Self-adjoint extensions as controls
- 13:15-13:45: Jaime Santos Rodríguez: Geometry and optimal transport
- 15:30-16:30: Cristina Brändle Cerqueira: On unbounded solutions of ergodic problems for non-local Hamilton-Jacobi equations
- 16:30-17:00: Javier Alejandro Quintero Roba: Asymptotic behaviour of rational modified orthogonal polynomials
- 17:00-17:30: Café
- 17:30-18:00: Iván Caamaño Ademunde: Espacios de Sobolev-Reshetnyak
- 18:00-18:30: Ana Soledad Meroño Moreno: La realcompactificación de Samuel de D^{ω_0}

Miércoles 25:

- 10:00-11:00: Ignacio Villanueva Díez: Valuations on star bodies and function lattices
- 11:00-11:45: Davide Barbieri: Group representations and signal processing
- 11:45-12:15: Café
- 12:15-12:30: Presentación Red de Doctorandos
- 12:30-13:00: Rosalío Reyes Guillermo: Hyperbolicity constant of circular-arc graphs
- 13:00-13:30: Juan Carlos Sampedro Pascual: Métodos topológicos para operadores Fredholm de índice cero
- 13:30-14:00: Francisco Mengual Bretón: On the vortex sheet problem

Abstracts

Combinatorial models for homotopy types

Fernando Abellán García
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Abstract

Since the begginings of Algebraic Topology mathematicians have pursued to construct algebraic invariants for topological spaces. This leads to the question: Can we construct a combinatorial model for homotopy types? The answer is yes, and this model provides a meeting point for algebraic topology, algebraic geometry and category theory. This talk is aimed as an informal introduction to infinity category theory and its relation to homotopy theory through the Kan-Quillen model structure.

Estructuras geométricas en superficies

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Abstract

El objetivo de esta charla será dar una descripción del conjunto de métricas de curvatura constante que uno puede poner en una superficie topológica orientable de género g.

Cualquier graduado en matemáticas podrá seguir la totalidad de la charla.

Self-adjoint extensions as controls

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Abstract

We consider the linear control problem on an infinite-dimensional Hilbert space with dynamics described by a family of self-adjoint operators H through the equation

$$\frac{d}{dt}\varphi(t) = -iH(t;u)\varphi(t),$$

where u is a control parameter. As the space of controls we will consider the space of self-adjoint extensions of a symmetric operator. This presents difficulties for the existence of dynamics that we need to handle. To prove the viability of this control scheme, we will consider the particular instance in which the Hilbert space is $L^2([0,1])$ and the dynamics are generated by a subfamily of the self-adjoint extensions of the Laplacian, $-\Delta = -\frac{d}{dx}$. For this system, we will provide conditions under which the dynamic is well defined and prove that the system is approximately controllable. Time permitting, we will discuss applications in Physics.

Group representations and signal processing

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Abstract

The design and application of results in abstract harmonic analysis for the study of symmetries in datasets like digital sounds and images constitutes an active research field with a long history. Besides abelian groups such as the group of translations, of special importance due to its natural relationship with convolution and Fourier analysis, several nonabelian group structures have proved to be relevant in applications. The simplest ones are the Heisenberg group of quantum mechanics and the affine group of the Euclidean space, which give rise to the so-called Gabor and wavelet systems. The purpose of this talk is to discuss some classical topics concerning group representations and Fourier analysis, and understand some of their applications such as the extraction of musical score-like diagrams from digital music.

Un zoo de variedades complejas, simplécticas y no Kähler

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Abstract

Toda variedad Kähler posee una estructura simpléctica. Hace 40 aos Thurston dio el primer ejemplo de una variedad compacta simpléctica sin métricas Kähler. Obtener ejemplos de tales variedades es un problema doble: se necesitan técnicas de construcción de variedades simplécticas compactas y herramientas para establecer que una variedad simpléctica no admite ninguna estructura Kähler. Toda variedad Kähler tiene también estructura de variedad compleja. Una pregunta más complicada es la existencia de variedades que son simultáneamente complejas y simplécticas, pero no admiten métricas Kähler. En esta charla revisaré los ejemplos de estas variedades en la literatura y presentaré la construcción de una 6-variedad compacta, compleja y simpléctica sin métricas Kähler.

On unbounded solutions of ergodic problems for non-local Hamilton-Jacobiequations

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Abstract

We study an ergodic problem associated to a non-local Hamilton-Jacobi equation defined on the whole space $\lambda - \mathcal{L}[u](x) + |Du(x)|^m = f(x)$ and determine whether (unbounded) solutions exist or not. We prove that there is a threshold growth of the function f, that

separates existence and non-existence of solutions, a phenomenon that does not appear in the local version of the problem. Moreover, we show that there exists a critical ergodic constant, λ_*, such that the ergodic problem has solutions for $\lambda \leq \lambda_*$ and such that the only solution bounded from below, which is unique up to an additive constant, is the one associated to λ_*.

Espacios de Sobolev-Reshetnyak

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Abstract

Estudiaremos, en primer lugar, las propiedades de medibilidad e integrabilidad de funciones que toman valores en espacios vectoriales, para posteriormente poder definir las clases de espacios de Sobolev-Reshetnyak. Nuestro objetivo es dar una generalización sobre las curvas de Peano, afirmando la existencia de curvas de Peano con regularidad de Sobolev-Reshetnyak que cubran un espacio métrico compacto de longitud dado. Por último analizaremos la relación existente entre los espacios de Sobolev-Reshetnyak y los espacios de Sobolev clásicos y cómo influye la propiedad de Radon-Nikodým en dicha relación.

Beyond the unique continuation: flat solutions for reactive slow diffusions and the infinite walls and confinement singular potentials for the Schrödinger equation

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Abstract

Solutions with compact support for some nonlinear elliptic and parabolic equations, and many other free boundary problems, are formulations for which the Unique Continuation Principle, in its several versions, fails. In such problems, which have attracted the attention of many specialists, the solution u and its normal derivative vanish on a region of the boundary (which leads to the definition of a "flat solution" of the corresponding equation).

In this talk I will present, in a very sketched way, some recent results in this direction, trying to show how many open problems of this nature still remain as a source of current research.

More specifically, I will report on some results concerning the following problems:

I). Stable flat solutions of $u_t - \Delta u^m + u^a = \lambda u^b$ for $0 < a < b < m$ under the stability condition $2(m+a)(m+b) - N(m-a)(m-b) < 0$ (joint work with J. Hernández and Y. Sh. Ilyasov),

II). Flat solutions to $\mathbf{i}\frac{\partial \psi}{\partial t} = -\Delta\psi + V(x)\psi$ in $\mathbb{R}^N$, for $V(x) \geq Cd(x, \partial\Omega)^{-\alpha}$, with $\alpha \geq 2$, for some bounded domain Ω (my research continued in collaboration with J. M. Rakotoson, D. Gómez-Castro, R. Temam and J. L. Vázquez).

Construction of cospectral graphs

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Abstract

The Laplacian is an operator acting on graphs. There several types of Laplacian (combinatorial, normalized, magnetic, etc.), it depends on the additional information about the graphs (weights on vertices, on edges, and vector potential). The spectrum of the Laplacian gives us important information about the graph (connected components, number of vertices, number of edges, bipartite, etc.). Two non-isomorphics graphs are cospectral or isospectral if they have the same spectrum. In this talk, we give a geometrical construction that generates cospectral graphs for the discrete normalized Magnetic Laplacian. The presented results are part of a work in process.

Higgs bundles twisted by a vector bundle

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Abstract

In this talk we explore a generalization of the theory of Higgs bundles over compact Riemann surfaces, in which the canonical line bundle is replaced by a vector bundle of arbitrary rank. This kind of situation appears naturally in the study of supersymmetric gauge theories. From the mathematical point of view, we study several topics: analogous of Hitchins equations, stability, HitchinKobayashi correspondence and a generalization of the spectral curve.

Takagi-Van der Waerden functions

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Abstract

Let $r \geq 2$. The Takagi-Van der Waerden function $f_r : [0,1] \to \mathbb{R}$ is defined as follows

$$f_r(x) = \sum_{n=0}^{\infty} \frac{1}{r^n} \phi(r^n x)$$

where $\phi(x)$ denotes the distance from the point x to the nearest integer. These functions are an immediate generalization of the Takagi function and they constitute a family of continuous nowhere differentiable functions. We characterize the set of points where the lateral derivatives of the Takagi-Van der Waerden function are infinite. Furthermore, we determine the Hausdorff dimension and the Lebesgue measure of this set. This is a joint work with J. Ferrera and J. Gómez Gil.

Terao's conjecture holds for RUA arrangements

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Abstract

One of the most famous open problems in the topic of hyperplane arrangements is the so calledTerao's conjecture,which statesthat the freedom of an arrangement depends on its combinatorics. After giving an introduction to the problem, we will focus our attention on the special case oftriangular arrangementsin the projective plane,i.e. with all the lines passing through three fixed points. We prove that the conjecture holds for a central family, calles Roots of Unity Arrangements, which also provides a counterexample to the weakconjecture. This is a joint work with Jean Vallès.

On the vortex sheet problem

Francisco Mengual
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Abstract

Although turbulence is commonly observed in everyday phenomena, it is still one of the greatest challenges in mathematical physics. In the last years the convex integration method, whose origin date back to works of Nash and Gromov in Differential Geometry, has allowed to verify the Onsager's conjecture about anomalous dissipation of energy of weak solutions to the incompressible Euler equations. This phenomenon can be triggered when the vorticity is initially concentrated on a curve z_0. In this case, the assumption that the system evolves as a free boundary turns out an ill-posed Cauchy problem for z_0 (Kelvin-Helmholtz instability). In the absence of analiticity in z_0, we will see in this talk how this method provides a mechanism that allows to construct weak solutions to the incompressible Euler equations which are smooth outside a strip which grows in time around z_0 (turbulence zone) where the energy is dissipated. Joint work with László Székelyhidi Jr.

La realcompactificacin de Samuel de D^{ω_0}

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Abstract

Para un espacio uniforme (X, μ) se define la realcompactificacin de Samuel como la complecin del espacio X con la uniformidad dbil inducida por la familia de funciones reales uniformemente continuas sobre (X, μ) . Cuando el cardinal de un espacio discreto D es no Ulam-medible se tiene trivialmente que la Realcompactificacin de Samuel del el espacio de Baire D^{ω_0} coincide con la conocida realcompactificacin de Hewitt inducida por la funciones reales continuas sobre X. En esta charla se ver que cuando el cardinal de D es ω_1-fuertemente compacto entonces este resultado ya no es cierto.

Asymptotic behaviour of rational modified orthogonal polynomials

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Abstract

Suppose ν is a positive Borel measure supported on $[0, \infty)$, with finite moments that satisfy a so called Carleman's condition, and such that its Radon-Nikodým derivative with respect to the Lebesgue measure is positive almost everywhere on $[0, \infty)$. If υ is a rational function with zeros on $\mathbb{C} \setminus [0, \infty)$ and poles on $(-\infty, 0]$, we denote by $\upsilon(x)d\nu(x)$ a rational modification of ν. We obtain the relative asymptotic behavior between the orthogonal polynomials with respect to $\upsilon d\nu$ and the orthogonal polynomials with respect to $d\nu$.

Hyperbolicity constant of circular-arc graphs

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Abstract

Gromov hyperbolicity is an interesting geometric property, and so it is natural to study it in the context of geometric graphs. It measures the tree-likeness of a graph from a metric viewpoint. In particular, we are interested in circular-arc graphs, which is an important class of geometric intersection graphs. In this paper we give sharp bounds for the hyperbolicity constant of (finite and infinite) circular-arc graphs. Moreover, we obtain bounds for the hyperbolicity constant of the complement and line of any circular-arc graph. In order to do that, we obtain new results about regular, chordal and line graphs which are interesting by themselves.

Reducción en Teoría Geométrica de Campos. Ecuaciones de Yang-Mills

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Abstract

El teorema de Utiyama caracteriza las densidades lagrangianas invariantes gauge en la teoría de Yang-Mills en el vacío. En esta charla se da una introducción a dicha teoría y se estudia en profundidad el teorema anterior, dando una demostración alternativa a la que existe en la literatura. Este nuevo enfoque para la demostración emplea la idea de reducción, ampliamente utilizada en Mecánica Geométrica y Teoría de Campos cuando el grupo de simetría es de dimensión finita.

Sin embargo, en el caso de Yang-Mills el grupo es de dimensión infinita por lo que se necesitan llevar a cabo modificaciones para aplicar la reducción en este contexto, lo que abre las puertas a la generalización de resultados de reducción para grupos arbitrarios.

El análisis de datos funcionales. Una introducción al PCA funcional

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Abstract

¿La muestra de contaminación de tu barrio tiene datos demasiado correlados? ¿El espacio euclídeo se queda pequeño para tus variables aleatorias? Quizá estás pensando en hacer estadística con curvas o en espacios de funciones.

El análisis de datos funcionales, FDA por sus siglas en inglés, nos provee de las herramientas necesarias para hacer estadística en espacios de dimensión infinita. Sí, has leído infinita. En esta charla haremos una revisión histórica de la evolución de la estadística desde sus más tiernos inicios en los modelos paramétricos de Fisher hasta estas sofisticadas técnicas. Como ejemplo mostraré como el teorema espectral en espacios de Hilbert se usa para el análisis de componentes principales en el contexto funcional.

Pencils of symplectic cubics

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Abstract

The dichotomy between symplectic and algebraic geometry is famous and well-known. There are many points in common as well as many diverging features between the two. In this talk we aim to show an instance of the latter case. It is a well-known result in algebraic geometry that if two plane cubics intersect in nine points, then any other cubic passing through eight of these points must also pass through the ninth point. This behaviour is, non surprisingly, specific to algebraic rigidity. We will show examples of symplectic cubics not satisfying this property.

Métodos topológicos para operadores Fredholm de índice cero

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Abstract

A finales del siglo pasado, P.M. Fitzpatrick y J. Pejsachowicz construyeron con técnicas de topología algebraica un análogo de grado topológico para operadores Fredholm de índice cero orientados. El objetivo de esta charla es poner en contexto el concepto de grado topológico y su utilidad en Análisis No Lineal vía su evolución histórica y presentar de manera breve la construcción de P.M. Fitzpatrick y J. Pejsachowicz, remarcando las dificultades de extensión para operadores Fredholm de índice cero.

Referencias:

P. M. Fitzpatrick, J. Pejsachowicz, P. J. Rabier. *Orientability of Fredholm families and topological degree for orientable nonlinear Fredholm mappings*, J. Functional Analysis, **124**, (1994), 1–39.

Geometry and optimal transport

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Abstract

The theory of optimal transport has proved to be quite useful in many areas of mathematics. Given a Riemannian manifold (M, d) and two probability measures μ, ν supported on M, the Monge-Kantorovich optimal transport problem consists of minimizing

$$\int d^2(x, y) d\pi(x, y)$$

among all probability measures π on $M \times M$ such that its marginals $p_{1\#}\pi = \mu, p_{2\#}\pi = \nu$.

Through optimal transport it is possible to define a distance, the so called Wasserstein distance, that metrizes the weak convergence of probability measures. Using this metric structure one can study its geodesics and the convexity of certain functionals. Perhaps one of the most famous applications of this theory comes from the definition of a lower Ricci curvature bound on non-smooth spaces given by Lott-Sturm-Villani.

In this talk we will discuss the optimal transport problem, geometric properties of the Wasserstein space and its relation to lower curvature bounds.

Valuations on star bodies and function lattices

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Abstract

After a short introduction to valuation theory in Convex Geometry, we will focus on valuations on star bodies, and present a recent integral characterization of these. The proof, which we will sketch, is based on tools from Measure Theory and Functional Analysis. After this, we will present recent results on valuations in a large class of function lattices.